d) $\cos 4\theta$

$\cos 4\theta = \cos(2\theta + 2\theta)$

$= \cos^2 2\theta - \sin^2 2\theta$

$= (\cos^2\theta - \sin^2\theta)^2 - (2\sin\theta\cos\theta)^2$

$= \cos^4\theta - 2\cos^2\theta\sin^2\theta + \sin^4\theta - 4\sin^2\theta\cos^2\theta$

$= \cos^4\theta - 6\sin^2\theta\cos^2\theta + (\sin^2\theta)^2$

$= \cos^4\theta - 6(1-\cos^2\theta)\cos^2\theta + (1-\cos^2\theta)^2$

$= \cos^4\theta - 6\cos^2\theta + 6\cos^4\theta + 1 - 2\cos^2\theta + \cos^4\theta$

$= 8\cos^4\theta - 8\cos^2\theta + 1$ ✓

36.

a) $\sin 67\frac{1}{2}^\circ = \sin 67.5^\circ$

Mas

$\sin 135^\circ = \sin(2\times 67.5^\circ)$

$\sin 45^\circ = 2\sin 67.5^\circ\cos 67.5^\circ$

$\frac{\sqrt{2}}{2} = 2\sin 67.5^\circ\cos 67.5^\circ$

Denotemos $\alpha = 67.5^\circ$,

$\frac{\sqrt{2}}{4} = \sin\alpha\cos\alpha$ (*)

Mas $\sin^2\alpha + \cos^2\alpha = 1$

$\cos\alpha = \sqrt{1-\sin^2\alpha}$

$(\alpha = 67.5^\circ \Rightarrow \cos\alpha > 0)$

$\therefore \frac{\sqrt{2}}{4} = \sin\alpha\sqrt{1-\sin^2\alpha}$

$\frac{2}{16} = \sin^2\alpha(1-\sin^2\alpha)$

$\frac{1}{8} = \sin^2\alpha - \sin^4\alpha$

$\sin^4\alpha - \sin^2\alpha + \frac{1}{8} = 0$

Seja $\sin^2\alpha = x$

$\therefore x^2 - x + \frac{1}{8} = 0$

$x = 1 \pm \sqrt{1-\frac{4}{8}}\,/2$

$x = \frac{1 \pm \frac{1}{\sqrt{2}}}{2}$

$\therefore \sin^2\alpha = \frac{1 \pm \frac{1}{\sqrt{2}}}{2}$

$\therefore \sin\alpha = \sqrt{\frac{1 \pm \frac{1}{\sqrt{2}}}{2}}$

Algo parece estar errado com a nossa solução pois