

Novamente, seguindo a análise que foi feita no item (a) vemos que o ponto pode representar

em $\frac{\pi}{8}$ quanto

$$\sin\left(\frac{\pi}{2} - \frac{\pi}{8}\right) = \sin\left(\frac{3\pi}{8}\right)$$

Logo, $\lim_{x \rightarrow \frac{\pi}{8}} \sin x < \lim_{x \rightarrow \frac{3\pi}{8}} \sin x$

Logo devemos pegar para $\lim_{x \rightarrow \frac{\pi}{8}} \sin x$ a menor dos valores.

$$\sin \frac{\pi}{8} = \sqrt{\frac{1 - \frac{1}{\sqrt{2}}}{2}}$$

Logo

$$\lim_{x \rightarrow \frac{\pi}{8}} -\frac{\pi}{8} = -\lim_{x \rightarrow \frac{\pi}{8}} \frac{\pi}{8}$$

$$\lim_{x \rightarrow \frac{\pi}{8}} -\frac{\pi}{8} = -\sqrt{\frac{1 - \frac{1}{\sqrt{2}}}{2}}$$

e) $\cos \frac{\pi}{16}$

$$\cos \frac{\pi}{4} = \cos 4 \frac{\pi}{16}$$

Mos do exercício 27(a)

Já vimos que

$$\begin{aligned} \cos 4x &= 2 \cos^2 2x - 1 \\ &= 2 (\cos^2 x - \sin^2 x)^2 - 1 \\ &= 2 (\cos^2 x - (1 - \cos^2 x))^2 - 1 \\ &= 2 (\cos^2 x - 1 + \cos^2 x)^2 - 1 \\ &= 2 (2 \cos^2 x - 1)^2 - 1 \\ &= 2 (4 \cos^4 x - 4 \cos^2 x + 1) - 1 \\ &= 8 \cos^4 x - 8 \cos^2 x + 2 - 1 \end{aligned}$$

$$\cos 4x = 8 \cos^4 x - 8 \cos^2 x + 1$$

$$\cos \frac{\pi}{4} = \cos 4 \frac{\pi}{16}$$

$$\frac{\sqrt{2}}{2} = 8 \cos^4 \frac{\pi}{16} - 8 \cos^2 \frac{\pi}{16} + 1$$

$$8 \cos^4 \frac{\pi}{16} - 8 \cos^2 \frac{\pi}{16} + (1 - \frac{\sqrt{2}}{2}) = 0$$

$$\cos^4 \frac{\pi}{16} - \cos^2 \frac{\pi}{16} + \left(\frac{1}{8} - \frac{\sqrt{2}}{16}\right) = 0$$