Análise do sinal

Se $0 \leq \theta < \frac{\pi}{2} \rightarrow 0 \leq 2\theta < \pi$

$tg\,\theta \geqslant 0$ $\quad$ $\sin 2\theta \geqslant 0$

Se $\frac{\pi}{2} < \theta \leq \pi \rightarrow \pi < 2\theta \leq 2\pi$

$tg\,\theta \leq 0$ $\quad$ $\sin 2\theta \leq 0$

Se $\pi \leq \theta < \frac{3\pi}{2} \rightarrow 2\pi \leq \theta < 3\pi$

$tg\,\theta \geqslant 0$ $\quad$ $\sin 2\theta \geqslant 0$

Se $\frac{3\pi}{2} < \theta \leq 2\pi \rightarrow 3\pi < 2\theta \leq 4\pi$

$tg\,\theta \leq 0$ $\quad$ $\sin 2\theta \leq 0$

Logo da análise dos sinais vimos que $tg\,\theta$ e $\sin 2\theta$ tem o mesmo sinal, logo devemos tomar a relação (*) com o sinal positivo, i.e.

$$\boxed{\sin 2\theta = \frac{2\,tg\,\theta}{(1+tg^2\theta)}}$$

39) cont.

$\cos 2\theta$

$$tg\,2\theta = \frac{\sin 2\theta}{\cos 2\theta} = \frac{\pm\sqrt{1-\cos^2 2\theta}}{\cos 2\theta}$$

$$\frac{2\,tg\,\theta}{1-tg^2\theta} = \frac{\pm\sqrt{1-\cos^2 2\theta}}{\cos 2\theta}$$

$$\frac{4\,tg^2\theta}{(1-tg^2\theta)^2} = \frac{1-\cos^2 2\theta}{\cos^2 2\theta}$$

$$\frac{4\,tg^2\theta}{(1-tg^2\theta)^2}\cos^2 2\theta = 1-\cos^2 2\theta$$

$$\left(\frac{4\,tg^2\theta}{(1-tg^2\theta)^2}+1\right)\cos^2 2\theta = 1$$

$$\left(\frac{4\,tg^2\theta + (1-tg^2\theta)^2}{(1-tg^2\theta)^2}\right)\cos^2 2\theta = 1$$

$$\left(\frac{4\,tg^2\theta + 1 - 2\,tg^2\theta + tg^4\theta}{(1-tg^2\theta)^2}\right)\cos^2 2\theta = 1$$

$$\frac{(1+tg^2\theta)^2}{(1-tg^2\theta)^2}\cos^2 2\theta = 1$$

$$\therefore \cos^2 2\theta = \frac{(1-tg^2\theta)^2}{(1+tg^2\theta)^2}$$