39. Cont. (18)

$$\cos 2\theta = \pm\left(\frac{1-tg^2\theta}{1+tg^2\theta}\right) \quad (**)$$

$1+tg^2\theta \to$ sempre positivo

Análise do sinal

$0 \leq \theta \leq \frac{\pi}{4} \to 0 \leq 2\theta \leq \frac{\pi}{2}$

$1-tg^2\theta \geq 0$ $\qquad$ $\cos 2\theta \geq 0$

$\frac{\pi}{4} \leq \theta < \frac{\pi}{2} \to \frac{\pi}{2} \leq 2\theta < \pi$

$1-tg^2\theta \leq 0$ $\qquad$ $\cos 2\theta \leq 0$

$\frac{\pi}{2} < \theta \leq \frac{3\pi}{4} \to \pi < 2\theta \leq \frac{3\pi}{2}$

$1-tg^2\theta \leq 0$ $\qquad$ $\cos 2\theta \leq 0$

$\frac{3\pi}{4} \leq \theta \leq \pi \to \frac{3\pi}{2} \leq 2\theta \leq 2\pi$

$1-tg^2\theta \geq 0$ $\qquad$ $\cos 2\theta \geq 0$

$\pi \leq \theta \leq \frac{5\pi}{4} \to 2\pi \leq 2\theta \leq \frac{5\pi}{2}$

$1-tg^2\theta \geq 0$ $\qquad$ $\cos 2\theta \geq 0$

$\frac{5\pi}{4} \leq \theta < \frac{3\pi}{2} \to \frac{5\pi}{2} \leq 2\theta < 3\pi$

$1-tg^2\theta \leq 0 \to \cos 2\theta \leq 0$

$\frac{3\pi}{2} < \theta \leq \frac{7\pi}{4} \to 3\pi < 2\theta \leq \frac{7\pi}{2}$

$1-tg^2\theta \leq 0$ $\qquad$ $\cos 2\theta \leq 0$

$\frac{7\pi}{4} \leq \theta \leq 2\pi \to \frac{7\pi}{2} \leq 2\theta \leq 4\pi$

$1-tg^2\theta \geq 0$ $\qquad$ $\cos 2\theta \geq 0$

A análise dos sinais nos mostra que devemos tomar o sinal positivo em (**) i.e.,

$$\boxed{\cos 2\theta = \frac{1-tg^2\theta}{1+tg^2\theta}}$$