40.

$$\frac{1-\cos 2x}{\sin 2x}$$

Do exercício 39 temos:

$$\begin{cases} \sin 2x = \dfrac{2\,tg\,x}{1+tg^2 x} \\[2ex] \cos 2x = \dfrac{1-tg^2 x}{1+tg^2 x} \end{cases}$$

Daí

$$\frac{1-\cos 2x}{\sin 2x} = \frac{1-\dfrac{1-tg^2}{1+tg^2 x}}{\dfrac{2\,tg\,x}{1+tg^2 x}}$$

$$= \frac{\dfrac{2\,tg^2 x}{\cancel{1+tg^2 x}}}{\dfrac{2\,tg\,x}{\cancel{1+tg^2 x}}}$$

$$= \frac{\cancel{2}\,tg^2 x}{\cancel{2\,tg\,x}} = tg\,x$$

$\therefore$

$$\boxed{\frac{1-\cos 2x}{\sin 2x} = tg\,x}$$

41.

$$\frac{1-tg^2 x}{1+tg^2 x}$$

$$\frac{1-tg^2 x}{1+tg^2 x} = \frac{1-\dfrac{\sin^2 x}{\cos^2 x}}{1+\dfrac{\sin^2 x}{\cos^2 x}}$$

$$= \frac{\dfrac{\cos^2 x - \sin^2 x}{\cancel{\cos^2 x}}}{\dfrac{\cos^2 x + \sin^2 x}{\cancel{\cos^2 x}}}$$

$$= \frac{\overbrace{\cos^2 x - \sin^2 x}^{\cos 2x}}{\underbrace{\cos^2 x + \sin^2 x}_{1}}$$

$\therefore$

$$\boxed{\frac{1-tg^2 x}{1+tg^2 x} = \cos 2x}$$

42. $\cos\theta + \sin\theta = \frac{2}{3}$ (*)

$$\sin 2\theta = 2\sin\theta\cos\theta$$

De (*):

$$(\cos\theta + \sin\theta)^2 = \frac{4}{9}$$

$$\cos^2\theta + 2\cos\theta\sin\theta + \sin^2\theta = \frac{4}{9}$$

$$1 + 2\cos\theta\sin\theta = \frac{4}{9}$$

$$2\cos\theta\sin\alpha = \frac{4}{9} - 1 = \frac{-5}{9}$$