

$$41) \sin^4 x + \cos^4 x = \sin^2 x (\csc^2 x - 2 \cos^2 x)$$

$$\rightarrow \left\{ \begin{aligned} \sin^2 x (\csc^2 x - 2 \cos^2 x) &= \sin^2 x \left(\frac{1}{\sin^2 x} - 2 \cos^2 x \right) \\ &= \boxed{1 - 2 \sin^2 x \cos^2 x} \end{aligned} \right.$$

$$\left\{ \begin{aligned} \sin^4 x + \cos^4 x &= (\underbrace{\sin^2 x + \cos^2 x})^2 - 2 \sin^2 x \cos^2 x \\ &= \boxed{1 - 2 \sin^2 x \cos^2 x} \end{aligned} \right.$$

$$\therefore \parallel \sin^4 x + \cos^4 x = \sin^2 x (\csc^2 x - 2 \cos^2 x) \parallel$$

$$42) \parallel \sin^3 x + \cos^3 x = (1 - \sin x \cos x) (\sin x + \cos x) \parallel$$

$$\rightarrow (1 - \sin x \cos x) (\sin x + \cos x) =$$

$$= \sin x + \cos x - \sin^2 x \cos x - \sin x \cos^2 x$$

$$= \sin x (1 - \cos^2 x) + \cos x (1 - \sin^2 x)$$

$$= \sin x \sin^2 x + \cos x \cos^2 x$$

$$= \sin^3 x + \cos^3 x \parallel$$