106. Cont.

e) $\begin{cases} tg\,x + sec\,2x = 1 \\ -\frac{\pi}{2} < x < \frac{\pi}{2} \end{cases}$

$tg\,x + sec\,2x = 1$

$\frac{\sin x}{\cos x} + \frac{1}{\cos 2x} = 1$

$\frac{\sin x}{\cos x} = 1 - \frac{1}{\cos 2x}$

$\frac{\sin x}{\cos x} = \frac{\cos 2x - 1}{\cos 2x}$ (*)

Mas $\begin{cases} \sin^2 x = \frac{1 - \cos 2x}{2} \\ \therefore \cos 2x - 1 = -2\sin^2 x \end{cases}$ (**)

(**) → (*) :

$\frac{\sin x}{\cos x} = \frac{-2\sin^2 x}{\cos 2x}$

$\sin x \cos 2x = -2 \underbrace{\sin^2 x \cos x}$

$= -2 \underbrace{\sin x \sin x \cos x}$

$\sin x \cos 2x = -\sin x \underbrace{\sin 2x}$

$\sin x (\cos 2x + \sin 2x) = 0$

$\Longrightarrow \begin{cases} \sin x = 0 \\ \text{ou} \\ \cos 2x + \sin 2x = 0 \end{cases}$

i) $\sin x = 0 \Longrightarrow x = n\pi,\ n \in \mathbb{Z}$

Mas estando $-\frac{\pi}{2} < x < \frac{\pi}{2}$

vemos que só podemos ter $\underline{\underline{x = 0}}$

ii) $\cos 2x + \sin 2x = 0$,

$-\frac{\pi}{2} < x < \frac{\pi}{2}$

$\therefore \underline{\underline{-\pi < 2x < \pi}}$

Se $2x \neq -\frac{\pi}{2}, \frac{\pi}{2}$

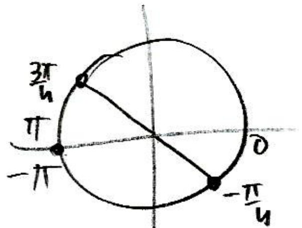
temos

$\cos 2x + \sin 2x = 0$

$\Leftrightarrow$

$\cos 2x = -\sin 2x$

$\therefore 1 = -tg\,2x,\ -\pi < 2x < \pi$



$\therefore$

$2x = -\frac{\pi}{4}, \frac{3\pi}{4}$

$\therefore$

$\boxed{x = -\frac{\pi}{8}, \frac{3\pi}{8}}$

Solução:

$\boxed{x = 0,\ x = -\frac{\pi}{8},\ x = \frac{3\pi}{8}}$