

12.

$$\left. \begin{array}{l} f: \mathbb{R} - \{0\} \rightarrow \mathbb{R} \\ f\left(\frac{a}{b}\right) = f(a) - f(b) \end{array} \right\}$$

Temos $f(-a) = f\left(\frac{a}{-1}\right) = f(a) - f(1)$

$$\therefore f(-a) = f(a) - f(1) \quad (*)$$

Se $a=1$, $(*)$ nos dá que

$$f(-1) = f(1) - f(1) = 0$$

$$\therefore f(-1) = 0 \quad (**)$$

Se $a=-1$, $(*)$ nos dá que

$$f(1) = f(-1) - f(1)$$

$$\therefore 2f(1) = f(-1)$$

$$f(1) = \frac{1}{2} f(-1) \stackrel{(**)}{=} 0$$

$$\therefore f(1) = 0 \quad (***)$$

Substituindo $(***)$ em $(*)$ obtemos que \rightarrow