Cálculo A

Limites laterais

Calcule os limites laterais das funções dadas

1.

$$f(x) = \begin{cases} 2 & \text{se } x < 1 \\ -1 & \text{se } x = 1 \\ -3 & \text{se } x > 1 \end{cases}$$

$\lim_{x\to 1^+} f(x), \quad \lim_{x\to 1^-} f(x), \quad \lim_{x\to 1} f(x)$

2.

$$f(x) = \begin{cases} -2 & \text{se } x < 0 \\ 2 & \text{se } x \geq 0 \end{cases}$$

$\lim_{x\to 0^+} f(x), \quad \lim_{x\to 0^-} f(x), \quad \lim_{x\to 0} f(x)$

3.

$$f(t) = \begin{cases} t+4 & \text{se } t \leq -4 \\ 4-t & \text{se } t > -4 \end{cases}$$

$\lim_{t\to -4^+} f(t), \quad \lim_{t\to -4^-} f(t), \quad \lim_{t\to -4} f(t)$

4.

$$g(s) = \begin{cases} s+3 & \text{se } s \leq -2 \\ 3-s & \text{se } s > -2 \end{cases}$$

$\lim_{s\to -2^+} g(s), \quad \lim_{s\to -2^-} g(s), \quad \lim_{s\to -2} g(s)$

5.

$$F(x) = \begin{cases} x^2 & \text{se } x \leq 2 \\ 8-2x & \text{se } x > 2 \end{cases}$$

$\lim_{x\to 2^+} F(x), \quad \lim_{x\to 2^-} F(x), \quad \lim_{x\to 2} F(x)$

6.

$$h(x) = \begin{cases} 2x+1 & \text{se } x < 3 \\ 10-x & \text{se } x \geq 3 \end{cases}$$

$\lim_{x\to 3^+} h(x), \quad \lim_{x\to 3^-} h(x), \quad \lim_{x\to 3} h(x)$

7.

$$g(r) = \begin{cases} 2r+3 & \text{se } r < 1 \\ 2 & \text{se } r = 1 \\ 7-2r & \text{se } r > 1 \end{cases}$$

$\lim_{r\to 1^+} g(r), \quad \lim_{r\to 1^-} g(r), \quad \lim_{r\to 1} g(r)$

8.

$$g(t) = \begin{cases} 3+t^2 & \text{se } t < -2 \\ 0 & \text{se } t = -2 \\ 11-t^2 & \text{se } t > -2 \end{cases}$$

$\lim_{t\to -2^+} g(t), \quad \lim_{t\to -2^-} g(t), \quad \lim_{t\to -2} g(t)$

9.

$$f(x) = \begin{cases} x^2-4 & \text{se } x < 2 \\ 4 & \text{se } x = 2 \\ 4-x^2 & \text{se } x > 2 \end{cases}$$

$\lim_{x\to 2^+} f(x), \quad \lim_{x\to 2^-} f(x), \quad \lim_{x\to 2} f(x)$

10.

$$f(x) = \begin{cases} 2x+3 & \text{se } x < 1 \\ 4 & \text{se } x = 1 \\ x^2+2 & \text{se } x > 1 \end{cases}$$

$\lim_{x\to 1^+} f(x), \quad \lim_{x\to 1^-} f(x), \quad \lim_{x\to 1} f(x)$

11. $F(x) = |x-5|$
$\lim_{x\to 5^+} F(x), \quad \lim_{x\to 5^-} F(x), \quad \lim_{x\to 5} F(x)$

12. $f(x) = 3 + |2x-4|$
$\lim_{x\to 2^+} f(x), \quad \lim_{x\to 2^-} f(x), \quad \lim_{x\to 2} f(x)$

13. $G(x) = |2x-3| - 4$
$\lim_{x\to \frac{3}{2}^+} G(x), \quad \lim_{x\to \frac{3}{2}^-} G(x), \quad \lim_{x\to \frac{3}{2}} G(x)$

14.

$$F(x) = \begin{cases} |x-1| & \text{se } x < -1 \\ 0 & \text{se } x = -1 \\ |1-x| & \text{se } x > -1 \end{cases}$$

$\lim_{x\to -1^+} F(x), \quad \lim_{x\to -1^-} F(x), \quad \lim_{x\to -1} F(x)$

15. $f(x) = \frac{|x|}{x}$
$\lim_{x\to 0^+} f(x), \quad \lim_{x\to 0^-} f(x), \quad \lim_{x\to 0} f(x)$

16. $f(x) = 2^{\frac{1}{x}}$
$\lim_{x\to 0^+} f(x), \lim_{x\to 0^-} f(x), \lim_{x\to 0} f(x),$
$\lim_{x\to -\infty} f(x), \lim_{x\to \infty} f(x)$

17. $f(x) = \frac{|x-3|}{x-3}$
$\lim_{x\to 3^+} f(x), \quad \lim_{x\to 3^-} f(x), \quad \lim_{x\to 3} f(x)$

Resposta

1. $-3, 2, \nexists$
2. $2, -2, \nexists$

3. $8, 0, \nexists$
4. $5, 1, \nexists$
5. $4, 4, 4$
6. $7, 7, 7$
7. $5, 5, 5$
8. $7, 7, 7$
9. $0, 0, 0$
10. $3, 5, \nexists$
11. $0, 0, 0$
12. $3, 3, 3$
13. $-4, -4, -4$
14. $2, 2, 2$
15. $1, -1, \nexists$
16. $+\infty, 0, \nexists, 1, 1$
17. $1, -1, \nexists$

1. $f(x) = \begin{cases} 2, & x < 1 \\ -1, & x = 1 \\ 4 - 3, & 1 < x \end{cases}$

a) $\lim_{x \to 1^+} f(x) = -3$

b) $\lim_{x \to 1^-} f(x) = 2$

c) $\lim_{x \to 1} f(x)$ não existe
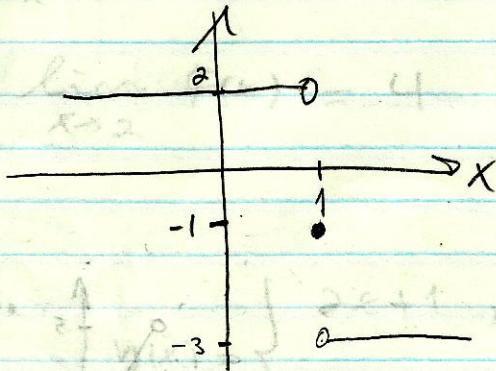
2. $f(x) = \begin{cases} -2, & x < 0 \\ 2, & 0 \leq x \end{cases}$
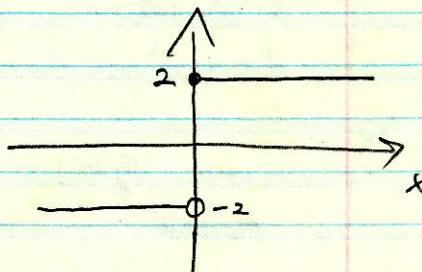
$\begin{cases} \lim_{x \to 0^+} f(x) = 2 \\ \lim_{x \to 0^-} f(x) = -2 \end{cases}$, $\lim_{x \to 0} f(x)$ não existe

3.

$$f(x) = \begin{cases} x+4, & x \le -4 \\ 4-x, & -4 < x \end{cases}$$

$$\lim_{x \to -4^+} f(x) = 8$$

$$\lim_{x \to -4^-} f(x) = 0$$

$$\lim_{x \to -4} f(x) \text{ não existe}$$
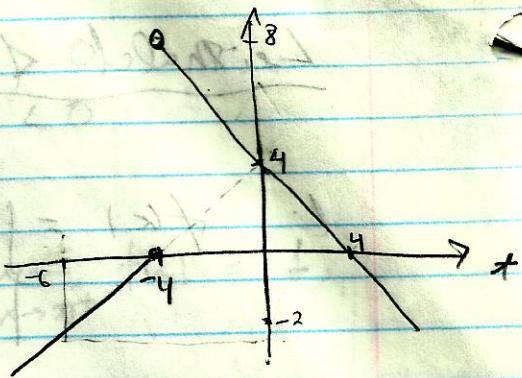
4.

$$g(s) = \begin{cases} s+3, & s \le -2 \\ 3-s, & -2 < s \end{cases}$$

$$\lim_{s \to -2^+} g(s) = 5 \quad , \quad \lim_{s \to -2^-} g(s) = 1$$

$$\lim_{s \to -2} g(s) = \text{não existe}$$
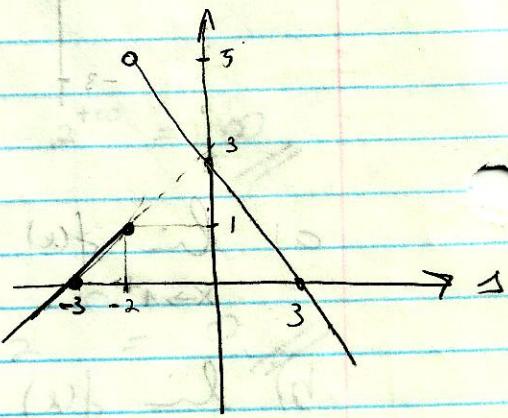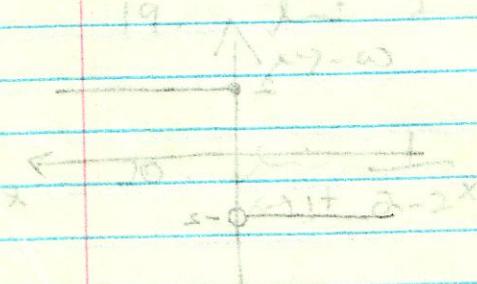
5.

$$F(x) = \begin{cases} x^2, & x \leq 2 \\ 8-2x, & 2 < x \end{cases}$$

$$\lim_{x \to 2^+} F(x) = 4, \quad \lim_{x \to 2^-} F(x) = 4$$

$$\lim_{x \to 2} F(x) = 4$$
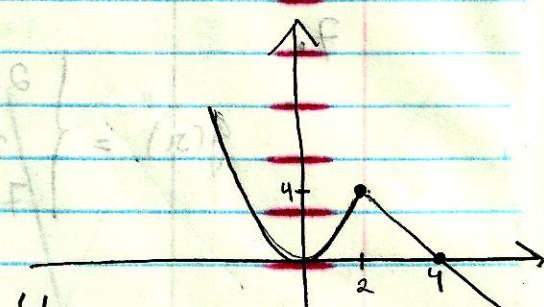
6.

$$h(x) = \begin{cases} 2x+1, & x < 3 \\ 10-x, & 3 \leq x \end{cases}$$

$$\lim_{x \to 3^+} h(x) = 7$$

$$\lim_{x \to 3^-} h(x) = 7$$

$$\lim_{x \to 3} h(x) = 7$$
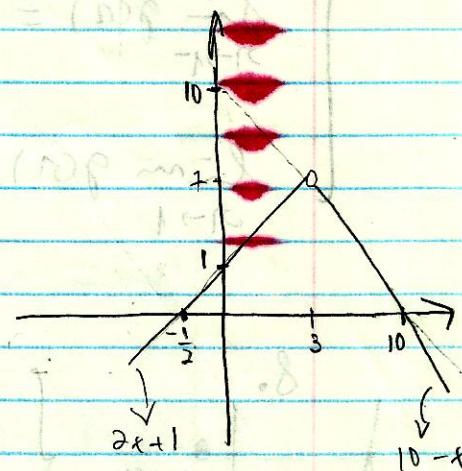
7.

$$g(x) = \begin{cases} 2x+3 & , \quad x < 1 \\ 2 & , \quad x = 1 \\ 7-2x & , \quad 1 < x \end{cases}$$

$$\lim_{x \to 1^+} g(x) = 5$$

$$\lim_{x \to 1^-} g(x) = 5$$
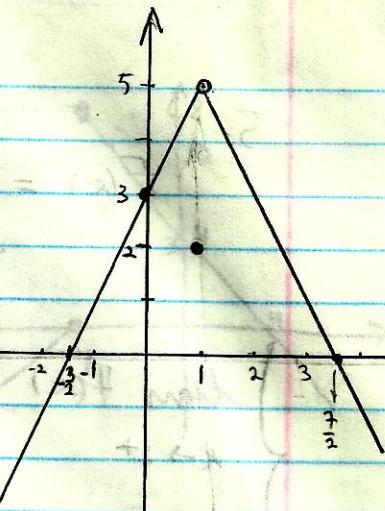
$$\lim_{x \to 1} g(x) = 5$$

8.

$$g(t) = \begin{cases} 3+t^2 & , \quad t < -2 \\ 0 & , \quad t = -2 \\ 11-t^2 & , \quad -2 < t \end{cases}$$
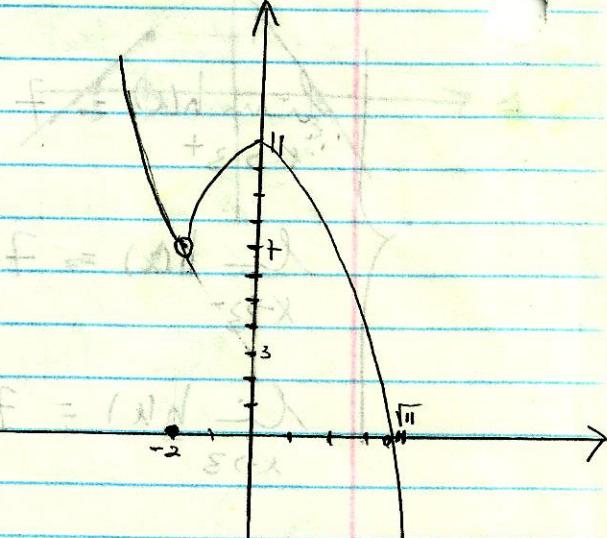

$$\lim_{t \to -2^+} g(t) = 7$$

$$\lim_{t \to -2^-} g(t) = 7$$

$$\lim_{t \to -2} g(t) = 7$$

9.

$$f(x) = \begin{cases} x^2-4, & x < 2 \\ 4, & x = 2 \\ 4-x^2, & 2 < x \end{cases}$$
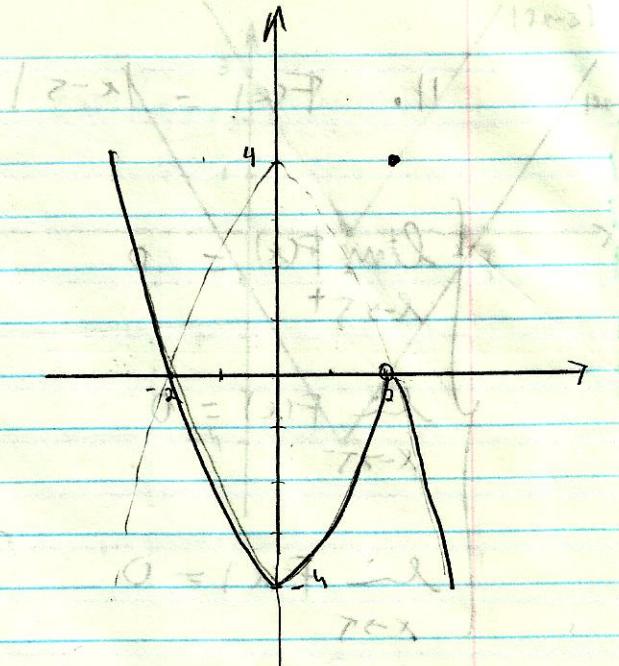
$$\begin{cases} \lim_{x \to 2^+} f(x) = 0 \\ \\ \lim_{x \to 2^-} f(x) = 0 \\ \\ \lim_{x \to 2} f(x) = 0 \end{cases}$$

10.

$$f(x) = \begin{cases} 2x+3, & x < 1 \\ 4, & x = 1 \\ x^2+2, & 1 < x \end{cases}$$

$$\begin{cases} \lim_{x \to 1^+} f(x) = 3 \\ \\ \lim_{x \to 1^-} f(x) = 5 \\ \\ \lim_{x \to 1} f(x) = \text{não existe} \end{cases}$$
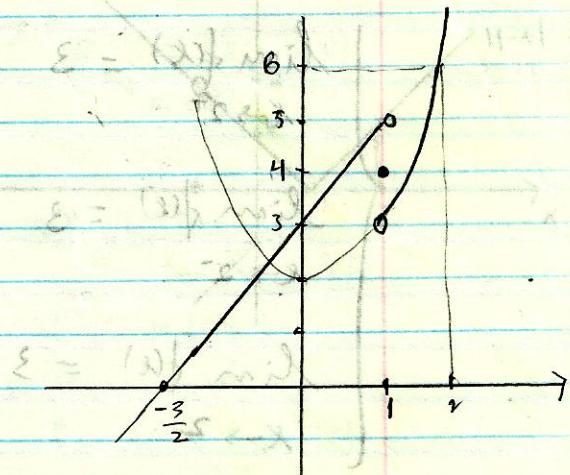
11. $F(x) = |x - 5|$

$\lim_{x \to 5^+} F(x) = 0$

$\lim_{x \to 5^-} F(x) = 0$

$\lim_{x \to 5} F(x) = 0$

12. $f(x) = 3 + |2x - 4|$

$\lim_{x \to 2^+} f(x) = 3$

$\lim_{x \to 2^-} f(x) = 3$

$\lim_{x \to 2} f(x) = 3$
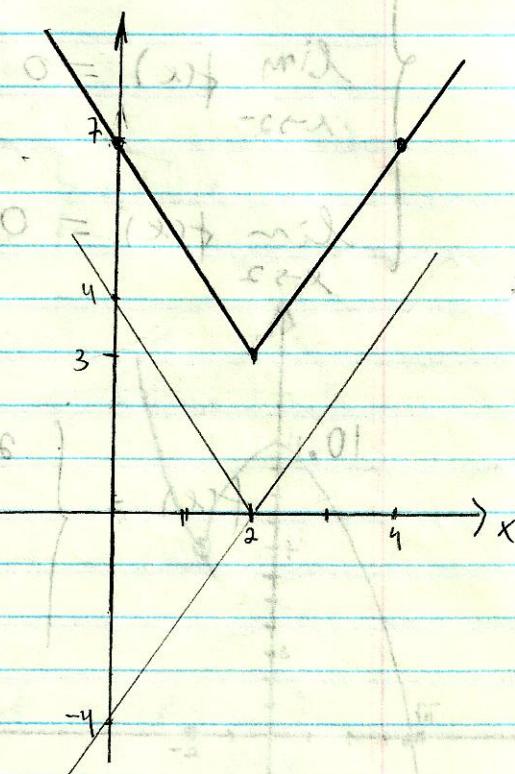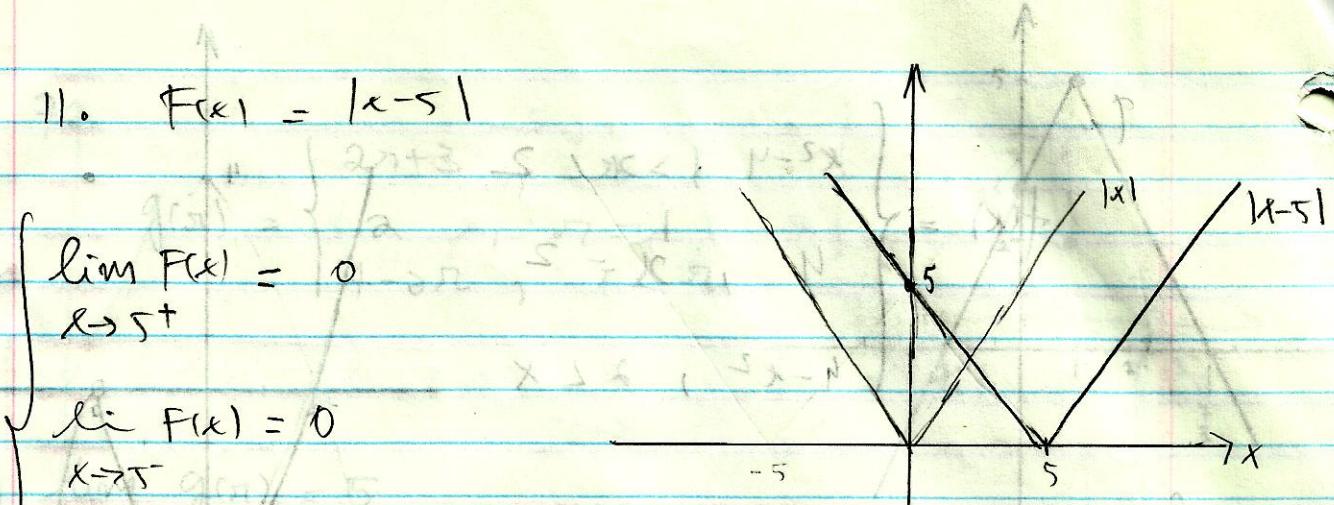
13.

$$G(x) = |2x-3| - 4$$
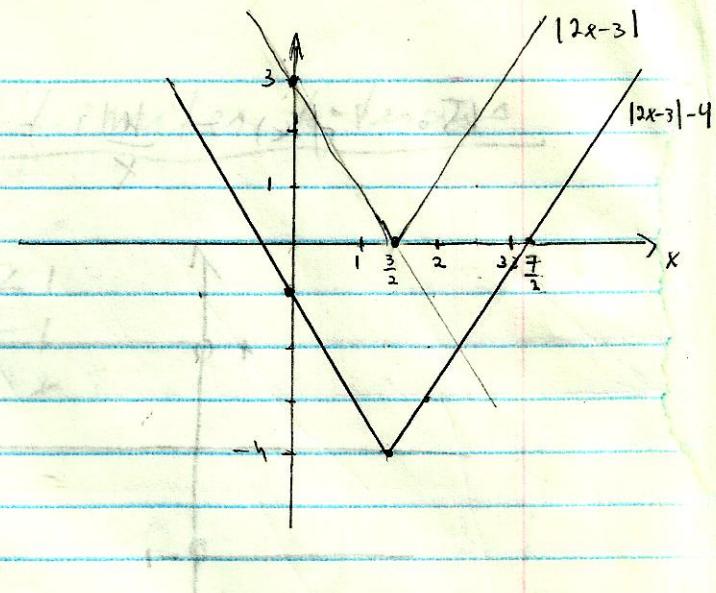

$$\lim_{x \to \frac{3}{2}^+} G(x) = -4$$

$$\lim_{x \to \frac{3}{2}^-} G(x) = -4$$

$$\lim_{x \to \frac{3}{2}} G(x) = -4$$

14.

$$F(x) = \begin{cases} |x-1| & , \ x < -1 \\ 0 & , \ x = -1 \\ |1-x| & , \ -1 < x \end{cases}$$
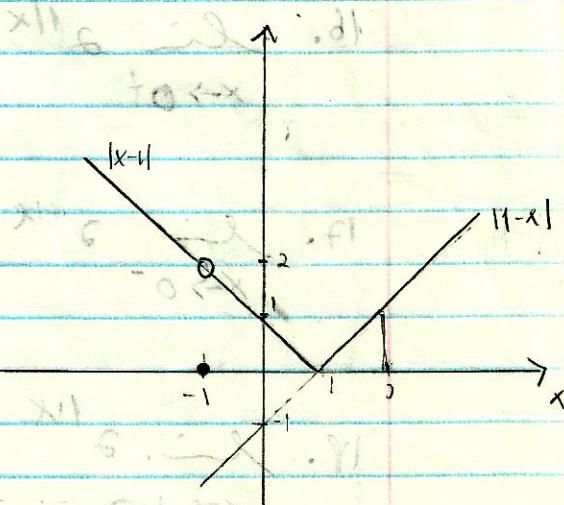

$$\lim_{x \to -1^+} F(x) = 2$$

$$\lim_{x \to -1^-} F(x) = 2$$

$$\lim_{x \to -1} F(x) = 2$$

15. $f(x) = \dfrac{|x|}{x} = \begin{cases} 1, & x > 0 \\ -1, & x < 0 \end{cases}$
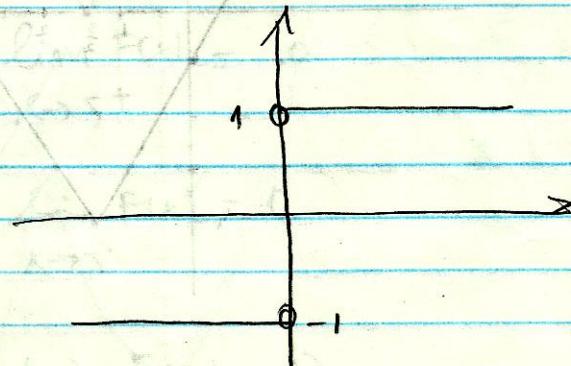
$\displaystyle\lim_{x \to 0^+} f(x) = 1$

$\displaystyle\lim_{x \to 0^-} f(x) = -1$

$\displaystyle\lim_{x \to 0} f(x) = $ não existe

16. $\displaystyle\lim_{x \to 0^+} 2^{1/x} = 2^{\lim_{x \to 0^+} \frac{1}{x}} = 2^{+\infty} = \infty$ //

$\displaystyle\lim_{x \to 0^-} 2^{1/x} = 2^{\lim_{x \to 0^-} \frac{1}{x}} = 2^{-\infty} = 0$ //

$\displaystyle\lim_{x \to +\infty} 2^{1/x} = 2^{\lim_{x \to +\infty} 1/x} = 2^{0^+} = 1$

$\displaystyle\lim_{x \to -\infty} 2^{1/x} = 2^{\lim_{x \to -\infty} 1/x} = 2^{0^-} = 1$ //

17. $\displaystyle\lim_{x \to 3^+} \frac{|x-3|}{x-3} = \lim_{x \to 3^+} \frac{x-3}{x-3} = 1$

$\displaystyle\lim_{x \to 3^-} \frac{|x-3|}{x-3} = \lim_{x \to 3^-} \frac{-(x-3)}{x-3} = -1$

$\displaystyle\lim_{x \to 3} \frac{|x-3|}{x-3} \;\not\equiv$