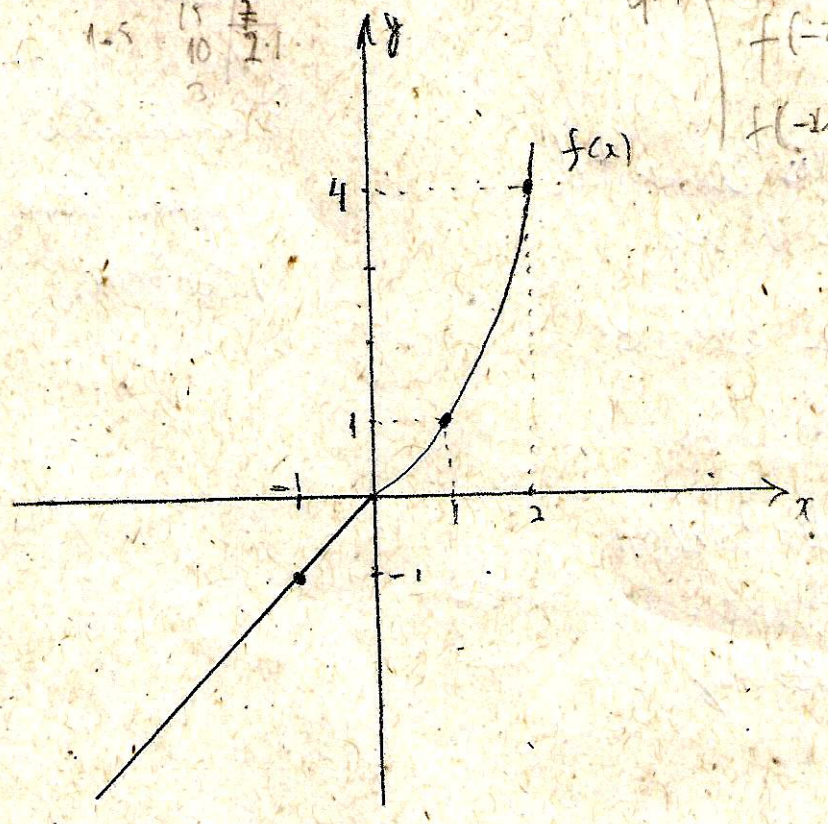
Cálculo A - Prova 1

1. Dê o domínio de $f(x) = \arcsin \sqrt{\frac{3x-1}{2}}$ 1.5

2. Dê o domínio $f(x) = \frac{\sqrt{x+5}}{\log_{10}(9-5x)}$ 1.5

3. Mostre que $\arctan x + \arctan y = \arctan \frac{x+y}{1-xy}$, $xy \neq 1$ 2.0

4. Dado o gráfico de $f(x)$ construa o gráfico de $f(-2x-1)+1$ 2.0



1. $f(x) = \arcsin\sqrt{\frac{3x-1}{2}}$

$$\begin{cases} \sqrt{\frac{3x-1}{2}} \in [-1,1] & (*) \\ \frac{3x-1}{2} \geq 0 & (**) \end{cases}$$

$(**)$: $\frac{3x-1}{2} \geq 0 \Rightarrow 3x-1 \geq 0$

$3x \geq 1$

$x \geq \frac{1}{3}$ 0.5

$(*)$: $\sqrt{\frac{3x-1}{2}} \in [-1,1]$

$\therefore -1 \leq \sqrt{\frac{3x-1}{2}} \leq 1$

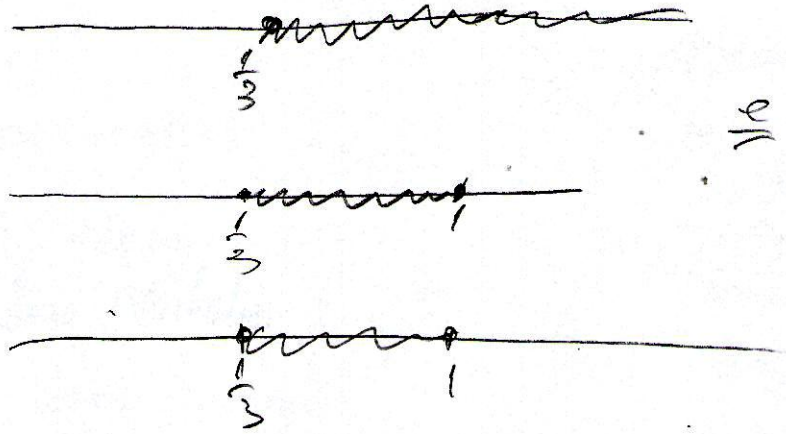
$\therefore 0 \leq \sqrt{\frac{3x-1}{2}} \leq 1$

$\therefore 0 \leq \frac{3x-1}{2} \leq 1$

$\therefore 0 \leq 3x-1 \leq 2$

$\therefore 1 \leq 3x \leq 3$

$\frac{1}{3} \leq x \leq 1$ 0.5



$\simeq$

$\| \operatorname{Dom} f = [\frac{1}{3}, 1] \|$ Q.E.D

2.

$$f(x) = \frac{\sqrt{x+5}}{\log_{10}(9-5x)}.$$

$$\begin{cases} x+5 \geq 0 & (*) \\ 9-5x > 0 & (**) \\ 9-5x \neq 1 & (***) \quad [\text{Anula o logaritmo}] \end{cases}$$

$(*)$: $x+5 \geq 0 \Rightarrow x \geq -5$ 0.5

$(**)$: $9-5x > 0 \Rightarrow 9 > 5x$

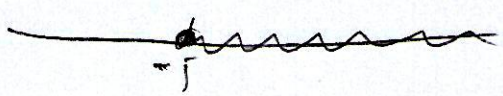
$\frac{9}{5} > x$ 0.5

$(***)$: $9-5x \neq 1$

$\therefore 5x \neq 8$

$\therefore x \neq \frac{8}{5}$ 0.25

$\text{Dom } f = [-5, \frac{8}{5}) \cup (\frac{8}{5}, \frac{9}{5})$



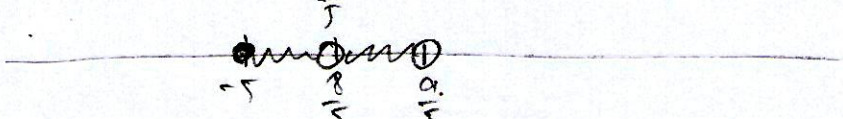
$x \geq -5$

$\wedge$

$x < \frac{9}{5}$

$\wedge$

$x \neq \frac{8}{5}$



$x \in [-5, \frac{8}{5}) \cup (\frac{8}{5}, \frac{9}{5})$ 0.25

3. $\operatorname{arctg} x + \operatorname{arctg} y = \operatorname{arctg} \frac{x+y}{1-xy}$; $xy \neq 1$

seja

$$\left.\begin{array}{l} w = \operatorname{arctg} x \\ \therefore \operatorname{tg} w = x \\ \quad w \in (-\frac{\pi}{2}, \frac{\pi}{2}) \\ z = \operatorname{arctg} y \\ \therefore \operatorname{tg} z = y \\ \quad z \in (-\frac{\pi}{2}, \frac{\pi}{2}) \end{array}\right\} 0.5$$

seja então

$$\operatorname{tg}(w+z) = \frac{\operatorname{tg} w + \operatorname{tg} z}{1 - \operatorname{tg} w \operatorname{tg} z}$$

$$\operatorname{tg}(w+z) = \frac{x+y}{1-xy} \quad \underline{1.0}$$

$$\therefore w + z = \operatorname{arctg} \frac{x+y}{1-xy}$$

$$\operatorname{arctg} x + \operatorname{arctg} y = \operatorname{arctg} \frac{x+y}{1-xy} \quad \underline{0.5}$$

4.

$$f(x) \longrightarrow f(-2x-1)+1$$

$$f(x) \to f_1(x) := f(-x) \longrightarrow f_2(x) := f_1(2x) = f(-2x)$$

$$\longrightarrow f_3(x) := f_2\left(x+\tfrac{1}{2}\right) = f\left(-2\left(x+\tfrac{1}{2}\right)\right) = f(-2x-1) \to$$

$$\longrightarrow f_4(x) := f_3(x)+1 = f(-2x-1)+1$$