Cálculo A - Prova 1

1. Dê o domínio de $f(x) = \arcsin(\frac{1}{2}x - 1) + \arccos(1 - \frac{1}{2}x)$ 1.0

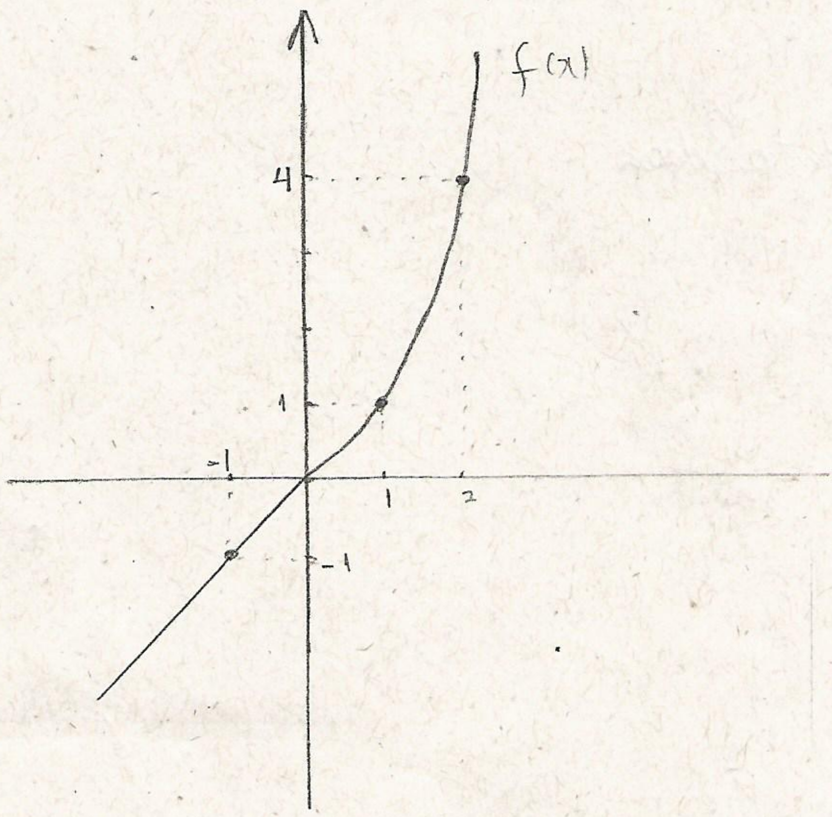
2. Dê o domínio de $f(x) = \log_{x+1}(x^2 - 3x + 2)$ 1.5

3. Mostre que

$$\arcsin x + \arcsin y = \arcsin\left(x\sqrt{1-y^2} + y\sqrt{1-x^2}\right)$$ 1.5

   sabendo que o lado esquerdo da expressão está no intervalo $[-\pi/2, \pi/2]$

4. Dado o gráfico de $f(x)$ construa o gráfico de $f(-2x+1) - 1$ 2.0



1. $f(x) = \arcsin\left(\frac{1}{2}x - 1\right) + \arccos\left(1 - \frac{1}{2}x\right)$

$\frac{1}{2}x - 1 \in [-1, 1]$ (*)

$1 - \frac{1}{2}x \in [-1, 1]$ (**)

(*) : $-1 \leq \frac{1}{2}x - 1 \leq 1$

$0 \leq \frac{1}{2}x \leq 2$

$0 \leq x \leq 4$ $\underline{0.5}$

(**) : $-1 \leq 1 - \frac{1}{2}x \leq 1$

$-2 \leq -\frac{1}{2}x \leq 0$ $\times(-2)$

$4 \geq x \geq 0$ $\underline{0.5}$

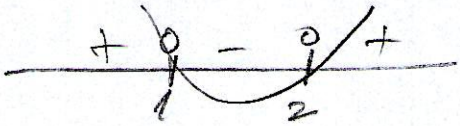
$\therefore$ $D_f = [0, 4]$

2.

$$f(x) = \log_{x+1}(x^2-3x+2)$$

$$\begin{cases} x^2-3x+2 > 0 & (*) \\ x+1 > 0 & (**) \\ x+1 \neq 1 & (***) \end{cases}$$

(*) x^2-3x+2



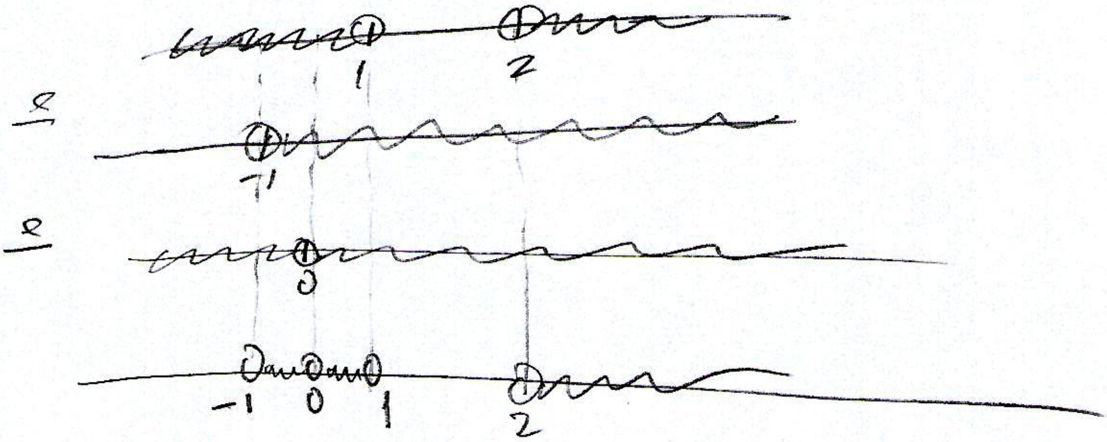
$x^2-3x+2>0 \Longrightarrow x<1$ ou $x>2$

0.5

(**) $x > -1$ 0.5

(***) $x \neq 0$ 0.25

Dom



$\text{Dom } f = (-1,0) \cup (0,1) \cup (2,+\infty)$

0.25

3. ...

$$\arcsin x + \arcsin y = \arcsin\left(x\sqrt{1-y^2} + y\sqrt{1-x^2}\right)$$

Seja

$$\left.\begin{array}{l} w = \arcsin x \\ \therefore \quad \sin w = x \\ \qquad w \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \\ \\ z = \arcsin y \\ \therefore \quad \sin z = y \\ \qquad z \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \end{array}\right\} \underline{0.5}$$

Temos

$$\sin(w+z) = \sin w \cos z + \sin z \cos w$$

$$= x \cos z + y \cos w$$

$$= x\sqrt{1-\sin^2 z} + y\sqrt{1-\sin^2 w}$$

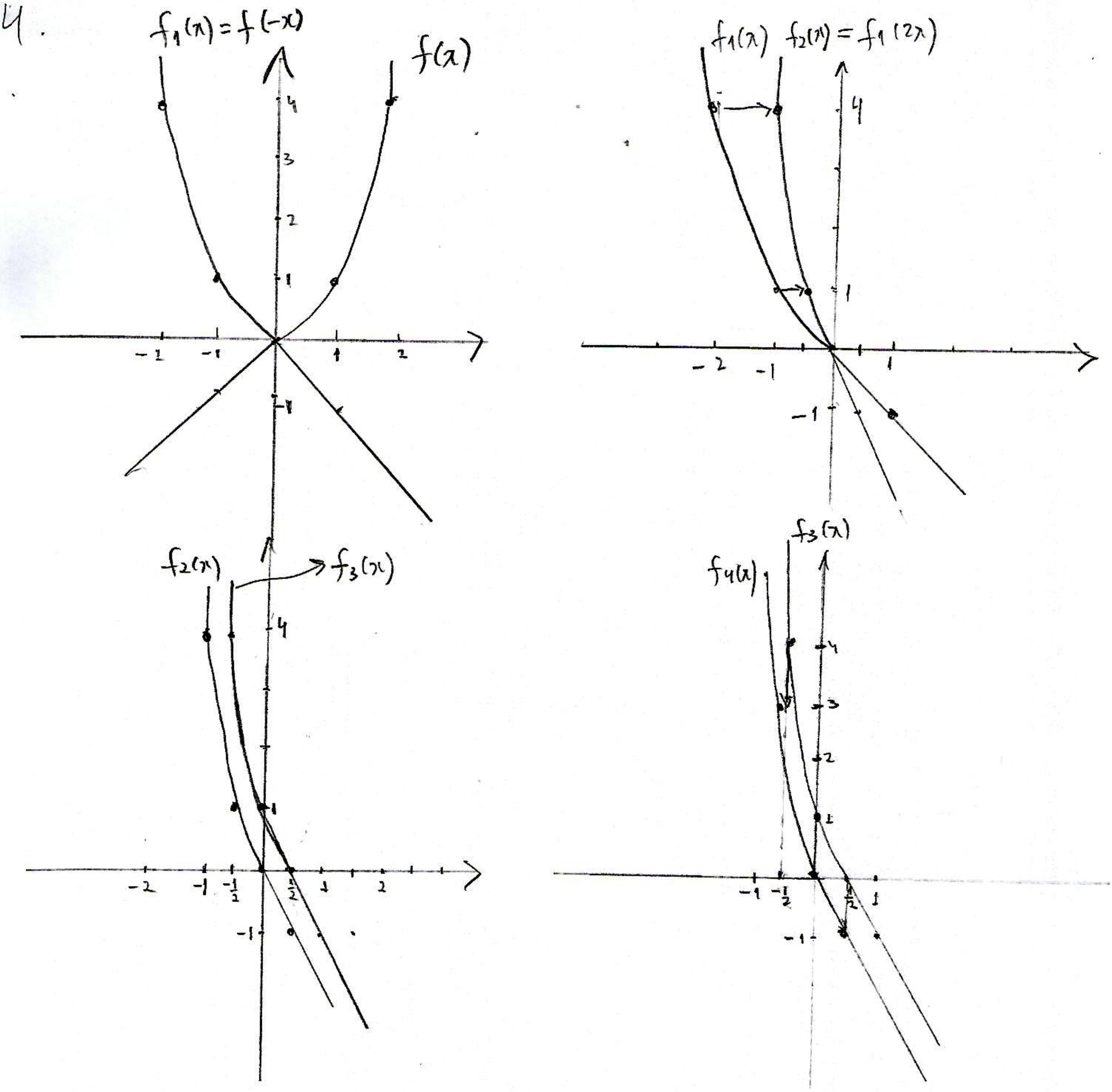
$$\sin(w+z) = x\sqrt{1-y^2} + y\sqrt{1-x^2} \quad 0.5$$

$$\therefore \quad \underline{w+z} = \arcsin\left(x\sqrt{1-y^2} + y\sqrt{1-x^2}\right)$$

$$//\arcsin x + \arcsin y = \arcsin\left(x\sqrt{1-y^2} + y\sqrt{1-x^2}\right)//$$

0.5

4.



$$f(x) \to f_1(x) = f(-x) \longrightarrow f_2(x) = f_1(2x) = f(-2x) \longrightarrow$$

$$\longrightarrow f_3(x) = f_2\left(x - \tfrac{1}{2}\right) = f\left(-2\left(x - \tfrac{1}{2}\right)\right) = f(-2x+1) \longrightarrow$$

$$\longrightarrow f_4(x) = f_3(x) - 1 = f(-2x+1) - 1$$