

Cálculo A - Prova 1

1. Se $f(2) = 1$ e $f(xy) = f(x) + f(y)$, determine $f(16)$. 1.0
2. Seja $f : \mathbb{R} \rightarrow \mathbb{R}$ e $g : \mathbb{R} \rightarrow \mathbb{R}$ funções ímpares. Se a composta $f \circ g$ estiver definida mostre que $f \circ g$ é ímpar. 1.0
3. Sejam

$$f(x) = \begin{cases} x^2 - 4x + 3 & \text{se } x \geq 2 \\ 2x - 3 & \text{se } x < 2 \end{cases}, \quad g(x) = 2x + 3$$

1.1

Determine $f \circ g$.

4. Sejam a, b, c números reais positivos satisfazendo $a^2 + b^2 = c^2$.
Mostre que

$$\log_{b+c} a + \log_{c-b} a = 2 \log_{c+b} a \log_{c-b} a$$

2.0

Propriedades:

$$\log_p q = 1 / \log_q p$$

$$\log_p(xy) = \log_p x + \log_p y$$

$$\log_p(x/y) = \log_p x - \log_p y$$

$$\log_a x^n = n \log_a x$$

$$\log_{a^n} x = \frac{1}{n} \log_a x$$

$$\frac{\log_c a}{\log_c b} = \log_a b$$

$$\left. \begin{array}{l} f(2) = 1 \\ f(x+y) = f(x) + f(y) \quad (\neq) \end{array} \right\}$$

$$(1) : f(2+2) = f(2) + f(2)$$

$$f(4) = \overbrace{2} + \overbrace{2} = 2 \quad (\text{**})$$

↓ 0.5

$$(2) : f(4 \cdot 4) = f(4) + f(4)$$

$$f(16) = \overbrace{2} + \overbrace{2} \quad) \text{Se (**)}$$

$$= 4$$

↓ 1.0

$$\underline{\underline{f(16) = 4}}$$

$$2. \quad \begin{cases} f: \mathbb{R} \rightarrow \mathbb{R} \\ g: \mathbb{R} \rightarrow \mathbb{R} \end{cases} \quad \left. \begin{array}{l} \text{impar} \\ \text{impar} \end{array} \right\}$$

$$\begin{aligned}
 (f \circ g)(-x) &= f(g(-x)) \\
 &= f(-g(x)) \quad \downarrow g \text{ e impar} \\
 &= -f(g(x)) \quad \downarrow f \text{ e impar} \\
 &= -(f \circ g)(x)
 \end{aligned}$$

↓ 0.05

$$\therefore (f \circ g)(-x) = -(f \circ g)(x)$$

$f \circ g$ é impar

1.0

3.

$$f(x) = \begin{cases} x^2 - 4x + 3, & x \geq 2 \\ 2x - 3, & x < 2 \end{cases}$$

$$g(x) = 2x + 3$$

$$(f \circ g)(x) = f(g(x)) = \begin{cases} g(x)^2 - 4g(x) + 3, & g(x) \geq 2 \\ 2g(x) - 3, & g(x) < 2 \end{cases}$$

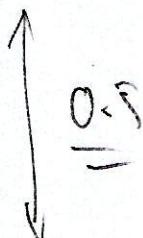
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0.5

$$g(x) \geq 2 \Rightarrow 2x + 3 \geq 2$$

$$2x \geq -1$$

$$x \geq -\frac{1}{2}$$



$$g(x) < 2 \Rightarrow x < -\frac{1}{2}$$

Dai

$$(f \circ g)(x) = \begin{cases} \underbrace{(2x+3)^2 - 4(2x+3) + 3}_{4x^2 + 12x + 9 - 8x - 12 + 3}, & x \geq -\frac{1}{2} \\ 4x^2 + 4x \\ 2(2x+3) - 3, & x < -\frac{1}{2} \\ 4x + 6 - 3 \\ 4x + 3 \end{cases}$$

$$(f \circ g)(x) = \begin{cases} 4x^2 + 4x & , x > -\frac{1}{2} \\ 4x + 3 & , x < -\frac{1}{2} \end{cases}$$

↑
0.5

$$4. \left\{ \begin{array}{l} a, b, c \text{ positivos} \\ a^2 + b^2 = c^2 \end{array} \right.$$

$$\underbrace{\log_{b+c} a + \log_{c-b} a} =$$

$$= \frac{1}{\log_a^{b+c}} + \frac{1}{\log_a^{c-b}}$$

$$= \frac{\log_a^{c-b} + \log_a^{b+c}}{\log_a^{b+c} \log_a^{c-b}}$$

$$= \frac{\log_a (c-b)(c+b)}{\log_a^{b+c} \log_a^{c-b}}$$

$$= \frac{\log_a (c^2 - b^2)}{\log_a^{b+c} \log_a^{c-b}}$$

$$= \frac{\log_a a^2}{\log_a^{b+c} \log_a^{c-b}} = \frac{2 \overbrace{\log_a a}^1}{\log_a^{b+c} \log_a^{c-b}} =$$

$$\log_{b+c} a + \log_{c-b} a = \frac{2}{\log_a b+c \log_a c-b}$$

$$= 2 \log_{b+c} a \log_{c-b} a$$

// $\log_{b+c} a + \log_{c-b} a = 2 \log_{c+b} a \log_{c-b} a //$