Cálculo A - Prova 1

1. Se $f(2) = 1$ e $f(xy) = f(x) + f(y)$, determine $f(16)$. 1.0

2. Seja $f : \mathbb{R} \to \mathbb{R}$ e $g : \mathbb{R} \to \mathbb{R}$ funções ímpares. Se a composta $f \circ g$ estiver definida mostre que $f \circ g$ é ímpar. 1.0

3. Sejam

$$f(x) = \begin{cases} x^2 - 4x + 3 & \text{se } x \geq 2 \\ 2x - 3 & \text{se } x < 2 \end{cases}, \qquad g(x) = 2x + 3$$

1.7

Determine $f \circ g$.

4. Sejam a, b, c números reais positivos satisfazendo $a^2 + b^2 = c^2$. Mostre que

$$\log_{b+c} a + \log_{c-b} a = 2 \log_{c+b} a \, \log_{c-b} a$$

2.0

Propriedades:

$\log_p q = 1/\log_q p$

$\log_p(xy) = \log_p x + \log_p y$

$\log_p(x/y) = \log_p x - \log_p y$

$\log_a x^n = n \log_a x$

$\log_{a^n} x = \frac{1}{n} \log_a x$

$\frac{\log_c a}{\log_c b} = \log_a b$

1.) $\begin{cases} f(2) = 1 \\ f(xy) = f(x) + f(y) \quad (*) \end{cases}$.

$(*)$: $f(2.2) = f(2) + f(2)$

$f(4) = \underbrace{1}_{} + \underbrace{1}_{} = 2 \quad (**)$

0.5

$(*)$: $f(4.4) = f(4) + f(4)$

$f(16) = 2 + 2$ De $(**)$

$= 4$

$\therefore \quad \underline{\underline{f(16) = 4}}$

$\underline{1.0}$

2.

$$\left.\begin{array}{l} f: \mathbb{R} \to \mathbb{R} \\ g: \mathbb{R} \to \mathbb{R} \end{array}\right\} \text{ímpar}$$

$$\begin{aligned}
(f \circ g)(-x) &= f(g(-x)) \\
&\downarrow g \text{ é ímpar} \\
&= f(-g(x)) \\
&\downarrow f \text{ é ímpar} \\
&= -f(g(x)) \\
&= -(f \circ g)(x)
\end{aligned}$$

0,5

$\therefore \quad (f \circ g)(-x) = -(f \circ g)(x)$

$\therefore \quad$ <u>$f \circ g$ é ímpar</u>.

1,0

3.

$$f(x) = \begin{cases} x^2 - 4x + 3, & x \geq 2 \\ 2x - 3, & x < 2 \end{cases}$$

$$g(x) = 2x + 3$$

$$(f \circ g)(x) = f(g(x)) = \begin{cases} g(x)^2 - 4g(x) + 3, & g(x) \geq 2 \\ 2g(x) - 3, & g(x) < 2 \end{cases}$$

0.5

Mes

$$g(x) \geq 2 \Rightarrow 2x + 3 \geq 2$$

$$2x \geq -1$$

$$x \geq -\frac{1}{2}$$

0.5

$$g(x) < 2 \Rightarrow x < -\frac{1}{2}$$

Dai

$$(f \circ g)(x) = \begin{cases} (2x+3)^2 - 4(2x+3) + 3, & x \geq -1/2 \\ 4x^2 + 12x + \cancel{9} - 8x - \cancel{12} + \cancel{3} \\ 4x^2 + 4x \\ 2(2x+3) - 3, & x < -\frac{1}{2} \\ 4x + 6 - 3 \\ 4x + 3 \end{cases}$$

$$(f \circ g)(x) = \begin{cases} 4x^2 + 4x, & x \geqslant -\frac{1}{2} \\ 4x + 3, & x < -\frac{1}{2} \end{cases}$$

0.5

4. $\begin{cases} a, b, c \text{ positivos} \\ a^2+b^2=c^2 \end{cases}$

$$\underline{\log_{b+c} a} + \log_{c-b} a =$$

$$= \frac{1}{\log_a b+c} + \frac{1}{\log_a c-b}$$

$$= \frac{\log_a c-b + \log_a b+c}{\log_a b+c \ \log_a c-b}$$

$$= \frac{\log_a (c-b)(c+b)}{\log_a b+c \ \log_a c-b}$$

$$= \frac{\log_a (c^2-b^2)}{\log_a b+c \ \log_a c-b}$$

$$= \frac{\log_a a^2}{\log_a b+c \ \log_a c-b} = \frac{2 \overbrace{\log_a a}^{1}}{\log_a b+c \ \log_a c-b} =$$

$\therefore$

$$\log_{b+c} a + \log_{c-b} a = \frac{2}{\log_a b+c \; \log_a c-b}$$

$$= 2 \log_{b+c} a \, \log_{c-b} a$$

$\therefore$

$$\log_{b+c} a + \log_{c-b} a = 2 \log_{c+b} a \, \log_{c-b} a$$