Cálculo A - Prova 1

1. Dê o domínio da função $f(x) = \sqrt{-\log_3 \frac{2x-3}{x-1}}$ 2.9

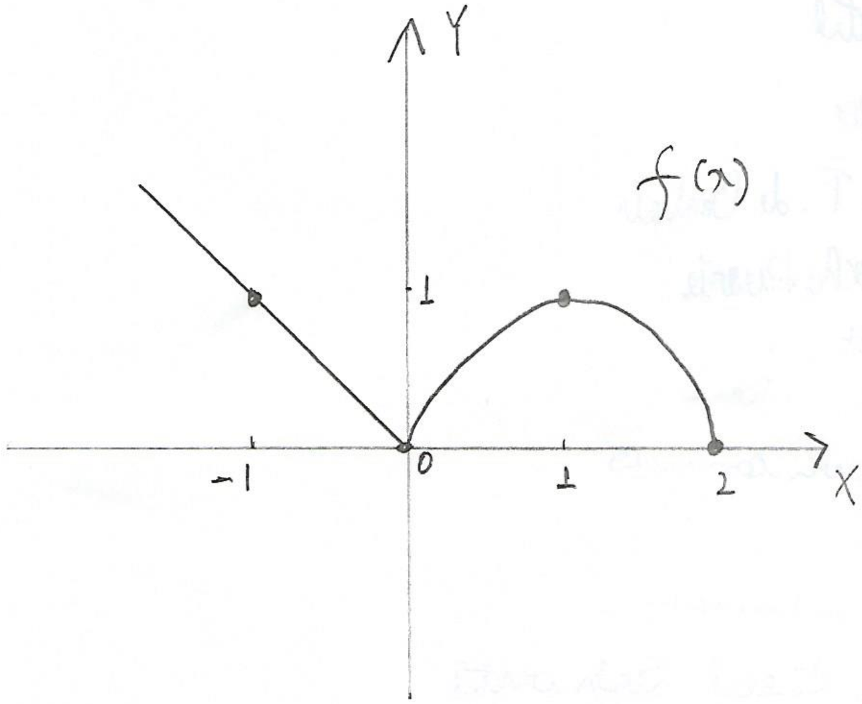
2. Resolva a equação 1.9

$$\arctan \frac{1+x}{2} + \arctan \frac{1-x}{2} = \frac{\pi}{6}$$

3. Mostre que 1.9

$$\frac{\log_a n}{\log_{am} n} = 1 + \log_a m$$

4. Seja o gráfico de $f(x)$ como mostrado na figura. Construa o gráfico de $2f(|x|+1)$. 2.5



1. $f(x) = \sqrt{-\log_3 \frac{2x-3}{x-1}}$

$\frac{2x-3}{x-1} > 0 \qquad (*)$

$-\log_3 \frac{2x-3}{x-1} \geq 0 \quad \therefore \quad \log_3 \frac{2x-3}{x-1} \leq 0$

$\therefore \quad 0 < \frac{2x-3}{x-1} \leq 1 \qquad (**)$

$\underline{\underline{0.5}}$

Vemos que (**) inclui (*).

$0 < \frac{2x-3}{x-1} \leq 1$

$\therefore \quad 0 < \frac{2x-3}{x-1} \quad \underline{\underline{e}} \quad \frac{2x-3}{x-1} \leq 1$

$$\begin{array}{cccccc} - & - & \overset{0}{|} & + & + & 2x-3 \\ & & \frac{3}{2} & & & \end{array}$$

$$\begin{array}{cccccc} - & - & \overset{0}{|} & + & + \; + & x-1 \\ & & 1 & & & \end{array}$$

$$\begin{array}{ccccccc} + & \overset{\nexists}{|} & - & \overset{0}{|} & + & + & \frac{2x-3}{x-1} \\ & 1 & & \frac{3}{2} & & & \end{array}$$

$\frac{2x-3}{x-1} > 0 \Rightarrow x < 1$ ou $x > \frac{3}{2}$

$\underline{\underline{0.75}}$

$\frac{2x-3}{x-1} - 1 \leq 0$

$\frac{2x-3-x+1}{x-1} \leq 0$

$\frac{x-2}{x-1} \leq 0$

$$\begin{array}{cccccc} - & - & \overset{0}{|} & + & + & x-2 \\ & & 2 & & & \end{array}$$

$$\begin{array}{cccccc} - & - & \overset{0}{|} & + & + \; + & x-1 \\ & & 1 & & & \end{array}$$

$$\begin{array}{cccccc} + & \overset{\nexists}{|} & - & \overset{0}{|} & + & \frac{x-2}{x-1} \\ & 1 & & 2 & & \end{array}$$

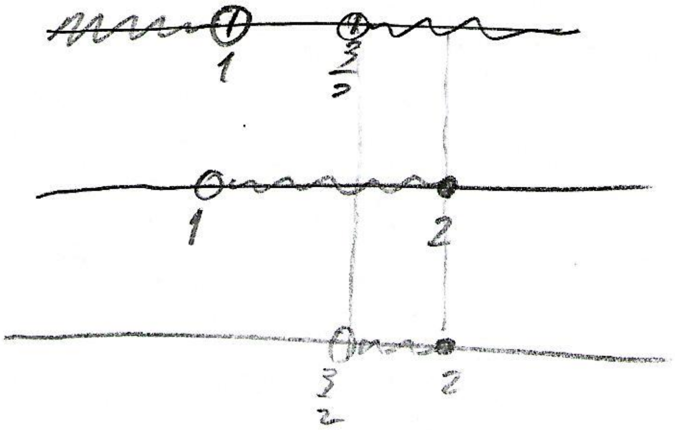
$\underline{\underline{0.75}}$

$x-2 < 0 \Longrightarrow 1 < x < 2$

Isto é, devemos ter

$$\left(x < 1 \text{ ou } x > \frac{3}{2}\right) \underline{\underline{e}} \left(1 < x \leq 2\right)$$

$$\left((-\infty, 1) \cup \left(\frac{3}{2}, +\infty\right)\right) \cap \left((1, 2]\right)$$



$\therefore$ $\boxed{\text{Dom } f = \left(\frac{3}{2}, 2\right]}$

$\underline{\underline{0.5}}$

2. $\operatorname{arctg} \frac{1+x}{2} + \operatorname{arctg} \frac{1-x}{2} = \frac{\pi}{6}$

$$\operatorname{tg}\left(\operatorname{arctg} \frac{1+x}{2} + \operatorname{arctg} \frac{1-x}{2}\right) = \operatorname{tg} \frac{\pi}{6}$$

$$\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}$$

$$\frac{\operatorname{tg}\left(\operatorname{arctg} \frac{1+x}{2}\right) + \operatorname{tg}\left(\operatorname{arctg} \frac{1-x}{2}\right)}{1 - \operatorname{tg}\left(\operatorname{arctg} \frac{1+x}{2}\right) \operatorname{tg} \operatorname{arctg}\left(\frac{1-x}{2}\right)} = \frac{\sqrt{3}}{3}$$

$$\frac{\frac{1+x}{2} + \frac{1-x}{2}}{1 - \left(\frac{1+x}{2}\right)\left(\frac{1-x}{2}\right)} = \frac{\sqrt{3}}{3}$$

0.5

$$\frac{1}{1 - \left(\frac{1-x^2}{4}\right)} = \frac{\sqrt{3}}{3}$$

$$\frac{4}{4-1+x^2} = \frac{\sqrt{3}}{3}$$

$$\frac{4}{3+x^2} = \frac{\sqrt{3}}{3} \quad \therefore \quad 12 = (3+x^2)\sqrt{3}$$

$$\frac{12}{\sqrt{3}} = 3+x^2$$

$$\therefore \quad x^2 = \frac{12}{\sqrt{3}} - 3 = 12\frac{\sqrt{3}}{3} - 3 = 4\sqrt{3} - 3$$

$$x = \pm\sqrt{4\sqrt{3}-3}$$

1.5

3. $$\frac{\log_a n}{\log_{am} n} = 1 + \log_a m$$

Temos

$$\frac{\log_a n}{\log_{am} n} = \frac{\frac{1}{\log_n a}}{\frac{1}{\log_n am}} = \frac{\log_n am}{\log_n a}$$

$$= \log_a am$$

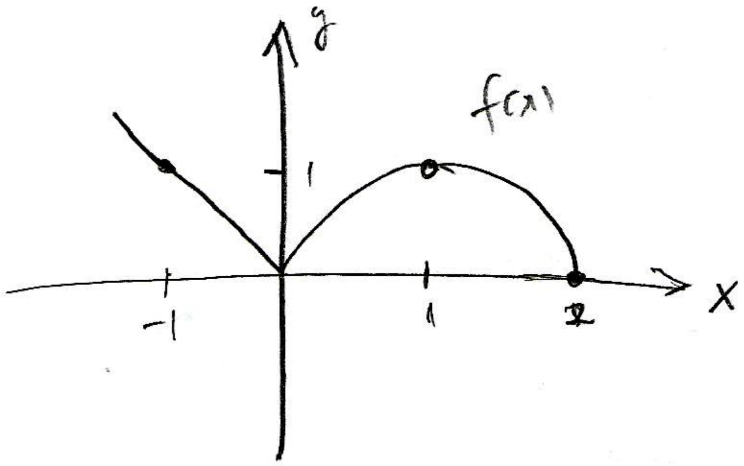
$$= \log_a a + \log_a m$$

$$= 1 + \log_a m$$

$$\therefore \boxed{\frac{\log_a n}{\log_{am} n} = 1 + \log_a m}$$

$\underline{\underline{1,5}}$

4.

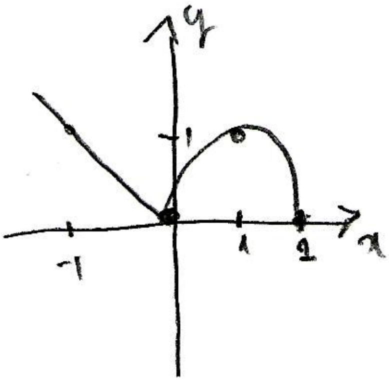


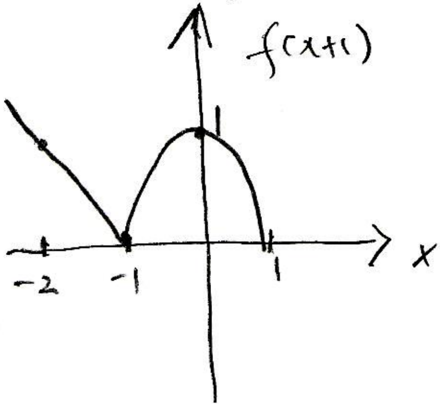
$2f(|x|+1)$:

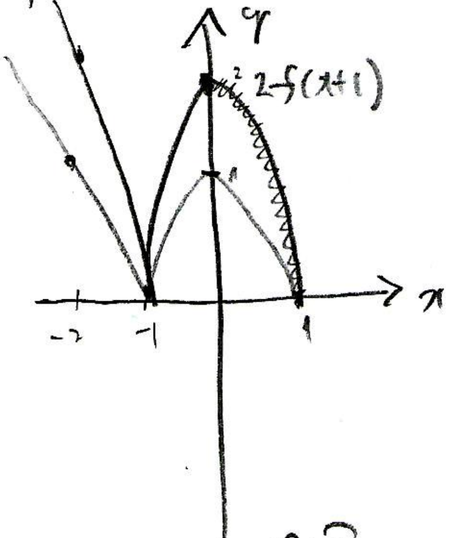
$$2f(|x|+1) = \begin{cases} 2f(x+1) & \text{se} \quad x \geq 0 \\ 2f(-x+1) & \text{se} \quad x < 0 \end{cases}$$

Construindo o gráfico de $2f(x+1)$:

$f(x) \longrightarrow f(x+1) \longrightarrow 2f(x+1)$







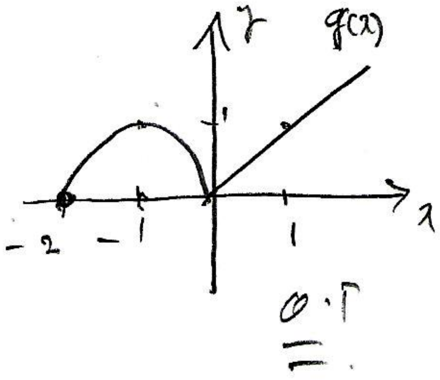
0.5

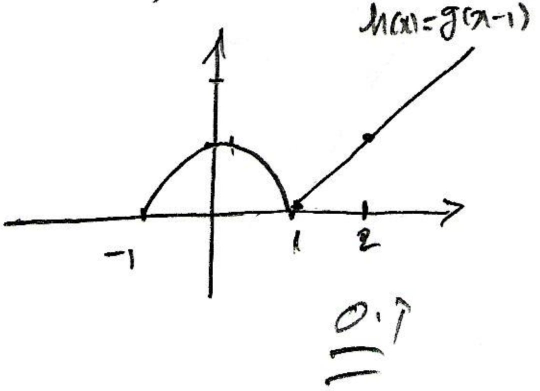
0.5

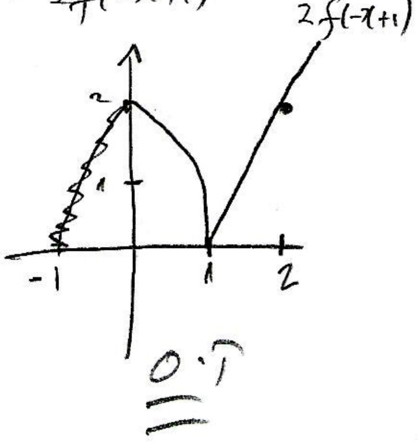
Construindo o gráfico de $2f(-x+1)$:

$f(x) \longrightarrow f(-x) = g(x) \longrightarrow g(x-1) = h(x) \longrightarrow 2h(x)$

$= f(-x+1)$ $\qquad$ $= 2f(-x+1)$







Daí