Cálculo A - Prova 1 [2014(1)] (Questões Adicionais)

1. Determine $f(x)$ sabendo que $2f(x) + f(1-x) = x^2$ 1.5

2. Mostre que 1.5

$$\arctan \frac{x}{\sqrt{1-x^2}} = \arcsin x, \qquad -1 < x < 1$$

3. Seja $f(x) = \frac{1}{2}(a^x + a^{-x})$, $\quad (a > 0)$. Mostrar que 1.0

$$f(x+y) + f(x-y) = 2f(x)f(y)$$

1. $2f(x) + f(1-x) = x^2$

Temos

$$2f(-x) + f(1+x) = (-x)^2 = x^2 \quad (*)$$

e

$$2f(x+1) + f(1-(x+1)) = (x+1)^2$$

$\therefore$

$$2f(x+1) + f(-x) = x^2+2x+1 \quad (**)$$

0.5

Daí

$2\times(*)$: $4f(-x) + 2f(1+x) = 2x^2$

$(**)$: $2f(x+1) + f(-x) = x^2+2x+1$

Daí, subtraindo essas duas eqs. obtemos

$$3f(-x) = 2x^2 - x^2 - 2x - 1$$

$$3f(-x) = x^2 - 2x - 1$$

$\therefore$

$$f(-x) = \frac{1}{3}(x^2 - 2x - 1)$$

$\therefore$

$$f(x) = \frac{1}{3}((-x)^2 - 2(-x) - 1)$$

$$f(x) = \frac{1}{3}(x^2 + 2x - 1)$$

1.5

2. $\operatorname{arctg} \frac{x}{\sqrt{1-x^2}} = \operatorname{arc\,sen} x; \quad -1 < x < 1$

Seja

$$z = \operatorname{arc\,sen} x \quad \therefore \quad \begin{cases} \operatorname{sen} z = x \\ z \in [-\frac{\pi}{2}, \frac{\pi}{2}] \end{cases} (*)$$

Uma vez que $-1 < x < 1$ temos que

$$z \in (-\frac{\pi}{2}, \frac{\pi}{2}) .$$ $\underline{\underline{0.5}}$

Mas, $\operatorname{tg} z = \frac{\operatorname{sen} z}{\cos z} = \frac{\operatorname{sen} z}{\sqrt{1-\operatorname{sen}^2 z}}$

pois $z \in (-\frac{\pi}{2}, \frac{\pi}{2})$

$\therefore$

$$\operatorname{tg} z = \frac{\operatorname{sen} z}{\sqrt{1-\operatorname{sen}^2 z}}$$

de $(*)$

$$\operatorname{tg} z = \frac{x}{\sqrt{1-x^2}}$$

$$\therefore \; \left\| \; z = \operatorname{arc\,tg} \frac{x}{\sqrt{1-x^2}} \; \right\|$$ $\underline{\underline{(1.0)}}$

3. $f(x) = \frac{1}{2}(a^x + a^{-x})$, $a > 0$

temos

$f(x+y) + f(x-y) =$

$= \frac{1}{2}(a^{x+y} + a^{-(x+y)}) + \frac{1}{2}(a^{x-y} + a^{-(x-y)})$

$= \frac{1}{2}(a^{x+y} + a^{-x-y}) + \frac{1}{2}(a^{x-y} + a^{y-x})$

0.5

e

$2f(x)f(y) = \cancel{2} \cdot \frac{1}{\cancel{2}}(a^x + a^{-x}) \frac{1}{2}(a^y + a^{-y})$

$= \frac{1}{2}(a^x a^y + a^x a^{-y} + a^{-x} a^y + a^{-x} a^{-y})$

$= \frac{1}{2}(a^{x+y} + a^{x-y} + a^{-x+y} + a^{-x-y})$

$= \frac{1}{2}(a^{x+y} + a^{-x-y}) + \frac{1}{2}(a^{x-y} + a^{y-x})$

$= f(x+y) + f(x-y)$

0.5

$\therefore$ $\| f(x+y) + f(x-y) = 2f(x)f(y) \|$