Cálculo A - Prova 2

Calcule os limites das questões 1-2

1. $\lim_{x\to 6} \frac{\sqrt{x-2}-2}{x-6}$

2. $\lim_{x\to\infty}(\sqrt{x^2-1}-\sqrt{x^2+1})$

Usando as regras de derivação, calcule as derivadas das funções nas questões 3-6

3. $f(x)=e^{\sqrt{\frac{1-x}{1+x}}}$

4. $f(x)=\ln\tan\frac{x}{2}-(\cos x)\ln\tan x$

5. $f(x)=\arcsin(\frac{2x}{1+x^2})$

6. $f(x)=x^{x^2}$

7. Um avião voa horizontalmente com velocidade de 800 km/h e a uma altitude de 2 km, e passa diretamente sobre uma estação de radar. Encontre a taxa segundo a qual a distância entre o avião e a estação aumenta qundo ele está a 3km além da estação.

1. $\lim_{x \to 6} \dfrac{\sqrt{x-2} - 2}{x-6} =$

$= \lim_{x \to 6} \dfrac{(\sqrt{x-2} - 2)}{(x-6)} \dfrac{(\sqrt{x-2} + 2)}{(\sqrt{x-2} + 2)}$

$= \lim_{x \to 6} \dfrac{x-2-4}{(x-6)(\sqrt{x-2} + 2)}$

$= \lim_{x \to 6} \dfrac{\cancel{(x-6)}}{\cancel{(x-6)}(\sqrt{x-2} + 2)}$

$= \lim_{x \to 6} \dfrac{1}{\sqrt{x-2} + 2} = \dfrac{1}{4}$. $\underline{1.0}$

2. $\lim_{x \to \infty} \left(\sqrt{x^2-1} - \sqrt{x^2+1} \right)$

$= \lim_{x \to \infty} \left(\sqrt{x^2-1} - \sqrt{x^2+1}\right) \dfrac{\left(\sqrt{x^2-1} + \sqrt{x^2+1}\right)}{\left(\sqrt{x^2-1} + \sqrt{x^2+1}\right)}$

$= \lim_{x \to \infty} \dfrac{x^2-1-(x^2+1)}{\sqrt{x^2-1}+\sqrt{x^2+1}}$

$= \lim_{x \to \infty} \dfrac{-2}{\underbrace{\sqrt{x^2-1}}_{\infty} + \underbrace{\sqrt{x^2+1}}_{\infty}}$

$= 0$ //

1.0

3.

$$f(x) = e^{\sqrt{\frac{1-x}{1+x}}}$$

0.5

$$f'(x) = e^{\sqrt{\frac{1-x}{1+x}}} \cdot \left(\sqrt{\frac{1-x}{1+x}}\right)'$$

$$= e^{\sqrt{\frac{1-x}{1+x}}} \cdot \frac{1}{2\sqrt{\frac{1-x}{1+x}}} \cdot \left(\frac{1-x}{1+x}\right)'$$

0.25

$$= e^{\sqrt{\frac{1-x}{1+x}}} \frac{1}{2\sqrt{\frac{1-x}{1+x}}} \cdot \left\{ \frac{(1-x)'(1+x) - (1-x)(1+x)'}{(1+x)^2} \right\}$$

$$= e^{\sqrt{\frac{1-x}{1+x}}} \frac{1}{2\sqrt{\frac{1-x}{1+x}}} \cdot \left\{ \frac{-1(1+x) - (1-x)}{(1+x)^2} \right\}$$

$$= e^{\sqrt{\frac{1-x}{1+x}}} \frac{1}{2\sqrt{\frac{1-x}{1+x}}} \left\{ \frac{-1 - \cancel{x} - 1 + \cancel{x}}{(1+x)^2} \right\}$$

$$= e^{\sqrt{\frac{1-x}{1+x}}} \frac{1}{\cancel{2}\sqrt{\frac{1-x}{1+x}}} \left\{ \frac{-2}{(1+x)^2} \right\}$$

0.5

$$= -e^{\sqrt{\frac{1-x}{1+x}}} \sqrt{\frac{1+x}{1-x}} \frac{1}{(1+x)^2} \quad //$$

0.5

4. $f(x) = \ln \operatorname{tg}\frac{x}{2} - (\cos x)\ln \operatorname{tg} x$

$$f'(x) = \overbrace{\frac{1}{\operatorname{tg}\frac{x}{2}}\sec^2\frac{x}{2}\cdot\frac{1}{2}}^{0.5} + \sin x \ln \operatorname{tg} x$$

$$- \cos x \frac{1}{\operatorname{tg} x}\sec^2 x$$

0.5

$$= \frac{1}{2}\,\frac{1}{\frac{\sin\frac{x}{2}}{\cancel{\cos\frac{x}{2}}}}\,\frac{1}{\cos^2\frac{x}{2}} + \sin x \ln \operatorname{tg} x$$

$$- \frac{\cancel{\cos x}}{\frac{\sin x}{\cancel{\cos x}}}\,\frac{1}{\cancel{\cos^2 x}}$$

$$= \frac{1}{2}\,\frac{1}{\sin\frac{x}{2}\cos\frac{x}{2}} + \sin x \ln \operatorname{tg} x - \frac{1}{\sin x}$$

$$= \cancel{\frac{1}{\sin x}} + \sin x \ln \operatorname{tg} x - \cancel{\frac{1}{\sin x}}$$

$$= \sin x \ln \operatorname{tg} x$$

5. $f(x) = \arcsin\left(\frac{2x}{1+x^2}\right)$

$$f'(x) = \frac{1}{\sqrt{1-\left(\frac{2x}{1+x^2}\right)^2}} \cdot \left(\frac{2x}{1+x^2}\right)' \quad 0.25$$

$$= \frac{1}{\sqrt{1-\frac{4x^2}{(1+x^2)^2}}} \left\{ \frac{2(1+x^2)-2x\cdot 2x}{(1+x^2)^2} \right\}$$

$-1 \le \frac{2x}{1+x^2} \le 1$

$-1-x^2 \le 2x \le 1+x^2$

$-1-x^2 \le 2x$ e $2x \le 1+x^2$

$0 \le x^2+2x+1 \Rightarrow x \in \mathbb{R}$

$0 \le x^2-2x+1 \Rightarrow x \in \mathbb{R}$

$$= \frac{1}{\sqrt{\frac{(1+x^2)^2-4x^2}{(1+x^2)^2}}} \left\{ \frac{2+2x^2-4x^2}{(1+x^2)^2} \right\}$$

$$= \frac{\cancel{(1+x^2)}}{\sqrt{1+2x^2+x^4-4x^2}} \left\{ \frac{2-2x^2}{(1+x^2)\cancel{^2}} \right\}$$

$$= \frac{1}{\sqrt{1-2x^2+x^4}} \quad \frac{2(1-x^2)}{(1+x^2)} \quad 0.75$$

$$= \frac{1}{\sqrt{(1-x^2)^2}} \quad \frac{2(1-x^2)}{(1+x^2)} = \frac{2(1-x^2)}{|1-x^2|\,(1+x^2)}$$

6.

$$f(x) = x^{x^2}$$

$$= e^{\ln x^{x^2}} = e^{x^2 \ln x}$$

$$f'(x) = \left(e^{x^2 \ln x}\right)'$$

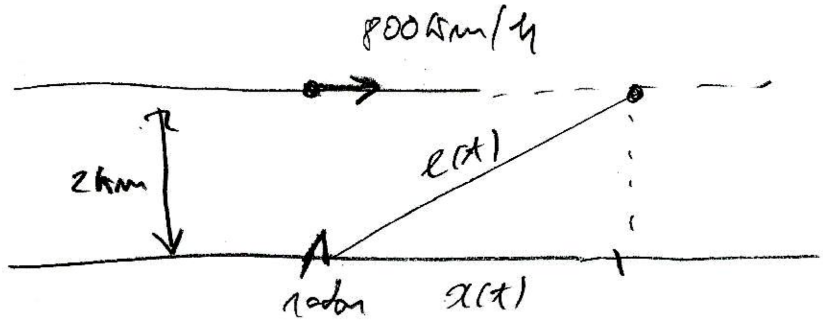
$$= \underbrace{e^{x^2 \ln x}} \left(x^2 \ln x\right)'$$

$$= x^{x^2}\left(2x \ln x + x^2 \frac{1}{x}\right)$$

$$= x^{x^2}\left(2x \ln x + x\right)$$

1.0

7.



Seja

$$\begin{cases} l(t) : \text{distância entre o avião e o radar} \\ x(t) : \text{afastamento horizontal do avião e o radar.} \end{cases}$$

Temos $\begin{cases} \dfrac{dx}{dt} = 800 \text{ km/h} \\ l^2(t) = x^2(t) + 4 \end{cases}$

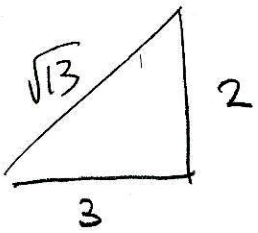
Queremos obter $\left.\dfrac{dl}{dt}\right|_{x=3km}$.

$$l^2(t) = x^2(t) + 4 \implies \frac{d}{dt} l^2(t) = \frac{d}{dt}\left(x^2(t)+4\right)$$

$$\cancel{2} l(t) \frac{dl}{dt} = \cancel{2} x(t) \frac{dx}{dt}$$

$$l(t) \frac{dl}{dt} = x(t)\, 800$$

$$\Bigg\| \frac{dl}{dt} = \frac{x(t)}{l(t)} 800 \Bigg\|$$



$$\left.\frac{dl}{dt}\right|_{\substack{x=3\\ l=\sqrt{13}}} = \frac{3}{\sqrt{13}}\, 800 \text{ km/h}$$

$$\left.\frac{dl}{dt}\right|_{\substack{x=3\\ l=\sqrt{13}}} = \frac{2400}{\sqrt{13}} \text{ km/h}$$