1694. $\int \frac{dx}{x \ln x \; \ln(\ln x)} \quad (x>0)$

$u = \ln \ln x \longrightarrow du = \frac{1}{\ln x} \frac{1}{x} dx$

$\therefore$

$$\int \frac{dx}{x \ln x \; \ln(\ln x)} = \int \frac{du}{u} = \ln|u|$$

$$= \ln|\ln \ln x| \; //$$

17.16 $\int \frac{1}{1-x^2} \ln\left(\frac{1+x}{1-x}\right) dx$

$u = \ln \frac{1+x}{1-x} \quad \therefore$

$\therefore$

$$du = \frac{1}{\frac{1+x}{1-x}} \left(\frac{1+x}{1-x}\right)' dx$$

$$= \left(\frac{1-x}{1+x}\right) \cdot \left(\frac{(1+x)'(1-x) - (1+x)(1-x)'}{(1-x)^2} \right) dx$$

$$= \left(\frac{1-x}{1+x}\right) \cdot \left(\frac{1-x - (1+x)(-1)}{(1-x)^2} \right) dx$$

$$= \left(\frac{1-x}{1+x}\right) \cdot \left(\frac{1 \cancel{-x} + 1 \cancel{+x}}{(1-x)^2} \right) dx$$

$$= \left(\frac{\cancel{1-x}}{1+x}\right) \frac{2}{(1-x)^{\cancel{2}}} dx = \frac{2}{1-x^2} dx$$

$\therefore$

$\therefore \quad \frac{dx}{1-x^2} = \frac{1}{2} du$

$\therefore$

$$\int \frac{1}{1-x^2} \ln \frac{1+x}{1-x} \, dx =$$

$$= \int u \frac{1}{2} du = \frac{1}{2} \int u \, du$$

$$= \frac{1}{2} \frac{u^2}{2} = \frac{1}{4} u^2 = \frac{1}{4} \left(\ln \left(\frac{1+x}{1-x} \right) \right)^2 \quad //$$

1711. $\displaystyle \int \sqrt{\frac{\ln(x+\sqrt{1+x^2})}{1+x^2}} \, dx =$

$$= \int \frac{\sqrt{\ln(x+\sqrt{1+x^2})}}{\sqrt{1+x^2}} \, dx$$

$u = \ln(x+\sqrt{1+x^2})$

$$du = \frac{1}{x+\sqrt{1+x^2}} (x+\sqrt{1+x^2})' \, dx$$

$$= \frac{1}{x+\sqrt{1+x^2}} \left(1 + \frac{x}{\sqrt{1+x^2}} \right) dx$$

$$= \frac{1}{(\cancel{x+\sqrt{1+x^2}})} \left(\frac{\cancel{\sqrt{1+x^2}+x}}{\sqrt{1+x^2}} \right) dx$$

$$du = \frac{1}{\sqrt{1+x^2}}\,dx$$

$$\int \sqrt{\frac{\ln(x+\sqrt{1+x^2})}{1+x^2}}\,dx =$$

$$= \int \sqrt{u}\,du$$

$$= \frac{u^{3/2}}{\frac{3}{2}}$$

$$= \frac{2}{3}u^{3/2} = \frac{2}{3}\ln^{3/2}(x+\sqrt{1+x^2}) \;//$$

1718. $\displaystyle\int \frac{\sin x \cos x}{\sin^4 x + \cos^4 x}\,dx$

$$\frac{\sin x \cos x}{\sin^4 x + \cos^4 x} = \frac{\frac{1}{2}\sin 2x}{(\sin^2 x)^2 + (\cos^2 x)^2}$$

$\sin^2 x = \dfrac{1-\cos 2x}{2}$

$\cos^2 x = \dfrac{1+\cos 2x}{2}$

$$= \frac{\frac{1}{2}\sin 2x}{\left(\frac{1-\cos 2x}{2}\right)^2 + \left(\frac{1+\cos 2x}{2}\right)^2}$$

$$= \frac{\frac{1}{2}\sin 2x}{\frac{1-\cancel{2\cos 2x}+\cos^2 2x+1+\cancel{2\cos 2x}+\cos^2 2x}{4}}$$

$$= \frac{\frac{1}{2}\sin 2x}{\frac{2+2\cos^2 2x}{4}}$$

$$= \frac{\frac{1}{2}\sin 2x}{\frac{1+\cos^2 2x}{2}}$$

$$= \frac{\sin 2x}{1+\cos^2 2x}$$

$\therefore$

$$\int \frac{\sin x \cos x}{\sin^4 x + \cos^4 x}\,dx = \int \frac{\sin 2x}{1+\cos^2 2x}\,dx \quad (*)$$

$$u = \cos 2x \longrightarrow du = -2\sin 2x\,dx$$

$\therefore$

$$-\frac{1}{2}\,du = \sin 2x\,dx \quad (**)$$

$(**) \rightarrow (*)$:

$$\int \frac{\sin x \cos x \, dx}{\sin^4 x + \cos^4 x} = \int \frac{-\frac{1}{2} du}{1+u^2} =$$

$$= -\frac{1}{2} \int \frac{du}{1+u^2}$$

$\int \frac{du}{1+u^2} = \operatorname{arctg} u$

$$= -\frac{1}{2} \operatorname{arctg} u$$

$$= -\frac{1}{2} \operatorname{arctg} \cos 2x$$

1720. $\displaystyle\int \frac{x\,dx}{\sqrt{1+x^2}+\sqrt{(1+x^2)^3}}$

$$u = 1+x^2 \;\; (*) \longrightarrow du = 2x\,dx$$

$$\frac{1}{2}du = x\,dx$$

$\therefore$

$$\int \frac{x\,dx}{\sqrt{1+x^2}+\sqrt{(1+x^2)^3}} = \int \frac{\frac{1}{2}du}{\sqrt{u}+u^{3/2}}$$

$$= \frac{1}{2}\int \frac{du}{\sqrt{u}(1+u^{1/2})}$$

$$= \frac{1}{2}\int \frac{du}{\sqrt{u}\sqrt{1+u^{1/2}}}$$

$\therefore$

$$w = 1+u^{1/2} \;\; (**) \longrightarrow dw = \frac{1}{2\sqrt{u}}\,du$$

$$= \int \frac{dw}{\sqrt{w}} = \int w^{-1/2}\,dw$$

$$= \frac{w^{1/2}}{\frac{1}{2}} = 2\sqrt{w}$$

$$\overset{(**)}{=} 2\sqrt{1+u^{1/2}}$$

$$\overset{(*)}{=} 2\sqrt{1+\sqrt{1+x^2}}$$

$\therefore$

$$\int \frac{x\,dx}{\sqrt{1+x^2}+\sqrt{(1+x^2)^3}} = 2\sqrt{1+\sqrt{1+x^2}} \;\; //$$

1719. $\displaystyle\int \frac{2^x 3^x}{9^x - 4^x}\,dx =$

$$\int a^x dx = \frac{a^x}{\ln a}$$

$$= \int \frac{1}{\frac{9^x}{2^x 3^x} - \frac{4^x}{2^x 3^x}}\,dx$$

$$\frac{9^x}{2^x 3^x} = \frac{(3^2)^x}{2^x 3^x} = \frac{3^{2x}}{2^x 3^x} = \frac{3^x}{2^x} = \left(\frac{3}{2}\right)^x$$

$$= \int \frac{1}{\left(\frac{3}{2}\right)^x - \left(\frac{2}{3}\right)^x}\,dx$$

$$= \int \frac{1}{\left(\frac{3}{2}\right)^x - \frac{1}{\left(\frac{3}{2}\right)^x}}\,dx$$

$$= \int \frac{1}{\frac{\left(\frac{3}{2}\right)^{2x} - 1}{\left(\frac{3}{2}\right)^x}}\,dx$$

$$= \int \frac{\left(\frac{3}{2}\right)^x dx}{\left(\frac{3}{2}\right)^{2x} - 1}$$

$$u = \left(\frac{3}{2}\right)^x \longrightarrow du = \ln\frac{3}{2}\left(\frac{3}{2}\right)^x dx$$

$$\therefore \left(\frac{3}{2}\right)^x dx = \frac{1}{\ln\frac{3}{2}}\,du$$

$$= \int \frac{\frac{1}{\ln\frac{3}{2}}\,du}{u^2 - 1} = -\frac{1}{\ln\frac{3}{2}} \int \frac{du}{1-u^2}$$

$$\int \frac{dx}{1-x^2} = \frac{1}{2}\ln\left|\frac{1+x}{1-x}\right|$$

$$= -\frac{1}{\ln\frac{3}{2}}\,\frac{1}{2}\ln\left|\frac{1+u}{1-u}\right|$$

$$= -\frac{1}{2\ln\frac{3}{2}}\ln\left|\frac{1+\left(\frac{3}{2}\right)^x}{1-\left(\frac{3}{2}\right)^x}\right| \quad //$$

- $\displaystyle\int \frac{e^x}{e^x+1}\,dx$

$u = e^x + 1 \longrightarrow du = e^x\,dx$

$\therefore$

$$\int \frac{e^x\,dx}{e^x+1} = \int \frac{1}{u}\,du$$

$$= \ln|u|$$

$$= \ln|e^x+1|$$

$$= \ln(e^x+1) \; //$$

puis $e^x+1>0$ $\forall x$.

- $\displaystyle\int \frac{\cos x}{\sin^2 x}\,dx$

$u = \sin x \longrightarrow du = \cos x\,dx$

$\therefore$

$$\int \frac{\cos x\,dx}{\sin^2 x} = \int \frac{1}{u^2}\,du$$

$$= \int u^{-2}\,du$$

$$= \frac{u^{-1}}{-1}$$

$$= -\frac{1}{u}$$

$$= -\frac{1}{\sin x} \; //$$

- $\displaystyle\int \frac{\sin(\ln x)}{x}\, dx$

$u = \ln x \longrightarrow du = \frac{1}{x}\, dx$

$\therefore$

$$\int \frac{\sin(\ln x)}{x}\, dx = \int \sin u\, du$$

$$= -\cos u$$

$$= -\cos \ln x \; //$$

- $\displaystyle\int e^{tg\, x} \sec^2 x\, dx$

$u = tg\, x \longrightarrow du = \sec^2 x\, dx$

$\therefore$

$$\int e^{tg\, x} \sec^2 x\, dx = \int e^{u}\, du$$

$$= e^{u}$$

$$= e^{tg\, x} \; //$$

- $\displaystyle\int \frac{\operatorname{arc\,tg} x}{1+x^2}\, dx$

$u = \operatorname{arc\,tg} x \longrightarrow du = \frac{1}{1+x^2}\, dx$

$\therefore$

$$\int \frac{\operatorname{arc\,tg} x}{1+x^2}\, dx = \int u\, du = \frac{u^2}{2} =$$

$$= \frac{1}{2} \operatorname{arc\,tg}^2 x \; //$$

- $\displaystyle\int \frac{z^2}{\sqrt[3]{1+z^3}}\,dz$

$u = 1+z^3 \longrightarrow du = 3z^2\,dz$

$\therefore \quad \frac{1}{3}\,du = z^2\,dz$

$\therefore$

$$\int \frac{z^2}{\sqrt[3]{1+z^3}}\,dz = \int \frac{1}{\sqrt[3]{u}}\,\frac{1}{3}\,du$$

$$= \frac{1}{3}\int u^{-1/3}\,du$$

$-\frac{1}{3}+1 = \frac{2}{3}$

$$= \frac{1}{3}\,\frac{u^{2/3}}{\frac{2}{3}}$$

$$= \frac{1}{2}\,u^{2/3} = \frac{1}{2}\,(1+z^3)^{2/3} \,//$$

- $\displaystyle\int e^{\cos t}\sin t\,dt$

$u = \cos t \longrightarrow du = -\sin t\,dt$

$\therefore \quad -du = \sin t\,dt$

$\therefore$

$$\int e^{\cos t}\underline{\sin t\,dt} = \int e^{u}\,(-)\,du$$

$$= -\int e^{u}\,du$$

$$= -e^{u} = -e^{\cos t} \,//$$

- $\int e^x \sqrt{1+e^x}\, dx$

$u = 1+e^x \longrightarrow du = e^x dx$

$\therefore$

$$\int \underbrace{e^x \sqrt{1+e^x}}\, \underline{dx} = \int \sqrt{u}\, \underline{du}$$

$$= \frac{u^{3/2}}{\frac{3}{2}}$$

$$= \frac{2}{3} u^{3/2} = \frac{2}{3}(1+e^x)^{3/2} \;//$$

- $\int (1+tg\theta)^5 \sec^2\theta\, d\theta$

$u = 1+tg\theta \longrightarrow du = \sec^2\theta\, d\theta$

$\therefore$

$$\int (1+tg\theta)^5 \underline{\sec^2\theta\, d\theta} = \int u^5\, du$$

$$= \frac{u^6}{6}$$

$$= \frac{1}{6}(1+tg\theta)^6 \;//$$

- $\int \cos\theta \sin^6\theta\, d\theta$

$u = \sin\theta \longrightarrow du = \cos\theta\, d\theta$

$\therefore$

$$\int \underline{\cos\theta} \sin^6\theta\, \underline{d\theta} = \int u^6\, du = \frac{u^7}{7} =$$

$$= \frac{1}{7}\sin^7\theta \;//$$

- $\int \sin t \sec^2(\cos t)\, dt =$

$u = \cos t \longrightarrow du = -\sin t\, dt$

$\therefore$

$\int \sin t \sec^2(\cos t)\, dt =$

$= \int \sec^2 u \; (-)\, du$

$= -\int \sec^2 u\, du$

$= -\operatorname{tg} u$

$= -\operatorname{tg} \cos t$

- $\int \sec^3 x \operatorname{tg} x\, dx$

$u = \sec x \longrightarrow du = \sec x \operatorname{tg} x\, dx$

$\therefore$

$\int \sec^3 x \operatorname{tg} x\, dx = \int \sec x \sec x \operatorname{tg} x\, dx$

$= \int u\, du$

$= \frac{u^2}{2}$

$= \frac{1}{2}\sec^2 x$

$$\int \frac{dt}{\cos^2 t \sqrt{1+tg\,t}}$$

$$u = \sqrt{1+tg\,t} \rightarrow du = \frac{1}{2\sqrt{1+tg\,t}} \underbrace{(1+tg\,t)'}\, dt$$

$$= \frac{1}{2\sqrt{1+tg\,t}} \underbrace{sec^2 t}\, dt$$

$$= \frac{1}{2} \frac{1}{\sqrt{1+tg\,t}} \frac{1}{\cos^2 t}\, dt$$

$$\therefore \quad 2\,du = \frac{1}{\cos^2 t \sqrt{1+tg\,t}}\, dt$$

$\therefore$

$$\int \frac{dt}{\cos^2 t \sqrt{1+tg\,t}} = \int 2\,du$$

$$= 2\int du$$

$$= 2u$$

$$= 2\sqrt{1+tg\,t} \quad //$$

- $\int \frac{\sin x}{1+\cos^2 x}\, dx$

$u = \cos x \longrightarrow du = -\sin x\, dx$

$\therefore -du = \sin x\, dx$

$\therefore$

$$\int \frac{\sin x}{1+\cos^2 x}\, dx = \int \frac{-du}{1+u^2}$$

$$= -\int \frac{du}{1+u^2}$$

$$= -\operatorname{arctg} u$$

$$= -\operatorname{arctg} \cos x$$

- $\int \frac{\sin 2x}{1+\cos^2 x}\, dx$

$$\int \frac{\sin 2x}{1+\cos^2 x}\, dx = \int \frac{2\sin x \cos x}{1+\cos^2 x}\, dx \quad (*)$$

$u = 1+\cos^2 x \longrightarrow du = -2\cos x \sin x\, dx$

$\therefore -du = 2\sin x \cos x\, dx \quad (**)$

$(**) \to (*)$:

$\therefore$

$$\int \frac{\sin 2x}{1+\cos^2 x}\, dx = \int \frac{-du}{u} = -\ln|u|$$

$$= -\ln(1+\cos^2 x) \; //$$

- $$\int \frac{\cos(\pi/x)}{x^2}\, dx$$

$$u = \frac{\pi}{x} \longrightarrow du = -\frac{\pi}{x^2}\, dx$$

$$\therefore -\frac{1}{\pi}\, du = \frac{1}{x^2}\, dx$$

$$\therefore \int \frac{\cos(\pi/x)}{x^2}\, dx = \int \cos u \left(-\frac{1}{\pi}\right) du$$

$$= -\frac{1}{\pi}\int \cos u\, du$$

$$= -\frac{1}{\pi} \sin u$$

$$= -\frac{1}{\pi} \sin\frac{\pi}{x} \; //$$

- $$\int \sqrt{\text{cotg}\, x}\; \text{cosec}^2 x\, dx$$

$$u = \text{cotg}\, x \longrightarrow du = -\text{cosec}^2 x\, dx$$

$$\therefore -du = \text{cosec}^2 x\, dx$$

$$\therefore \int \sqrt{\text{cotg}\, x}\; \text{cosec}^2 x\, dx = \int \sqrt{u}\, (-)\, du$$

$$= -\int \sqrt{u}\, du$$

$$= -\frac{u^{3/2}}{\frac{3}{2}} = -\frac{2}{3} u^{3/2}$$

$$= -\frac{2}{3} (\text{cotg})^{3/2} \; //$$

$$\int x^3 \sqrt{x^2+1}\, dx$$

$$u = x^2+1 \quad\Bigg|\quad du = 2x\,dx$$

$$\therefore \quad x^2 = u - 1 \;(**) \quad\Bigg|\quad \frac{1}{2} du = x\,dx \quad (*)$$

$$\int x^3 \sqrt{x^2+1}\, dx = \int \underbrace{x^2}_{\downarrow (**)} \sqrt{x^2+1}\; \underbrace{x\,dx}_{(*)}$$

$$= \int (u-1)\sqrt{u}\; \frac{1}{2} du$$

$$= \frac{1}{2}\int (u\sqrt{u} - \sqrt{u})\, du$$

$$= \frac{1}{2}\int u\sqrt{u}\, du - \frac{1}{2}\int \sqrt{u}\, du$$

$$= \frac{1}{2}\int u^{3/2}\, du - \frac{1}{2}\int u^{1/2}\, du$$

$$= \frac{1}{2}\,\frac{u^{5/2}}{\frac{5}{2}} - \frac{1}{2}\,\frac{u^{3/2}}{\frac{3}{2}}$$

$$= \frac{1}{5} u^{5/2} - \frac{1}{3} u^{3/2}$$

$$= \frac{1}{5}(x^2+1)^{5/2} - \frac{1}{3}(x^2+1)^{3/2}$$

$$\bullet \quad \int \frac{1+x}{1+x^2}\,dx =$$

$$= \underbrace{\int \frac{1}{1+x^2}\,dx}_{} + \int \frac{x}{1+x^2}\,dx$$

Integral tabelada

$$= \operatorname{arctg} x + \underbrace{\int \frac{x}{1+x^2}\,dx}_{(*)}$$

$$(*) = \int \frac{x}{1+x^2}\,dx$$

$$u = 1+x^2 \rightarrow du = 2x\,dx$$

$$\therefore \quad \frac{1}{2}\,du = x\,dx$$

$$\therefore$$

$$(*) = \int \frac{x}{1+x^2}\,dx = \int \frac{1}{u}\,\frac{1}{2}\,du$$

$$= \frac{1}{2}\int \frac{1}{u}\,du$$

$$= \frac{1}{2}\ln|u|$$

$$= \frac{1}{2}\ln|1+x^2|$$

$$\therefore$$

$$\int \frac{1+x}{1+x^2}\,dx = \operatorname{arctg} x + \frac{1}{2}\ln(1+x^2)$$

$$\bullet \quad \int \frac{x^2}{\sqrt{1-x}}\,dx$$

$$\begin{cases} u = 1-x \longrightarrow du = -dx \\ x = 1-u \end{cases}$$

$$\int \frac{x^2}{\sqrt{1-x}}\,\underline{dx} = \int \frac{(1-u)^2}{\sqrt{u}}\,\underline{(-)du}$$

$$= -\int \frac{(1-u)^2}{\sqrt{u}}\,du$$

$$\frac{1}{\sqrt{u}} = u^{-1/2} \qquad = -\int \frac{1-2u+u^2}{\sqrt{u}}\,du$$

$$\frac{u}{\sqrt{u}} = u^{1/2} \qquad = -\int \frac{1}{\sqrt{u}}\,du + 2\int \frac{u}{\sqrt{u}}\,du - \int \frac{u^2}{\sqrt{u}}\,du$$

$$\frac{u^2}{\sqrt{u}} = u^{3/2} \qquad = -\int u^{-1/2}\,du + 2\int u^{1/2}\,du - \int u^{3/2}\,du$$

$$2-\frac{1}{2} = \frac{3}{2} \qquad = -\frac{u^{1/2}}{\frac{1}{2}} + 2\,\frac{u^{3/2}}{\frac{3}{2}} - \frac{u^{5/2}}{\frac{5}{2}}$$

$$= -2u^{1/2} + \frac{1}{3}u^{3/2} - \frac{2}{5}u^{5/2}$$

$$= -2\sqrt{1-x} + \frac{1}{3}(1-x)^{3/2} - \frac{2}{5}(1-x)^{5/2}$$

- $\displaystyle\int \frac{x}{1+x^4}\,dx$

$u = x^2 \longrightarrow du = 2x\,dx$

$\therefore \frac{1}{2}du = x\,dx$

$\therefore$

$$\int \frac{x}{1+x^4}\,dx = \int \frac{1}{1+u^2}\,\frac{1}{2}du$$
$$= \frac{1}{2}\int \frac{1}{1+u^2}\,du$$
$$= \frac{1}{2}\,\text{arctg}\,u$$
$$= \frac{1}{2}\,\text{arctg}\,x^2$$

- $\displaystyle\int \frac{dx}{\sqrt{1-x^2}\,\arcsin x}$

$u = \arcsin x \longrightarrow du = \frac{1}{\sqrt{1-x^2}}\,dx$

$\therefore$

$$\int \frac{dx}{\sqrt{1-x^2}\,\arcsin x} = \int \frac{1}{u}\,du$$
$$= \ln|u|$$
$$= \ln|\arcsin x|$$

- $\int \sqrt{x}\ \sin(1+x^{3/2})\, dx$

$u = 1+x^{3/2} \longrightarrow du = \frac{3}{2} x^{1/2} dx$

$\therefore \quad \frac{2}{3} du = \sqrt{x}\, dx$

$\therefore$

$\int \underbrace{\sqrt{x}}\ \sin(1+x^{3/2})\ \underline{dx} = \int \sin u\ \frac{2}{3} du$

$= \frac{2}{3} \int \sin u\ du$

$= -\frac{2}{3} \cos u$

$= -\frac{2}{3} \cos(1+x^{3/2})$ //

- $\int \frac{\cos\sqrt{t}}{\sqrt{t}}\ dt$

$u = \sqrt{t} \longrightarrow du = \frac{1}{2\sqrt{t}}\ dt$

$\therefore \quad 2\, du = \frac{1}{\sqrt{t}}\, dt$

$\therefore$

$\int \frac{\cos\sqrt{t}}{\sqrt{t}}\ dt = \int \cos u\ 2\, du$

$= 2 \int \cos u\ du$

$= 2 \sin u$

$= 2 \sin\sqrt{t}$ //

- $\displaystyle\int \frac{dx}{ax+b} \quad (a \neq 0)$

$u = ax+b \longrightarrow du = a\,dx$

$\therefore \quad \frac{1}{a}\,du = dx$

$$\therefore \quad \int \frac{dx}{ax+b} = \int \frac{1}{u}\,\frac{1}{a}\,du$$

$$= \frac{1}{a}\int \frac{1}{u}\,du$$

$$= \frac{1}{a}\ln|u|$$

$$= \frac{1}{a}\ln|ax+b| \quad //$$

- $\displaystyle\int \frac{(\ln x)^2}{x}\,dx$

$u = \ln x \longrightarrow du = \frac{1}{x}\,dx$

$$\therefore \quad \int \frac{(\ln x)^2}{x}\,dx = \int u^2\,du$$

$$= \frac{u^3}{3}$$

$$= \frac{1}{3}(\ln x)^3$$

$\bullet \int \sec 2\theta \; tg\, 2\theta \; d\theta$

$u = \sec 2\theta \rightarrow du = \frac{du}{d\theta} d\theta$

$= \underline{\frac{d}{d\theta} \sec 2\theta} \; d\theta$

$= \sec 2\theta \; tg\, 2\theta \; 2 \; d\theta$

$\therefore$

$\frac{1}{2} du = \sec 2\theta \; tg\, 2\theta \; d\theta$

$\therefore$

$\int \sec 2\theta \; tg\, 2\theta \; d\theta = \int \frac{1}{2} du$

$= \frac{1}{2} \int du$

$= \frac{1}{2} u$

$= \frac{1}{2} \sec 2\theta$ //

$$\int \frac{x}{\sqrt[4]{x+2}}\,dx$$

$u = x+2 \rightarrow du = dx$

$x = u-2$

$$\int \frac{x}{\sqrt[4]{x+2}}\,dx = \int \frac{(u-2)}{\sqrt[4]{u}}\,du$$

$$= \int \frac{u}{\sqrt[4]{u}}\,du - 2\int \frac{1}{\sqrt[4]{u}}\,du$$

$1 - \frac{1}{4}$

$$= \int u^{3/4}\,du - 2\int u^{-1/4}\,du$$

$\frac{3}{4} + 1 = \frac{7}{4}$

$$= \frac{u^{7/4}}{\frac{7}{4}} - 2\,\frac{u^{3/4}}{\frac{3}{4}}$$

$\frac{-1}{4} + 1 = \frac{3}{4}$

$$= \frac{4}{7}u^{7/4} - \frac{8}{3}u^{3/4}$$

$$= \frac{4}{7}(x+2)^{7/4} - \frac{8}{3}(x+2)^{3/4}$$