

Cálculo B - Lista 2

Integração de funções trigonométricas envolvendo potências de tangente, secante, cotangente e cosecante

Resolva cada uma das integrais a seguir

1. $\int \tan^2 5x \, dx$

2. $\int \cot^2 4t \, dt$

3. $\int x \cot^2 2x^2 \, dx$

4. $\int e^x \tan^2(e^x) \, dx$

5. $\int \cot^3 t \, dt$

6. $\int \tan^4 x \, dx$

7. $\int \tan^6 3x \, dx$

8. $\int \cot^5 2x \, dx$

9. $\int \sec^4 x \, dx$

10. $\int \csc^4 x \, dx$

11. $\int \csc^3 x \, dx$

12. $\int \sec^5 x \, dx$

13. $\int e^x \tan^4(e^x) \, dx$

14. $\int \frac{\sec^4(\ln x)}{x} \, dx$

15. $\int \tan^6 x \sec^4 x \, dx$

16. $\int \tan^5 x \sec^3 x \, dx$

17. $\int \cot^2 3x \csc^4 3x \, dx$

18. $\int (\sec 5x + \csc 5x)^2 \, dx$

19. $\int (\tan 2x + \cot 2x)^2 \, dx$

20. $\int \frac{1}{1+\cos x} \, dx$

21. $\int \frac{2\sin w - 1}{\cos^2 w} \, dw$

22. $\int \frac{\tan^3 \sqrt{x}}{\sqrt{x}} \, dx$

23. $\int \tan^5 3x \, dx$

24. $\int \frac{\tan^4 y}{\sec^5 y} dy$
25. $\int \frac{du}{1+\sec \frac{1}{2}u}$
26. $\int \frac{\csc^4 x}{\cot^2 x} dx$
27. $\int \frac{\sec^3 x}{\tan^4 x} dx$
28. $\int \frac{\sin^2 \pi x}{\cos^6 \pi x} dx$
29. $\int \frac{\tan^3(\ln x) \sec^6(\ln x)}{x} dx$
30. $\int \frac{\sec^4 w}{\sqrt{\tan w}} dw$
31. Mostre que $\int \cot x \csc^n x dx = -\frac{\csc^n x}{n} + C$ se $n \neq 0$.
32. Mostre que $\int \tan^n x dx = \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x dx$ se $n \geq 1, n \in N$.
33. Deduza uma fórmula similar aquela do exercício (31) para $\int \tan x \sec^n x dx$, se $n \neq 0$.
34. Deduza uma fórmula similar aquela do exercício (32) para $\int \cot^n x dx$, se $n \geq 1, n \in N$.

Resposta

1. $\frac{\tan 5x}{5} - x + C$
2. $-\frac{1}{4} \cot 4t - t + C$
3. $-\frac{1}{4} \cot 2x^2 - \frac{1}{2}x^2 + C$
4. $\tan e^x - e^x + C$
5. $-\frac{1}{2} \cot^2 t + \ln |\csc t| + C$
6. $\frac{1}{3} \tan^3 x - \tan x + x + C$
7. $\frac{1}{15} \tan^5 3x - \frac{1}{9} \tan^3 3x + \frac{1}{3} \tan 3x - x + C$
8. $-\frac{1}{8} \cot^4 2x + \frac{1}{4} \cot^2 2x + \frac{1}{2} \ln |\sin 2x| + C$
9. $\tan x + \frac{1}{3} \tan^3 x + C$
10. $-\cot x - \frac{1}{3} \cot^3 x + C$
11. $-\frac{1}{2} \csc x \cot x + \frac{1}{2} \ln |\csc x - \cot x| + C$
12. $\frac{1}{4} \sec^3 x \tan x + \frac{3}{8} \sec x \tan x + \frac{3}{8} \ln |\sec x + \tan x| + C$
13. $\frac{1}{3} \tan^3 e^x - \tan e^x + e^x + C$
14. $\tan \ln x + \frac{1}{3} \tan^3 \ln x + C$
15. $\frac{1}{7} \tan^7 x + \frac{1}{9} \tan^9 x + C$

16. $\frac{1}{7} \sec^7 x - \frac{2}{5} \sec^5 x + \frac{1}{3} \sec^3 x + C$
17. $-\frac{1}{15} \cot^5 3x - \frac{1}{9} \cot^3 3x + C$
18. $\frac{1}{5} \tan 5x - \frac{1}{5} \cot 5x + \frac{2}{5} \ln |\csc 10x - \cot 10x| + C$
19. $\frac{1}{2} \tan 2x - \frac{1}{2} \cot 2x + C$
20. $\tan \frac{x}{2} + C$
21. $2 \sec w - \tan w + C$
22. $\tan^2 \sqrt{x} - 2 \ln |\sec \sqrt{x}| + C$
23. $\frac{1}{12} \tan^4 3x - \frac{1}{6} \tan^2 3x + \frac{1}{3} \ln |\sec 3x| + C$
24. $\frac{1}{5} \sin^5 y + C$
25. $2(\cot \frac{u}{2} - \csc \frac{u}{2}) + u + C$
26. $-2 \cot 2x + C$
27. $-\frac{1}{3} \csc^3 x + C$
28. $\frac{1}{3\pi} \tan^3 \pi x + \frac{1}{5\pi} \tan^5 \pi x + C$
29. $\frac{1}{4} \tan^4 \ln x + \frac{1}{3} \tan^6 \ln x + \frac{1}{8} \tan^8 \ln x + C$
30. $2\sqrt{\tan w} + \frac{2}{5} \tan^{5/2} w + C$

Int. Funções Trigonométricas

1 1

Exercícios: Leithold pg 545)

$$1. \int \operatorname{tg}^2 5x \, dx = \quad (\operatorname{tg}^2 x = \sec^2 x - 1)$$

$$= \int (\sec^2 5x - 1) \, dx \quad \int \sec^2 x \, dx = \operatorname{tg} x$$

$$= \int \sec^2 5x \, dx - \int dx$$

$$\boxed{\int \operatorname{tg}^2 5x \, dx = \frac{\operatorname{tg} 5x}{5} - x + C}$$

$$2. \int \operatorname{cotg}^2 4t \, dt =$$

$$\operatorname{cotg}^2 x = \operatorname{cosec}^2 x - 1$$

$$= \int (\operatorname{cosec}^2 4t - 1) \, dt$$

$$\int \operatorname{cosec}^2 x \, dx = -\operatorname{cotg} x$$

$$= \int \operatorname{cosec}^2 4t \, dt - \int dt$$

$$\boxed{\int \operatorname{cotg}^2 4t \, dt = -\frac{\operatorname{cotg} 4t}{4} - t + C}$$

$$3. \int x \operatorname{cotg}^2 2x^2 \, dx$$

$$2x^2 = u \quad \rightarrow \quad du = 4x \, dx$$

$$\therefore \int x \operatorname{cotg}^2 2x^2 \, dx = \int \operatorname{cotg}^2 u \, \frac{du}{4}$$

$$= \frac{1}{4} \int \operatorname{cotg}^2 u \, du$$

$$\begin{aligned}
 &= \frac{1}{4} \int \cot^2 u \, du \\
 &= \frac{1}{4} \int (\csc^2 u - 1) \, du \\
 &= \frac{1}{4} \int \csc^2 u \, du - \frac{1}{4} \int du \\
 &= \frac{1}{4} (-\cot u) - \frac{1}{4} u + C \\
 &= -\frac{1}{4} \cot 2x^2 - \frac{1}{4} 2x^2 + C
 \end{aligned}$$

$$\int x \cot^2 2x^2 \, dx = -\frac{1}{4} \cot 2x^2 - \frac{1}{8} x^2 + C$$

$$4. \int e^x \log^2(e^x) \, dx$$

$$\text{Seja } u = e^x \rightarrow du = e^x \, dx$$

Seja,

$$\int e^x \log^2(e^x) \, dx = \int \log^2 u \, du$$

$$= \int (\csc^2 u - 1) \, du$$

$$= \int \csc^2 u \, du - \int du$$

$$= -\cot u - u + C$$

$$\int e^x \log^2(e^x) \, dx = -\log e^x - e^x + C$$

5. $\int \cot^3 t \, dt$

$\int \cot^n t \operatorname{cosec}^2 t \, dt = -\frac{\cot^{n+1} t}{n+1}$

$= \int \operatorname{cosec}^2 t \cot t \, dt$

$= \int (\operatorname{cosec}^2 t - 1) \cot t \, dt$

$= \int \operatorname{cosec}^2 t \cot t \, dt - \int \cot t \, dt$

$= -\frac{1}{2} \cot^2 t - (-\ln |\operatorname{cosec} t|) + C$

$\int \cot^3 t \, dt = -\frac{1}{2} \cot^2 t + \ln |\operatorname{cosec} t| + C$

6. $\int \operatorname{tg}^4 x \, dx = \int \operatorname{tg}^2 x \operatorname{tg}^2 x \, dx$

$= \int (\operatorname{sec}^2 x - 1) \operatorname{tg}^2 x \, dx$

$= \int \operatorname{sec}^2 x \operatorname{tg}^2 x \, dx - \int \operatorname{tg}^2 x \, dx$

$= \frac{\operatorname{tg}^3 x}{3} - \int (\operatorname{sec}^2 x - 1) \, dx$

$= \frac{\operatorname{tg}^3 x}{3} - \int \operatorname{sec}^2 x \, dx + \int dx$

$\int \operatorname{tg}^4 x \, dx = \frac{\operatorname{tg}^3 x}{3} - \operatorname{tg} x + x + C$

Ans: $\int \operatorname{tg}^{2m} x \, dx = \frac{\operatorname{tg}^{2m-1} x}{2m-1} - \frac{\operatorname{tg}^{2m-3} x}{2m-3} + x$

1/7

$$7. \int \log^6 3x dx = \left. \int \log^m kx \cdot \log^2 kx dx \right\} = \frac{\log^{m+1} kx}{k(m+1)}$$

$$= \int \log^2 3x \log^4 3x dx$$

$$= \int (\log^2 3x - 1) \log^4 3x dx$$

$$= \int \log^2 3x \log^4 3x dx - \int \log^4 3x dx$$

$$= \frac{\log^5 3x}{15} - \int \log^2 3x \log^2 3x dx$$

exercício 6

$$= \frac{\log^5 3x}{15} - \int (\log^2 3x - 1) \log^2 3x dx$$

$$= \frac{\log^5 3x}{15} - \int \log^2 3x \log^2 3x dx + \int \log^2 3x dx$$

$$= \frac{\log^5 3x}{15} - \frac{\log^3 3x}{9} + \int (\log^2 3x - 1) dx$$

$$= \frac{\log^5 3x}{15} - \frac{\log^3 3x}{9} + \int \log^2 3x dx - \int dx$$

$$\int \log^6 3x dx = \frac{\log^5 3x}{15} - \frac{\log^3 3x}{9} + \frac{\log 3x}{3} - x + C$$

$$\begin{aligned}
 8. \int \cot^5 2x \, dx & \quad \left. \int \cot^m kx \operatorname{cosec}^2 kx \, dx = \right. \\
 & = \int \cot^2 2x \cot^3 2x \, dx \quad \left. = -\frac{\cot^{m+1} kx}{(m+1)k} \right. \\
 & = \int (\operatorname{cosec}^2 2x - 1) \cot^3 2x \, dx \\
 & = \int \operatorname{cosec}^2 2x \cot^3 2x \, dx - \int \cot^3 2x \, dx \\
 & = -\frac{\cot^4 2x}{8} - \int \cot^2 2x \cot 2x \, dx \\
 & = -\frac{\cot^4 2x}{8} - \int (\operatorname{cosec}^2 2x - 1) \cot 2x \, dx \\
 & = -\frac{\cot^4 2x}{8} - \int \operatorname{cosec}^2 2x \cot 2x \, dx + \int \cot 2x \, dx \\
 & = -\frac{\cot^4 2x}{8} - \frac{1}{4} \cot^2 2x + \frac{\ln |\sin 2x|}{2} + C
 \end{aligned}$$

$$\int \cot^5 2x \, dx = -\frac{\cot^4 2x}{8} + \frac{\cot^2 2x}{4} + \frac{\ln |\sin 2x|}{2} + C$$

$$\begin{aligned}
 9. \int \operatorname{cosec}^4 x \, dx & \quad \left. \operatorname{cosec}^{2k} x = \operatorname{cosec}^{2k-2} x \operatorname{cosec}^2 x \right. \\
 & \quad \left. = (\operatorname{cosec}^2 x)^{k-1} \operatorname{cosec}^2 x \right. \\
 & = \int \operatorname{cosec}^2 x \operatorname{cosec}^2 x \, dx \\
 & = \int (1 + \tan^2 x) \operatorname{cosec}^2 x \, dx \\
 & = \int \operatorname{cosec}^2 x \, dx + \int \tan^2 x \operatorname{cosec}^2 x \, dx
 \end{aligned}$$

$$\int \operatorname{cosec}^4 x \, dx = \tan x + \frac{\tan^3 x}{3} + C$$

$$10. \int \operatorname{cosec}^4 x \, dx =$$

$$= \int \operatorname{cosec}^2 x \operatorname{cosec}^2 x \, dx$$

$$= \int (1 + \cot^2 x) \operatorname{cosec}^2 x \, dx$$

$$= \int \operatorname{cosec}^2 x \, dx + \int \cot^2 x \operatorname{cosec}^2 x \, dx$$

$$= -\cot x + \frac{\cot^3 x}{3} + C$$

$$\int \operatorname{cosec}^4 x \, dx = -\cot x - \frac{1}{3} \cot^3 x + C$$

(caso 3 : usar anti. por partes)

$$11. \int \operatorname{cosec}^3 x \, dx$$

$$u = \operatorname{cosec} x \rightarrow du = -\operatorname{cosec} x \cot x \, dx$$

$$dv = \operatorname{cosec}^2 x \, dx \rightarrow v = -\cot x$$

$$\int \operatorname{cosec}^3 x \, dx = -\operatorname{cosec} x \cot x - \int \cot x \operatorname{cosec} x \cot x \, dx$$

$$= -\operatorname{cosec} x \cot x - \int \operatorname{cosec} x \cot^2 x \, dx$$

$$= -\operatorname{cosec} x \cot x - \int \operatorname{cosec} x (\operatorname{cosec}^2 x - 1) \, dx$$

$$\int \operatorname{cosec}^3 x \, dx = -\operatorname{cosec} x \cot x - \int \operatorname{cosec}^3 x \, dx + \int \operatorname{cosec} x \, dx$$

$\frac{1}{7}$

$$4 \int \sec^5 x \, dx = \sec^3 x \operatorname{tg} x + 3 \int \sec^3 x \, dx$$

(für obliche)

$$4 \int \sec^5 x \, dx = \sec^3 x \operatorname{tg} x + 3 \left(\frac{1}{2} \sec x \operatorname{tg} x + \frac{1}{2} \ln |\sec x + \operatorname{tg} x| \right)$$

$$4 \int \sec^5 x \, dx = \sec^3 x \operatorname{tg} x + \frac{3}{2} \sec x \operatorname{tg} x + \frac{3}{2} \ln |\sec x + \operatorname{tg} x|$$

$$\int \sec^5 x \, dx = \frac{1}{4} \sec^3 x \operatorname{tg} x + \frac{3}{8} \sec x \operatorname{tg} x + \frac{3}{8} \ln |\sec x + \operatorname{tg} x| + C$$

 $\frac{1}{5}$

$$13) \int e^x \operatorname{tg}^4(e^x) \, dx$$

$$u = e^x \rightarrow du = e^x \, dx$$

$$\int \underbrace{e^x}_{du} \operatorname{tg}^4(\underbrace{e^x}_u) \, dx = \int \operatorname{tg}^4(u) \, du$$

$$= \left(\frac{\operatorname{tg}^3 u}{3} - \operatorname{tg} u + u \right) + C \quad \left. \vphantom{\int} \right\} \text{ex. 6}$$

$$\int e^x \operatorname{tg}^4(e^x) \, dx = \frac{\operatorname{tg}^3 e^x}{3} - \operatorname{tg} e^x + e^x + C$$

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$$14. \int \frac{\sec^4(\ln x)}{x} dx$$

$$u = \ln x \rightarrow du = \frac{1}{x} dx$$

$$\int \frac{\sec^4(\ln x)}{x} dx = \int \sec^4 u du$$

ec. 9

$$= \left(\operatorname{tg} u + \frac{\operatorname{tg}^3 u}{3} + C \right)$$

$$\int \frac{\sec^4(\ln x)}{x} dx = \operatorname{tg} \ln x + \frac{\operatorname{tg}^3 \ln x}{3} + C$$

(104) $\int \operatorname{tg}^m x \sec^{2m} x dx$

$$15. \int \operatorname{tg}^6 x \sec^8 x dx$$

$$= \int \operatorname{tg}^6 x \sec^2 x \sec^2 x dx$$

$$= \int \operatorname{tg}^6 x \sec^2 x (1 + \operatorname{tg}^2 x) dx$$

$$= \int \operatorname{tg}^6 x \sec^2 x dx + \int \operatorname{tg}^8 x \sec^2 x dx$$

$$= \frac{\operatorname{tg}^7 x}{7} + \frac{\operatorname{tg}^9 x}{9} + C$$

$$\int \operatorname{tg}^6 x \sec^8 x dx = \frac{\operatorname{tg}^7 x}{7} + \frac{\operatorname{tg}^9 x}{9} + C$$

$$16. \int \operatorname{tg}^5 x \operatorname{sec}^3 x dx =$$

$$= \int \underbrace{\operatorname{tg}^4 x \operatorname{sec}^2 x}_{\substack{d \operatorname{sec} x = \operatorname{sec} x \operatorname{tg} x \\ dx}} \operatorname{tg} x \operatorname{sec} x dx$$

$$= \int (\operatorname{sec}^2 x - 1)^2 \operatorname{sec}^2 x \operatorname{tg} x \operatorname{sec} x dx$$

$$= \int (\operatorname{sec}^4 x - 2 \operatorname{sec}^2 x + 1) \operatorname{sec}^2 x \operatorname{tg} x \operatorname{sec} x dx$$

$$= \int \underbrace{\operatorname{sec}^6 x \operatorname{tg} x \operatorname{sec} x dx}_{\substack{d \operatorname{sec} x = \operatorname{sec} x \operatorname{tg} x \\ dx}} - 2 \int \operatorname{sec}^4 x \operatorname{tg} x \operatorname{sec} x dx + \int \operatorname{sec}^2 x \operatorname{tg} x \operatorname{sec} x dx$$

$$= \frac{\operatorname{sec}^7 x}{7} - 2 \left(\frac{\operatorname{sec}^5 x}{5} \right) + \frac{\operatorname{sec}^3 x}{3} + C$$

$$\int \operatorname{tg}^5 x \operatorname{sec}^3 x dx = \frac{1}{7} \operatorname{sec}^7 x - \frac{2}{5} \operatorname{sec}^5 x + \frac{1}{3} \operatorname{sec}^3 x + C$$

$$17. \int \operatorname{cotg}^2 3x \operatorname{cosec}^4 3x dx =$$

$$= \int \operatorname{cotg}^2 3x \operatorname{cosec}^2 3x \operatorname{cosec}^2 3x dx$$

$$= \int \operatorname{cotg}^2 3x \operatorname{cosec}^2 3x (1 + \operatorname{cotg}^2 3x) dx$$

$$= \int \operatorname{cotg}^2 3x \operatorname{cosec}^2 3x dx + \int \operatorname{cotg}^4 3x \operatorname{cosec}^2 3x dx$$

$$= -\frac{\operatorname{cotg}^3 3x}{9} - \frac{\operatorname{cotg}^5 3x}{15} + C$$

tilibra

$$\therefore \int \operatorname{cotg}^2 3x \operatorname{cosec}^4 3x dx = -\frac{1}{15} \operatorname{cotg}^5 3x - \frac{1}{9} \operatorname{cotg}^3 3x + C$$

18. $\int (\sec 5x + \csc 5x)^2 dx$

$= \int (\sec^2 5x + 2 \sec 5x \csc 5x + \csc^2 5x) dx$

$= \int \sec^2 5x dx + 2 \int \sec 5x \csc 5x dx + \int \csc^2 5x dx$

$(*) = \int \sec^2 5x dx = \frac{\tan 5x}{5}$

$(**) = 2 \int \sec 5x \csc 5x dx$

$(2 \cos 2x = \sec x \csc x)$

$= 2 \int 2 \operatorname{cosec} 10x dx$

$\sin 2x = 2 \sin x \cos x$

$= 4 \int \operatorname{cosec} 10x dx$

$\frac{1}{\sin 2x} = \frac{1}{2 \sin x \cos x}$

$= 4 \frac{\ln |\operatorname{cosec} 10x - \cot 10x|}{10}$

$\operatorname{cosec} 2x = \frac{1}{\sin 2x}$

$2 \operatorname{cosec} 2x = \operatorname{cosec} 4x$

$= \frac{2}{5} \ln |\operatorname{cosec} 10x - \cot 10x|$

$(***) = \int \csc^2 5x dx = -\frac{\cot 5x}{5} + C$

$\int (\sec 5x + \csc 5x)^2 dx = \frac{\tan 5x}{5} - \frac{\cot 5x}{5} + \frac{2}{5} \ln |\operatorname{cosec} 10x - \cot 10x| + C$

19. $\int (\operatorname{tg} 2x + \operatorname{ctg} 2x)^2 dx =$

$$= \int (\operatorname{tg}^2 2x + 2 \operatorname{tg} 2x \operatorname{ctg} 2x + \operatorname{ctg}^2 2x) dx$$

$$= \int \operatorname{tg}^2 2x dx + 2 \int \operatorname{tg} 2x \operatorname{ctg} 2x dx + \int \operatorname{ctg}^2 2x dx$$

(*)

(**)

(***)

$$(*) = \int \operatorname{tg}^2 2x dx = \int (\operatorname{ctg}^2 2x - 1) dx$$

$$= \int \operatorname{ctg}^2 2x dx - \int dx$$

$$= \frac{\operatorname{ctg} 2x}{2} - x //$$

$$(**) = 2 \int \operatorname{tg} 2x \operatorname{ctg} 2x dx$$

$$= 2 \int dx = 2x //$$

$$(***) = \int \operatorname{ctg}^2 2x dx = \int (\operatorname{tg}^2 2x - 1) dx$$

$$= -\frac{\operatorname{ctg} 2x}{2} - x //$$

$$\int (\operatorname{tg} 2x + \operatorname{ctg} 2x)^2 dx = \frac{\operatorname{ctg} 2x}{2} - x + 2x - \frac{\operatorname{ctg} 2x}{2} - x$$

$$\int (\operatorname{tg} 2x + \operatorname{ctg} 2x)^2 dx = \frac{\operatorname{ctg} 2x}{2} - \frac{\operatorname{ctg} 2x}{2} + C$$

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20. $\int \frac{dx}{1+\cos x}$

Veremos que $\frac{1+\cos 2x}{2} = \cos^2 x$

Dasí, $\| 1+\cos x = 2 \cos^2 \frac{x}{2} \|$

$\therefore \int \frac{dx}{1+\cos x} = \int \frac{dx}{2 \cos^2 \frac{x}{2}} = \frac{1}{2} \int \sec^2 \frac{x}{2} dx$
 $= \frac{1}{2} (\tan \frac{x}{2}) + C$

$\int \frac{dx}{1+\cos x} = \tan \frac{x}{2} + C$

21. $\int \frac{2 \sin w - 1}{\cos^2 w} dw$

$= 2 \int \frac{\sin w}{\cos^2 w} dw - \int \frac{1}{\cos^2 w} dw$

$= 2 \int \tan w \sec w dw - \int \sec^2 w dw$

$= 2 \sec w - \tan w$

$\int \frac{2 \sin w - 1}{\cos^2 w} dw = 2 \sec w - \tan w + C$

16. 22. $\int \frac{t_0^3 \sqrt{x} dx}{\sqrt{x}}$

17. Sep $u = \sqrt{x} \rightarrow du = \frac{1}{2\sqrt{x}} dx$

18. $\int \frac{t_0^3 \sqrt{x} dx}{\sqrt{x}} = \int t_0^3 u^2 du$

19. $= 2 \int t_0^3 u du$

20. $\textcircled{*} = \int t_0^3 u du = \int t_0^2 u t_0 u du$

21. $= \int (u^2 u - 1) t_0 u du$

22. $= \int u^2 u t_0 u du - \int t_0 u du$

23. $= \frac{t_0^2 u^3}{3} - \ln|u| + C$

24. $= \frac{t_0^2 \sqrt{x}^3}{3} - \ln|\sqrt{x}| + C$

2 $\int \frac{t_0^3 \sqrt{x} dx}{\sqrt{x}} = 2 \left[\frac{t_0^2 \sqrt{x}^3}{3} - \ln|\sqrt{x}| \right] + C$

2 $\int \frac{t_0^3 \sqrt{x} dx}{\sqrt{x}} = t_0^2 \sqrt{x}^3 - 2 \ln|\sqrt{x}| + C$

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23. $\int \log^5 3x \, dx$

$$= \int \log^2 3x \cdot \log^3 3x \, dx$$

$$= \int (2x^2 3x - 1) \log^3 3x \, dx$$

$$= \int 2x^2 3x \log^3 3x - \int \log^3 3x \, dx$$

$$= \frac{\log^4 3x}{12} - \int \log^3 3x \, dx$$

$$= \frac{\log^4 3x}{12} - \int \log^2 3x \cdot \log 3x \, dx$$

$$= \frac{\log^4 3x}{12} - \int (2x^2 3x - 1) \log 3x \, dx$$

$$= \frac{\log^4 3x}{12} - \int 2x^2 3x \log 3x \, dx + \int \log 3x \, dx$$

$$\int \log^5 3x \, dx = \frac{\log^4 3x}{12} - \frac{\log^2 3x}{6} + \frac{\ln |2x^2 3x + 1|}{3} + C$$

24. $\int \frac{\log^4 y}{\cos y} \, dy$

$$= \int \log^4 y \cdot \frac{1}{\cos y} \, dy$$

$$= \int \frac{\sin^4 y}{\cos^4 y} \cdot \frac{1}{\cos y} \, dy$$

$$= \int \frac{\sin^4 y}{\cos^5 y} \, dy = \int \sin^4 y \cdot \sec^5 y \, dy$$

$$= \frac{\sin^5 y}{5} + C$$

25. $\int \frac{du}{1 + \operatorname{sech} \frac{u}{2}}$

18. $= \int \frac{(1 - \operatorname{sech} \frac{u}{2})}{(1 + \operatorname{sech} \frac{u}{2})(1 - \operatorname{sech} \frac{u}{2})} du$

19. $= \int \frac{(1 - \operatorname{sech} \frac{u}{2})}{1 - \operatorname{sech}^2 \frac{u}{2}} du$ ($1 + \tanh^2 \frac{u}{2} = \operatorname{sech}^2 \frac{u}{2}$)

20. $= \int \frac{(1 - \operatorname{sech} \frac{u}{2})}{-\tanh^2 \frac{u}{2}} du$

21. $= - \int \frac{1 - \operatorname{sech} \frac{u}{2}}{\tanh^2 \frac{u}{2}} du$

22. $= - \int \frac{1}{\tanh^2 \frac{u}{2}} du + \int \frac{\operatorname{sech} \frac{u}{2}}{\tanh^2 \frac{u}{2}} du$

23. $= - \int \operatorname{cotg}^2 \frac{u}{2} du + \int \underbrace{\operatorname{sech} \frac{u}{2} \operatorname{cotg}^2 \frac{u}{2}} du$

24. $= - \int \operatorname{cotg}^2 \frac{u}{2} du + \int \frac{1}{\cosh \frac{u}{2}} \frac{\operatorname{cotg} \frac{u}{2}}{\sin^2 \frac{u}{2}} du$

25. $= - \int \operatorname{cotg}^2 \frac{u}{2} du + \int \operatorname{cotg} \frac{u}{2} \operatorname{cosec} \frac{u}{2} du$

Mos

26. $\int \textcircled{*} = - \int \operatorname{cotg}^2 \frac{u}{2} du = - \int (\operatorname{cosec}^2 \frac{u}{2} - 1) du$

27. $= - \int \operatorname{cosec}^2 \frac{u}{2} du + \int du$

28. $= - (-\operatorname{cotg} \frac{u}{2}) + u$

29. $= 2 \operatorname{cotg} \frac{u}{2} + u //$

$$\textcircled{*} = \int \cotg \frac{u}{2} \operatorname{cosec} \frac{u}{2} du =$$

$$= -2 \operatorname{cosec} \frac{u}{2}$$

Par

$$\int \frac{du}{1 + \operatorname{cosec} \frac{u}{2}} = 2 \cotg \frac{u}{2} + u - 2 \operatorname{cosec} \frac{u}{2} + C$$

$$\int \frac{du}{1 + \operatorname{cosec} \frac{u}{2}} = 2 \left(\cotg \frac{u}{2} - \operatorname{cosec} \frac{u}{2} \right) + u + C$$

obs.:

$$\cotg \frac{u}{2} - \operatorname{cosec} \frac{u}{2} = \frac{\cos \frac{u}{2}}{\sin \frac{u}{2}} - \frac{1}{\sin \frac{u}{2}} = \frac{\cos \frac{u}{2} - 1}{\sin \frac{u}{2}}$$

$$\left. \begin{aligned} \frac{1 - (\cos) 2x}{2} &= \sin^2 x \\ &= - \frac{1 - (\cos) \frac{u}{2}}{\sin \frac{u}{2}} \end{aligned} \right\}$$

$$= - \frac{2 \sin^2 \frac{u}{4}}{\sin \frac{u}{2}}$$

$$\sin \frac{u}{2} = 2 \sin \frac{u}{4} \cos \frac{u}{4}$$

$$= - \frac{\cancel{2} \sin^2 \frac{u}{4}}{\cancel{2} \sin \frac{u}{4} \cos \frac{u}{4}} = - \frac{\sin \frac{u}{4}}{\cos \frac{u}{4}}$$

Por

$$\int \frac{du}{1 + \operatorname{cosec} \frac{u}{2}} = -2 \operatorname{tg} \frac{u}{4} + u + C$$

1 1

$$26. \int \frac{\operatorname{cosec}^4 x}{\cot^2 x} dx$$

$$= \int \operatorname{cosec}^4 x \tan^2 x dx$$

$$= \int \frac{1}{\sin^4 x} \frac{\sin^2 x}{\cos^2 x} dx$$

$$= \int \frac{1}{\sin^2 x \cos^2 x} dx =$$

$$\sin 2x = 2 \sin x \cos x$$

$$= \int \frac{1}{(\sin x \cos x)^2} dx$$

$$= \int \frac{1}{\left(\frac{\sin 2x}{2}\right)^2} dx$$

$$= 4 \int \frac{1}{\sin^2 2x} dx$$

$$= 4 \int \operatorname{cosec}^2 2x dx$$

$$= 4 (-) \frac{\cot 2x}{2}$$

$$\int \frac{\operatorname{cosec}^4 x}{\cot^2 x} dx \equiv -2 \cot 2x + C$$

11

$$27. \int \frac{\sec^3 x}{\tan^4 x} dx$$

$$= \int \sec^3 x \cot^4 x dx$$

$$= \int \frac{1}{\cos^3 x} \frac{\cos^4 x}{\sin^4 x} dx$$

$$= \int \frac{\cos x}{\sin x} \frac{1}{\sin^3 x} dx$$

$$= \int \cot x \operatorname{cosec}^3 x dx$$

$$= \int \cot x \operatorname{cosec} x \operatorname{cosec}^2 x dx$$

$$\int \frac{\sec^3 x}{\tan^4 x} dx = -\frac{\operatorname{cosec}^3 x}{3} + C$$

$$28. \int \frac{\sin^2 \pi x}{\cos^6 \pi x} dx$$

$$= \int \frac{\sin^2 \pi x}{\cos^2 \pi x} \frac{1}{\cos^4 \pi x} dx$$

$$= \int \tan^2 \pi x \sec^4 \pi x dx$$

$$= \int \tan^2 \pi x \sec^2 \pi x \sec^2 \pi x dx$$

$$= \int \tan^2 \pi x \sec^2 \pi x (1 + \tan^2 \pi x) dx$$

$$= \int \tan^2 \pi x \sec^2 \pi x dx + \int \tan^4 \pi x \sec^2 \pi x dx$$

$$= \frac{\tan^3 \pi x}{3\pi} + \frac{\tan^5 \pi x}{5\pi} + C //$$

16. 1 1

29. $\int \frac{\tan^3(\ln x) \sec^6(\ln x)}{x} dx$

8. Let $u = \ln x \rightarrow du = \frac{1}{x} dx$

9 $\int \frac{\tan^3(\ln x) \sec^6(\ln x)}{x} dx =$

$= \int \tan^3 u \sec^6 u du$

$= \int \tan^2 u \sec^4 u \sec^2 u du$

$= \int \tan^2 u (1 + \tan^2 u)^2 \sec^2 u du$

$= \int \tan^2 u (1 + 2\tan^2 u + \tan^4 u) \sec^2 u du$

$= \int (\tan^2 u \sec^2 u + 2 \tan^4 u \sec^2 u + \tan^6 u \sec^2 u) du$

$= \frac{\tan^4 u}{4} + 2 \frac{\tan^6 u}{6} + \frac{\tan^8 u}{8} + C$

$= \frac{\tan^4 u}{4} + \frac{\tan^6 u}{3} + \frac{\tan^8 u}{8} + C$

$= \frac{\tan^4 \ln x}{4} + \frac{\tan^6 \ln x}{3} + \frac{\tan^8 \ln x}{8} + C$

1 / 1

30. $\int \frac{\sec^4 w}{\sqrt{\tan w}} dw$

$$= \int \sec^4 w \tan^{-1/2} w dw$$

$$= \int \sec^2 w \underbrace{\sec^2 w}_{1 + \tan^2 w} \tan^{-1/2} w dw$$

$$= \int \sec^2 w (1 + \tan^2 w) \tan^{-1/2} w dw$$

$$= \int \sec^2 w \tan^{-1/2} w dw + \int \sec^2 w \tan^{3/2} w dw$$

$$\int \frac{\sec^4 w}{\sqrt{\tan w}} dw = 2 \tan^{1/2} w + \frac{2}{5} \tan^{5/2} w + C$$

40. $\int \cot x \csc^n x dx = -\frac{\csc^{n-1} x}{n} + C, n \neq 0$

De fato:

$$\int \cot x \csc^n x dx = \int \cot x \csc x \csc^{n-1} x dx$$

$$= -\frac{\csc^{n-1} x}{n} + C$$

$$41. \int \operatorname{tg}^n x \, dx = \frac{\operatorname{tg}^{n-1} x}{n-1} - \int \operatorname{tg}^{n-2} x \, dx, \quad n > 1$$

Do fato

$$n > 1 \rightarrow n \geq 2.$$

Portanto,

$$\int \operatorname{tg}^n x \, dx = \int \operatorname{tg}^{n-2} x \operatorname{tg}^2 x \, dx$$

$$= \int \operatorname{tg}^{n-2} x (\sec^2 x - 1) \, dx$$

$$= \int \operatorname{tg}^{n-2} x \sec^2 x \, dx - \int \operatorname{tg}^{n-2} x \, dx$$

$$\boxed{\int \operatorname{tg}^n x \, dx = \frac{\operatorname{tg}^{n-1} x}{n-1} - \int \operatorname{tg}^{n-2} x \, dx}$$

$$42. \int \operatorname{tg} x \sec^n x \, dx, \quad n \neq 0$$

tenho:
$$\int \operatorname{tg} x \sec^n x \, dx = \int \operatorname{tg} x \sec x \sec^{n-1} x \, dx$$

$$= \frac{\sec^n x}{n} + C$$

$$\begin{aligned} 43. \int \operatorname{cotg}^n x \, dx & \stackrel{n \geq 2}{=} \int \operatorname{cotg}^{n-2} x \operatorname{cotg}^2 x \, dx = \int \operatorname{cotg}^{n-2} x (\operatorname{cosec}^2 x - 1) \, dx \\ & = \int \operatorname{cotg}^{n-2} x \operatorname{cosec}^2 x \, dx - \int \operatorname{cotg}^{n-2} x \, dx \\ & = -\frac{\operatorname{cotg}^{n-1} x}{n-1} - \int \operatorname{cotg}^{n-2} x \, dx \end{aligned}$$

$$\boxed{\int \operatorname{cotg}^n x \, dx = -\frac{\operatorname{cotg}^{n-1} x}{n-1} - \int \operatorname{cotg}^{n-2} x \, dx}$$