1. Seja $f(x) = \dfrac{x^2+1}{x^2-1}$.

1.5 i) Determine os extremos locais da função $f(x)$, caso existam.

ii) Determine os intervalos onde $f(x)$ é crescente, decrescente.

1.5 iii) Determine os intervalos onde o gráfico de $f(x)$ tem concavidade para cima, e concavidade para baixo.

1.0 iv) Análise o comportamento da função $f(x)$ quando $x \to +\infty$, $x \to -\infty$, $x \to 1$, $x \to -1$.

0.5 v) A partir dos dados obtidos em (i) a (iv) construa o gráfico da função $f(x)$

1.0 2. Seja $f(x) = x\sqrt{x+1}$; $-1 \leq x \leq 1$.
Determine os extremos globais da função $f(x)$.

1.0 3. Usando a regra de L'Hopital, calcule $\lim_{x \to -\infty} x e^x$.

1.5 4. Calcule $\int \sin\sqrt{x}\, dx$

1.5 5. Calcule $\int \frac{1}{x\sqrt{\ln x}}\,dx$.

1.1 6. Calcule

$$\int_{-2}^{0} e^{|x+1|}\,dx$$

8.1

1.1

$$f(x) = \frac{x^2+1}{x^2-1}$$

$\text{Dom } f = \mathbb{R} - \{-1, 1\}$

$$\begin{cases} f(x) = 0 \ : \ \dfrac{x^2+1}{x^2-1} = 0 \ \Rightarrow \ \nexists\, x \\ f(0) = -1 \end{cases}$$

$$f'(x) = \frac{2x(x^2-1) - (x^2+1)\,2x}{(x^2-1)^2}$$

$$= \frac{\cancel{2x^3} - 2x - \cancel{2x^3} - 2x}{(x^2-1)^2} = \frac{-4x}{(x^2-1)^2}$$

$$\begin{cases} f'(x) = 0 \ : \ x = 0 \\ f'(x) \ \nexists \ : \ x = \pm 1 \end{cases}$$

$$\begin{array}{ccccccc} + & \nexists & + & 0 & - & \nexists & - \\ & -1 & & 0 & & 1 & \end{array}$$

1.0

$f(0) = -1$ é máximo local

$f(x)$ é crescente em $(-\infty, -1) \cup (-1, 0)$

$f(x)$ é decrescente em $(0, 1) \cup (1, +\infty)$

0.5

$$f''(x) = \frac{-4(x^2-1)^2 + 4x\,2(x^2-1)\,2x}{(x^2-1)^4}$$

$$= \frac{-4(x^2-1)^2 + 16x^2(x^2-1)}{(x^2-1)^4}$$

$$= \frac{4(x^2-1)\left[-(x^2-1) + 4x^2\right]}{(x^2-1)^4}$$

$$= \frac{4\cancel{(x^2-1)}\left[3x^2+1\right]}{(x^2-1)^{\cancel{4}\,3}}$$

$$= \frac{4(3x^2+1)}{(x^2-1)^3}$$ 0.5

$$\begin{cases} f''(x) = 0 & : \nexists x \\ f''(x) \nexists & : x = \pm 1 \end{cases}$$

$$\begin{array}{ccccccc} + & \nexists & - & \nexists & + & & f''(x),\ x^2-1 \\ & -1 & & 1 & & & \end{array}$$

$$\begin{cases} x = -1 & \text{pto. inflexión} \\ x = 1 & \text{pto inflexión} \end{cases}$$ 0.5

$f''(x) > 0$: $f(x)$ é côncava p/ cima em $(-\infty, -1) \cup (1, +\infty)$

0.5

$f''(x) < 0$: $f(x)$ é côncava p/ baixo em $(-1, 1)$

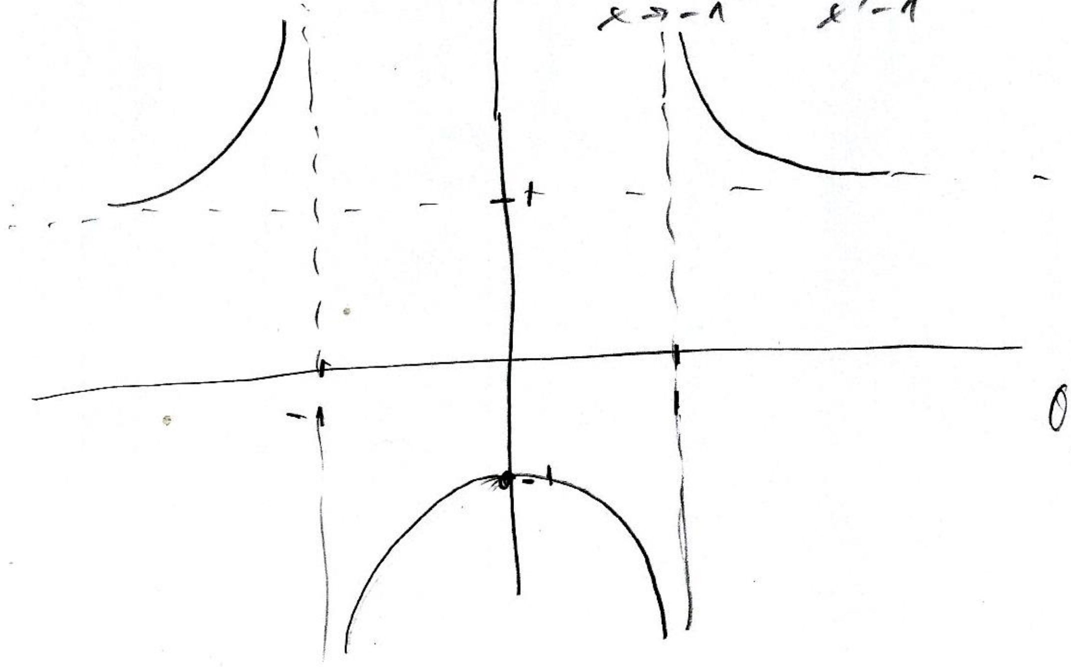
$$\lim_{x \to +\infty} \frac{x^2+1}{x^2-1} = \lim_{x \to \infty} \frac{1+\frac{1}{x^2}}{1-\frac{1}{x^2}} = 1$$

$$\lim_{x \to -\infty} \frac{x^2+1}{x^2-1} = \lim_{x \to -\infty} \frac{1+\frac{1}{x^2}}{1-\frac{1}{x^2}} = 1$$

$$\lim_{x \to 1} \frac{x^2+1}{x^2-1} \nearrow \lim_{x \to 1^+} \frac{x^2+1}{x^2-1} = +\infty$$
$$\searrow \lim_{x \to 1^-} \frac{x^2+1}{x^2-1} = -\infty$$

$$\lim_{x \to -1} \frac{x^2+1}{x^2-1} \nearrow \lim_{x \to -1^+} \frac{x^2+1}{x^2-1} = -\infty$$
$$\searrow \lim_{x \to -1^-} \frac{x^2+1}{x^2-1} = +\infty$$

1.0



0.5

2.

$f(x) = x\sqrt{x+1} \quad , \quad -1 \leq x \leq 1$

$f'(x) = \sqrt{x+1} + \frac{x}{2} \cdot \frac{1}{\sqrt{x+1}}$

$f'(x) = 0 \quad : \quad \sqrt{x+1} + \frac{x}{2\sqrt{x+1}} = 0$

$$\frac{2(x+1)+x}{2\sqrt{x+1}} = 0$$

$$\frac{3x+2}{2\sqrt{x+1}} = 0$$

$3x+2 = 0 \quad \therefore \quad // x = -\frac{2}{3} // \quad \underline{0.5}$

$f'(x) \nexists \quad : \quad \underline{\underline{x=-1}}$

x	$f(x) = x\sqrt{x+1}$
-1	0
1	$\sqrt{2}$ → max. absoluto
$-\frac{2}{3}$	$-\frac{2}{3\sqrt{3}}$ → min. absoluto

0.5 //

$f(-\frac{2}{3}) = -\frac{2}{3}\sqrt{-\frac{2}{3}+1}$

$= -\frac{2}{3}\sqrt{\frac{1}{3}}$

$= -\frac{2}{3\sqrt{3}}$

3.

L'Hopital

$$\lim_{x \to -\infty} x e^{x} = \lim_{x \to -\infty} \frac{x}{e^{-x}}$$

$$\overset{\text{L'Hop.}}{=} \lim_{x \to -\infty} \frac{1}{-e^{-x}}$$

$$= \lim_{x \to -\infty} -e^{x}$$

$$= 0$$

1.0

4.

$\int \sin\sqrt{x}\, dx$

$z = \sqrt{x}$, $dz = \frac{1}{2\sqrt{x}}\, dx$

$= \frac{1}{2z}\, dx$

$\therefore \quad 2z\, dz = dx$

$\int \sin\sqrt{x}\, dx = \int \sin z \; 2z\, dz$

$= 2\int z \sin z \, dz$

$\int z \sin z \, dz$ 0.5

$\begin{cases} u = z \longrightarrow du = dz \\ dv = \sin z\, dz \rightarrow v = -\cos z \end{cases}$

$\int z \sin z\, dz = -z\cos z + \int \cos z\, dz$

$= -z\cos z + \sin z$

$= -\sqrt{x}\cos\sqrt{x} + \sin\sqrt{x}$

$\therefore \quad \int \sin\sqrt{x}\, dx = -2\sqrt{x}\cos\sqrt{x} + 2\sin\sqrt{x} + C$.

1.0

5i

$$\int \frac{1}{x\sqrt{\ln x}}\,dx$$

$u = \ln x \longrightarrow du = \frac{1}{x}\,dx$ 0.5

$$\int \frac{1}{x\sqrt{\ln x}}\,dx = \int \frac{1}{\sqrt{u}}\,du = 2\sqrt{u}$$

$$= 2\sqrt{\ln x}$$

$$\therefore \int \frac{1}{x\sqrt{\ln x}}\,dx = 2\sqrt{\ln x}$$ 1.0

6.-

$$\int_{-2}^{0} e^{|x+1|}\,dx$$

$$|x+1| = \begin{cases} x+1 & \therefore \quad x \geq -1 \\ -(x+1) & \therefore \quad x < -1 \end{cases}$$

0.5 0.5

$$\int_{-2}^{0} e^{|x+1|}\,dx = \int_{-2}^{-1} e^{-(x+1)}\,dx + \int_{-1}^{0} e^{(x+1)}\,dx$$

$$= -e^{-(x+1)}\Big]_{x=-2}^{-1} + e^{(x+1)}\Big|_{x=-1}^{0}$$

$$= -e^{-0} + \underbrace{e^{1} + e^{1}} - e^{0}$$

$$= -1 + 2e - 1$$

$$= -2 + 2e$$

0.5