1. Obtenha uma fórmula para $\int \sin^n x\,dx$ e use esta fórmula para calcular $\int \sin^3 x\,dx$.

2. $$\int \frac{x}{(x-1)(x^2+x+1)}\,dx$$

3. $$\int \frac{1}{\sin x + 2\cos x}\,dx$$

4. Determine a área da região delimitada pelas curvas $y = x^3+8$, $y = 4x+8$

5. Calcule $\int_0^1 x^2 \ln x\,dx$.

1. $\int \sin^n x \, dx = \int \sin^{n-1} x \sin x \, dx$

$$u = \sin^{n-1} x \rightarrow du = (n-1) \sin^{n-2} x \cos x \, dx$$

$$dv = \sin x \, dx \rightarrow v = -\cos x$$

$$= -\cos x \sin^{n-1} x - \int (-) \cos x (n-1) \sin^{n-2} x \cos x \, dx$$

$$= -\cos x \sin^{n-1} x + (n-1) \int \sin^{n-2} x \cos^2 x \, dx$$

$$= -\cos x \sin^{n-1} x + (n-1) \int \sin^{n-2} x (1 - \sin^2 x) \, dx$$

$$\int \sin^n x \, dx = -\cos x \sin^{n-1} x + (n-1) \int \sin^{n-2} x \, dx -$$

$$- (n-1) \int \sin^n x \, dx$$

$$\int \sin^n x \, dx + (n-1) \int \sin^n x \, dx = -\cos x \sin^{n-1} x + $$
$$+ (n-1) \int \sin^{n-2} x \, dx$$

$$n \int \sin^n x \, dx = -\cos x \sin^{n-1} x + (n-1) \int \sin^{n-2} x \, dx$$

$$\boxed{\int \sin^n x \, dx = -\frac{1}{n} \cos x \sin^{n-1} x + \frac{(n-1)}{n} \int \sin^{n-2} x \, dx}$$

1.0

$n=3$:

$$\int \sin^3 x \, dx = -\frac{1}{3} \cos x \sin^2 x + \frac{2}{3} \int \sin x \, dx$$

$$\boxed{\int \sin^3 x \, dx = -\frac{1}{3} \cos x \sin^2 x - \frac{2}{3} \cos x}$$

No

$$2.\ \int \frac{\cancel{x}}{(x-1)(x^2+x+1)}\,dx$$

$$\frac{x}{(x-1)(x^2+x+1)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+x+1}$$

$$= \frac{A(x^2+x+1) + (Bx+C)(x-1)}{(x-1)(x^2+x+1)}$$

$$= \frac{Ax^2 + Ax + A + Bx^2 - Bx + Cx - C}{(x-1)(x^2+x+1)}$$

$$\frac{x}{(x-1)(x^2+x+1)} = \frac{(A+B)x^2 + (A-B+C)x + A - C}{(x-1)(x^2+x+1)}$$

$$\begin{cases} A+B=0 \implies B=-A & \therefore \ //B=-\frac{1}{3}// \\ A-B+C=1 \implies A+A+A=1 & \therefore \ \underline{\underline{A=1/3}} \\ A-C=0 \implies C=A & \therefore \ //C=\frac{1}{3}// \end{cases}$$

$$\frac{x}{(x-1)(x^2+x+1)} = \frac{1/3}{x-1} + \frac{-\frac{1}{3}x+\frac{1}{3}}{x^2+x+1}$$

~~0.75~~

$$\int \frac{x}{(x-1)(x^2+x+1)}\,dx = \frac{1}{3}\int \frac{1}{(x-1)}\,dx + \int \frac{\left(\frac{-1}{3}x + \frac{1}{3}\right)}{x^2+x+1}\,dx$$

$$= \underbrace{\frac{1}{3}\ln|x-1|}_{\text{O.2F}} - \frac{1}{3}\underbrace{\int \frac{x-1}{x^2+x+1}\,dx}_{(*)}$$

Mo7

$$(*) = \int \frac{x-1}{x^2+x+1}\,dx = \int \frac{x-1}{\left(x+\frac{1}{2}\right)^2 - \frac{1}{4} + 1}\,dx$$

$$= \int \frac{x-1}{\left(x+\frac{1}{2}\right)^2 + \frac{3}{4}}\,dx$$

$$\begin{cases} u = x + \frac{1}{2} \longrightarrow du = dx \\ x = u - \frac{1}{2} \end{cases}$$

$$= \int \frac{u - \frac{1}{2} - 1}{u^2 + \frac{3}{4}}\,du$$

$$= \int \frac{u - \frac{3}{2}}{u^2+\frac{3}{4}}\,du$$

$$= \underbrace{\int \frac{u}{u^2+\frac{3}{4}}\,du}_{} - \frac{3}{2}\int \frac{1}{u^2+\frac{3}{4}}\,du$$

$$= \frac{1}{2}\ln\left(u^2+\frac{3}{4}\right) - \frac{3}{2}\underbrace{\int \frac{1}{u^2+\frac{3}{4}}\,du}_{(**)}$$

$$= \frac{1}{2}\ln\left(\underbrace{\left(x+\frac{1}{2}\right)^2 + \frac{3}{4}}_{x^2+x+1}\right) - \frac{3}{2}\,(**)$$

$$(*) = \underbrace{\frac{1}{2}\ln(x^2+x+1)}_{0.25} - \frac{3}{2}(**)$$

Mes

$$(**) = \int \frac{1}{u^2+\frac{3}{4}}\,du$$

$$\left[\begin{array}{l} u = \frac{\sqrt{3}}{2}\operatorname{tg}\theta \longrightarrow du = \frac{\sqrt{3}}{2}\sec^2\theta\, d\theta \\ \theta = \operatorname{arctg}\frac{2}{\sqrt{3}}u \end{array}\right]$$

$$= \int \frac{1}{\frac{3}{4}\operatorname{tg}^2\theta + \frac{3}{4}}\,\frac{\sqrt{3}}{2}\sec^2\theta\, d\theta$$

$$= \int \frac{1}{\frac{3}{4}(\operatorname{tg}^2\theta+1)}\,\frac{\sqrt{3}}{2}\sec^2\theta\, d\theta$$

$$= \int \frac{1}{\frac{3}{4}\cancel{\sec^2\theta}}\,\frac{\sqrt{3}}{2}\cancel{\sec^2\theta}\, d\theta$$

$$= \int \frac{2}{\sqrt{3}}\,d\theta = \frac{2}{\sqrt{3}}\theta = \frac{2}{\sqrt{3}}\operatorname{arctg}\frac{2}{\sqrt{3}}u$$

$$= \frac{2}{\sqrt{3}}\operatorname{arctg}\frac{2}{\sqrt{3}}\left(x+\frac{1}{2}\right)$$

$$\therefore\quad (*) = \frac{1}{2}\ln(x^2+x+1) - \frac{3}{\cancel{2}}\,\frac{\cancel{2}}{\sqrt{3}}\operatorname{arctg}\frac{2}{\sqrt{3}}\left(x+\frac{1}{2}\right)$$

$$= \frac{1}{2}\ln(x^2+x+1) - \sqrt{3}\operatorname{arctg}\frac{2}{\sqrt{3}}\left(x+\frac{1}{2}\right)$$

$\underline{\underline{0.75}}$

$$\int \frac{x}{(x-1)(x^2+x+1)}\,dx =$$

$$= \frac{1}{3}\ln|x-1| - \frac{1}{3}\left(\overbrace{\frac{1}{2}\ln(x^2+x+1) - \sqrt{3}\,\mathrm{arctg}\,\frac{2}{\sqrt{3}}\left(x+\frac{1}{2}\right)}^{\cancel{A}\,1}\right)$$

$$= \frac{1}{3}\ln|x-1| - \frac{1}{6}\ln(x^2+x+1) + \frac{\sqrt{3}}{3}\,\mathrm{arctg}\left(\frac{2}{\sqrt{3}}\left(x+\frac{1}{2}\right)\right)$$

3. $\displaystyle\int \frac{1}{\sin x + 2\cos x}\,dx$

$dz = \frac{1}{2}\sec^2\frac{x}{2}\,dx$

$= \frac{1}{2}\left(1+tg^2\frac{x}{2}\right)dx$

$= \frac{1}{2}(1+z^2)\,dx$

$dx = \frac{2\,dz}{1+z^2}$

$$\begin{cases} z = tg\,\frac{x}{2}, & dx = 2\,\frac{dz}{1+z^2} \\ \sin x = \frac{2z}{1+z^2}, & \cos x = \frac{1-z^2}{1+z^2} \end{cases}$$

$$\int \frac{1}{\sin x + 2\cos x}\,dx = \int \frac{1}{\left(\frac{\cancel{2}z}{\cancel{1+z^2}} + \cancel{2}\frac{(1-z^2)}{\cancel{1+z^2}}\right)}\,\frac{\cancel{2}\,dz}{\cancel{1+z}^2}$$

$$= \int \frac{1}{z+1-z^2}\,dz$$

$$= \int \frac{-1}{z^2-z-1}\,dz \quad 0{,}5$$

$$= \int \frac{-1}{\left(z-\frac{1}{2}\right)^2-\frac{1}{4}-1}\,dz$$

$$= \int \frac{-1}{\left(z-\frac{1}{2}\right)^2-\frac{5}{4}}\,dz$$

$u = z - \frac{1}{2} \rightarrow du = dz$

$$= \int \frac{-1}{u^2-\frac{5}{4}}\,du \quad 1{,}0$$

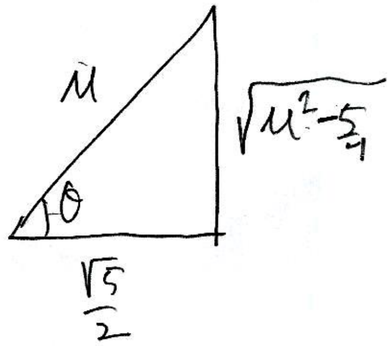
$$u = \frac{\sqrt{5}}{2}\sec\theta \rightarrow du = \frac{\sqrt{5}}{2}\sec\theta \operatorname{tg}\theta \, d\theta$$

$$= \int \frac{-1}{\frac{5}{4}\operatorname{tg}^2\theta} \frac{\sqrt{5}}{2}\sec\theta \cancel{\operatorname{tg}\theta}\, d\theta$$

$$= -\frac{2}{\sqrt{5}}\int \frac{\sec\theta}{\operatorname{tg}\theta}\, d\theta$$

$$= -\frac{2}{\sqrt{5}}\int \frac{\frac{1}{\cos\theta}}{\frac{\sin\theta}{\cos\theta}}\, d\theta \qquad \sec\theta = \frac{u}{\frac{\sqrt{5}}{2}}$$

$$= -\frac{2}{\sqrt{5}}\int \frac{1}{\sin\theta}\, d\theta$$



$$= -\frac{2}{\sqrt{5}}\int \operatorname{cosec}\theta\, d\theta$$

$$= -\frac{2}{\sqrt{5}}(-)\ln|\operatorname{cosec}\theta + \operatorname{cotg}\theta|$$

$$= \frac{2}{\sqrt{5}}\ln|\operatorname{cosec}\theta + \operatorname{cotg}\theta|$$

$$= \frac{2}{\sqrt{5}}\ln\left|\frac{u}{\sqrt{u^2-\frac{5}{4}}} + \frac{\frac{\sqrt{5}}{2}}{\sqrt{u^2-\frac{5}{4}}}\right|$$

$$= \frac{2}{\sqrt{5}}\ln\left|\frac{u+\frac{\sqrt{5}}{2}}{\sqrt{u^2-\frac{5}{4}}}\right| = \frac{2}{\sqrt{5}}\ln\left|\frac{u+\frac{\sqrt{5}}{2}}{\sqrt{u-\frac{\sqrt{5}}{2}}\sqrt{u+\frac{\sqrt{5}}{2}}}\right|$$

$$= \frac{2}{\sqrt{5}}\ln\left|\sqrt{\frac{u+\frac{\sqrt{5}}{2}}{u-\frac{\sqrt{5}}{2}}}\right| = \frac{1}{\sqrt{5}}\ln\left|\frac{u+\frac{\sqrt{5}}{2}}{u-\frac{\sqrt{5}}{2}}\right|$$

$$= \frac{1}{\sqrt{5}} \ln \left| \frac{3 - \frac{1}{2} + \frac{\sqrt{5}}{2}}{3 - \frac{1}{2} - \frac{\sqrt{5}}{2}} \right|$$

$$= \frac{1}{\sqrt{5}} \ln \left| \frac{tg\,\frac{x}{2} + \frac{\sqrt{5}-1}{2}}{tg\,\frac{x}{2} - \frac{\sqrt{5}+1}{2}} \right| + c$$

1,0

derivando

$$\left(\frac{1}{\sqrt{5}} \ln \left| \frac{tg\,\frac{x}{2} + \frac{\sqrt{5}-1}{2}}{tg\,\frac{x}{2} - \frac{\sqrt{5}+1}{2}} \right| \right)' = \frac{1}{\sqrt{5}} \left(\frac{tg\,\frac{x}{2} - \frac{\sqrt{5}+1}{2}}{tg\,\frac{x}{2} + \frac{\sqrt{5}-1}{2}} \right) \cdot$$

$$\cdot \left(\frac{tg\,\frac{x}{2} + \frac{\sqrt{5}-1}{2}}{tg\,\frac{x}{2} - \frac{\sqrt{5}+1}{2}} \right)' =$$

$$= \frac{1}{\sqrt{5}} \left(\frac{tg\,\frac{x}{2} - \frac{\sqrt{5}+1}{2}}{tg\,\frac{x}{2} + \frac{\sqrt{5}-1}{2}} \right) \cdot \left\{ \frac{\left(tg\,\frac{x}{2} + \frac{\sqrt{5}-1}{2}\right)' \left(tg\,\frac{x}{2} - \frac{\sqrt{5}+1}{2}\right) - \left(tg\,\frac{x}{2} + \frac{\sqrt{5}-1}{2}\right) \left(tg\,\frac{x}{2} - \frac{\sqrt{5}+1}{2}\right)'}{\left(tg\,\frac{x}{2} - \frac{\sqrt{5}+1}{2}\right)^2} \right\}$$

$$= \frac{1}{\sqrt{5}} \left(\frac{tg\,\frac{x}{2} - \frac{\sqrt{5}+1}{2}}{tg\,\frac{x}{2} + \frac{\sqrt{5}-1}{2}} \right) \left\{ \frac{\frac{1}{2} \sec^2 \frac{x}{2} \left(tg\,\frac{x}{2} - \frac{\sqrt{5}+1}{2}\right) - \left(tg\,\frac{x}{2} + \frac{\sqrt{5}-1}{2}\right) \left(\frac{1}{2} \sec^2 \frac{x}{2}\right)}{\left(tg\,\frac{x}{2} - \frac{\sqrt{5}+1}{2}\right)^2} \right\}$$

$$= \frac{1}{\sqrt{5}} \frac{\left(tg\,\frac{x}{2} - \frac{\sqrt{5}+1}{2}\right)}{\left(tg\,\frac{x}{2} + \frac{\sqrt{5}-1}{2}\right)} \left\{ \frac{\cancel{\frac{1}{2} \sec^2 \frac{x}{2}\, tg\,\frac{x}{2}} - \frac{1}{4}(\sqrt{5}+1) \sec^2 \frac{x}{2} - \cancel{\frac{1}{2} \sec^2 \frac{x}{2}\, tg\,\frac{x}{2}} - \frac{1}{4}(\sqrt{5}-1) \sec^2 \frac{x}{2}}{\left(tg\,\frac{x}{2} - \frac{\sqrt{5}+1}{2}\right)^2} \right\}$$

$$= \frac{1}{\sqrt{5}} \frac{\left(tg\,\frac{x}{2} - \frac{\sqrt{5}+1}{2}\right)}{\left(tg\,\frac{x}{2} + \frac{\sqrt{5}-1}{2}\right)} \left\{ \frac{\sec^2 \frac{x}{2} \left\{ -\frac{\sqrt{5}}{4} - \cancel{\frac{1}{4}} - \frac{1}{4}\sqrt{5} + \cancel{\frac{1}{4}} \right\}}{\left(tg\,\frac{x}{2} - \frac{\sqrt{5}+1}{2}\right)^2} \right\}$$

$$= \frac{1}{\cancel{\sqrt{5}}} \frac{\cancel{\left(\operatorname{tg}\frac{x}{2} - \frac{\sqrt{5}+1}{2}\right)}}{\left(\operatorname{tg}\frac{x}{2} + \frac{\sqrt{5}-1}{2}\right)} \frac{\left(-\frac{\cancel{\sqrt{5}}}{2} \sec^2\frac{x}{2}\right)}{\left(\operatorname{tg}\frac{x}{2} - \frac{\sqrt{5}+1}{2}\right)^{\cancel{2}}}$$

$$= -\frac{1}{2} \frac{\sec^2\frac{x}{2}}{\left\{\operatorname{tg}^2\frac{x}{2} - \frac{(\sqrt{5}+1)}{2}\operatorname{tg}\frac{x}{2} + \frac{(\sqrt{5}-1)}{2}\operatorname{tg}\frac{x}{2} - \left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)\left(\frac{\sqrt{5}}{2}+\frac{1}{2}\right)\right\}}$$

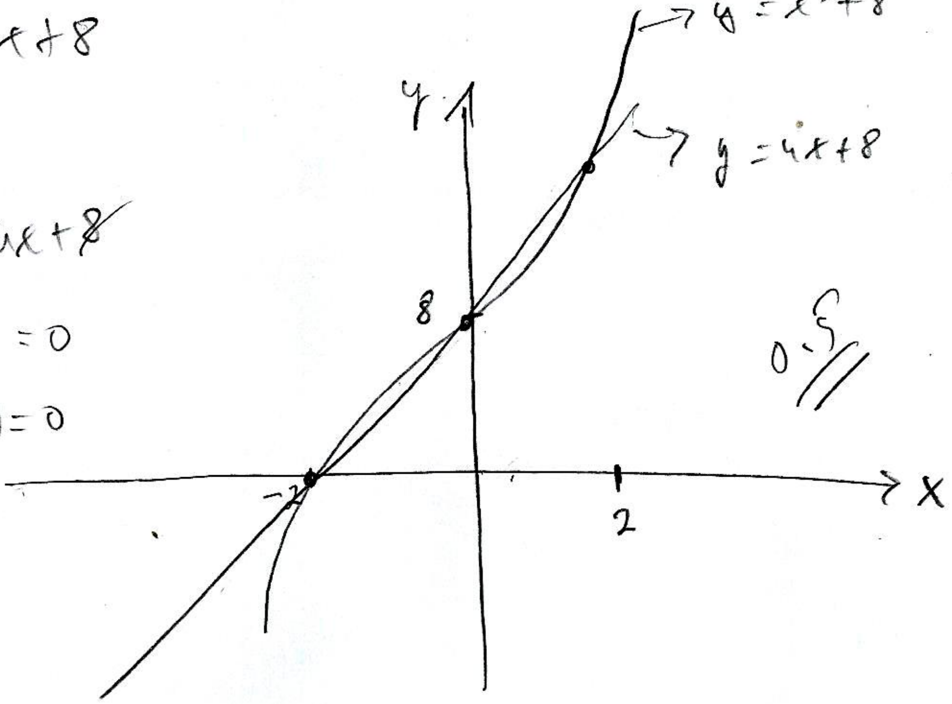
$$= -\frac{1}{2} \frac{\sec^2\frac{x}{2}}{\operatorname{tg}^2\frac{x}{2} + \operatorname{tg}\frac{x}{2}\left(-\cancel{\frac{\sqrt{5}}{2}}-\frac{1}{2} + \cancel{\frac{\sqrt{5}}{2}}-\frac{1}{2}\right) - \left(\frac{5}{4}-\frac{1}{4}\right)}$$

$$= -\frac{1}{2} \frac{\sec^2\frac{x}{2}}{\operatorname{tg}^2\frac{x}{2} - \operatorname{tg}\frac{x}{2} - 1}$$

$$= -\frac{1}{2} \frac{1+\operatorname{tg}^2\frac{x}{2}}{\operatorname{tg}^2\frac{x}{2} - \operatorname{tg}\frac{x}{2} - 1}$$

$$= -\frac{1}{2} \frac{1+z^2}{z^2-z-1} = \frac{1+z^2}{-z+z+1}$$

4.) $y = x^3+8$
$y = 4x+8$



$\therefore$

$\cancel{x^3+8} = 4x+\cancel{8}$

$x^3-4x=0$

$x(x^2-4)=0$

$\begin{cases} x=0 \\ x=\pm 2 \end{cases}$

0.5 //

$$S = \int_{-2}^{0} (x^3+8-4x-8)\,dx + \int_{0}^{2} (4x+8-x^3-8)\,dx$$

0.5 //

$$= \int_{-2}^{0} (x^3-4x)\,dx + \int_{0}^{2} (4x-x^3)\,dx$$

$$= \left(\frac{x^4}{4} - \frac{\cancel{4}^{2}x^2}{\cancel{2}}\right)\Bigg|_{-2}^{0} + \left(\frac{\cancel{4}^{2}x^2}{\cancel{2}} - \frac{x^4}{4}\right)\Bigg|_{0}^{2}$$

$$= -\left(\frac{16}{4} - 2.4\right) + 2.4 - \frac{16}{4}$$

$$= -(4-8) + 8-4$$

$$= -(-4) + 4$$

$$= 8 \,//$$

1.0 //

5. $\displaystyle\int_0^1 x^2 \ln x \, dx = \lim_{t \to 0^+} \int_t^1 x^2 \ln x \, dx$

0.5 //

$\displaystyle\int x^2 \ln x \, dx =$

$u = \ln x \rightarrow du = \frac{1}{x} dx$

$dv = x^2 dx \rightarrow v = \frac{1}{3} x^3$

$\displaystyle = \frac{1}{3} x^3 \ln x - \int \frac{1}{3} x^2 dx$

$\displaystyle = \frac{1}{3} x^3 \ln x - \frac{x^3}{9}$

0.5 //

$$= \lim_{t \to 0^+} \left(\frac{1}{3} x^3 \ln x - \frac{x^3}{9} \right) \Bigg|_t^1$$

$$= \lim_{t \to 0^+} \left\{ \frac{1}{3} \cancelto{0}{\ln 1} - \frac{1}{9} - \left(\frac{1}{3} t^3 \ln t - \frac{t^3}{9} \right) \right\}$$

$$= \lim_{t \to 0^+} \left\{ -\frac{1}{9} - \frac{1}{3} t^3 \ln t + \frac{t^3}{9} \right\}$$

$$\lim_{t \to 0^+} t^3 \ln t = \lim_{t \to 0^+} \frac{\ln t}{\frac{1}{t^3}} = \lim_{t \to 0^+} \frac{\frac{1}{t}}{-\frac{3}{t^4}}$$

$$= \lim_{t \to 0^+} -\frac{1}{3} t^3 = 0$$

$$= -\frac{1}{9} \quad //$$

1.0