Cálculo A - Limites (I)

Calcule os seguintes limites

1. $\lim_{x\to -1} \frac{x^3+1}{x^2+1}$
2. $\lim_{x\to 0} \frac{x^2-1}{2x^2-x-1}$
3. $\lim_{x\to -1} \frac{x^2-1}{x^2+3x+2}$
4. $\lim_{x\to 1} \frac{x^2-1}{2x^2-x-1}$
5. $\lim_{x\to 3} \frac{x^2-5x+6}{x^2-8x+15}$
6. $\lim_{x\to 1} \frac{x^3-3x+2}{x^4-4x+3}$
7. $\lim_{x\to 1} \frac{x^4-3x+2}{x^5-4x+3}$
8. $\lim_{x\to 2} \frac{x^3-2x^2-4x+8}{x^4-8x^2+16}$
9. $\lim_{x\to -1} \frac{x^3-2x-1}{x^5-2x-1}$
10. $\lim_{x\to 2} \frac{x^2-2x}{x^2-4x+4}$
11. $\lim_{x\to 1} \frac{x^3-3x+2}{x^4-4x+3}$
12. $\lim_{x\to a} \frac{x^2-(a+1)x+a}{x^3-a^3}$
13. $\lim_{x\to 1} \left(\frac{1}{1-x} - \frac{3}{1-x^3}\right)$
14. $\lim_{x\to 0} \frac{(1+x)(1+2x)(1+3x)-1}{x}$
15. $\lim_{x\to 0} \frac{(1+x)(1+2x)(1+3x)...(1+nx)-1}{x}$
16. $\lim_{x\to 0} \frac{(1+x)^5-(1+5x)}{x^2+x^5}$
17. $\lim_{x\to 0} \frac{(1+x)^n-(1+nx)}{x^2+x^n}$
18. $\lim_{x\to 0} \frac{(1+3x)^2-(1+2x)^3}{x^2}$
19. $\lim_{x\to 0} \frac{(1+mx)^n-(1+nx)^m}{x^2}$
20. $\lim_{x\to 1} \frac{x^{n+1}-x^n+x-1}{2x^n-x^{n-1}-1}$
21. $\lim_{x\to 1} \frac{x+x^2+...+x^n-n}{x-1}$
22. $\lim_{x\to 2} \frac{(x^2-x-2)^{20}}{(x^3-12x+16)^{10}}$
23. $\lim_{x\to 1} \frac{x^{100}-2x+1}{x^{50}-2x+1}$
24. $\lim_{x\to a} \frac{\sqrt{x}-\sqrt{a}}{x-a}$
25. $\lim_{x\to 1} \frac{\sqrt{x}-1}{x-1}$
26. $\lim_{x\to 64} \frac{\sqrt{x}-8}{\sqrt[3]{x}-4}$
27. $\lim_{x\to 1} \frac{\sqrt[3]{x}-1}{\sqrt[4]{x}-1}$
28. $\lim_{x\to 1} \frac{\sqrt[p]{x}-1}{\sqrt[n]{x}-1}$
29. $\lim_{x\to 7} \frac{2-\sqrt{x-3}}{x^2-49}$
30. $\lim_{x\to 8} \frac{x-8}{\sqrt[3]{x}-2}$
31. $\lim_{x\to 1} \frac{\sqrt{x}-1}{\sqrt[3]{x}-1}$
32. $\lim_{x\to 4} \frac{3-\sqrt{5+x}}{1-\sqrt{5-x}}$
33. $\lim_{x\to 0} \frac{\sqrt{1+x}-\sqrt{1-x}}{x}$
34. $\lim_{h\to 0} \frac{\sqrt{x+h}-\sqrt{x}}{h} \quad (x > 0)$
35. $\lim_{h\to 0} \frac{\sqrt[3]{x+h}-\sqrt[3]{x}}{h} \quad (x \neq 0)$
36. $\lim_{x\to 3} \frac{\sqrt{x^2-2x+6}-\sqrt{x^2+2x-6}}{x^2-4x+3}$
37. $\lim_{x\to 0} \frac{\sqrt{1+x}-1}{\sqrt[3]{1+x}-1}$
38. $\lim_{x\to 1} \frac{\sqrt[3]{x^2}-2\sqrt[3]{x}+1}{(x-1)^2}$
39. $\lim_{x\to 3} \frac{x^2-5x+6}{\sqrt{x+1}-\sqrt{3x-5}}$
40. $\lim_{x\to -3} \frac{x^2+2x-3}{x+1+\sqrt[3]{x^2-1}}$
41. $\lim_{x\to 1} \frac{\sqrt[3]{4x^5-3}-1}{3x^2-x-2}$
42. $\lim_{x\to 1} \frac{x^2-1}{\sqrt[5]{2x^5-1}-1}$
43. $\lim_{x\to a} \frac{\sqrt{a-x}+\sqrt{a}-\sqrt{x}}{\sqrt{a^2-x^2}}$

Limites quando $x \to \pm\infty$

44. $\lim_{x\to \infty} \frac{(x+1)^2}{x^2+1}$
45. $\lim_{x\to \infty} \frac{1000x}{x^2-1}$
46. $\lim_{x\to \infty} \frac{x^2-5x+1}{3x+7}$
47. $\lim_{x\to \infty} \frac{2x^2-x+3}{x^3-8x+5}$
48. $\lim_{x\to \infty} \frac{(2x+3)^3(3x-2)^2}{x^5+5}$
49. $\lim_{x\to \infty} \frac{2x^2-3x-4}{\sqrt{x^4+1}}$
50. $\lim_{x\to \infty} \frac{2x+3}{x+\sqrt[3]{x}}$
51. $\lim_{x\to \infty} \frac{x^2}{10+x\sqrt{x}}$
52. $\lim_{x\to \infty} \frac{\sqrt[3]{x^2+1}}{x+1}$
53. $\lim_{x\to \infty} \frac{\sqrt{x}}{\sqrt{x+\sqrt{x+\sqrt{x}}}}$
54. $\lim_{x\to \infty} \frac{x}{\sqrt[3]{x^3+10}}$

55. $\lim_{x\to\infty}(\sqrt{x+a}-\sqrt{x})$

56. $\lim_{x\to\infty}(\sqrt{x(x+a)}-x)$

57. $\lim_{x\to\infty}(\sqrt{x^2-5x+6}-x)$

58. $\lim_{x\to\infty} x(\sqrt{x^2+1}-x)$

59. $\lim_{x\to\infty}(x+\sqrt[3]{1-x^3})$

60. $\lim_{x\to\infty}(\sqrt{x^2+1}-\sqrt{x^2-1})$

61. $\lim_{x\to\infty}(\frac{1-2x}{\sqrt[3]{1+8x^3}}+2^{-x^2})$

62. $\lim_{x\to\infty}(\frac{x^3}{3x^2-4}-\frac{x^2}{3x+2})$

63. $\lim_{x\to\infty}(\sqrt{9x^2+1}-3x)$

64. $\lim_{x\to\infty}\frac{2\sqrt{x}+3\sqrt[3]{x}+5\sqrt[5]{x}}{\sqrt{3x-2}+\sqrt[3]{2x-3}}$

65. $\lim_{x\to-\infty}(\sqrt{2x^2-3}-5x)$

66. $\lim_{x\to\infty} x(\sqrt{x^2+1}-x)$

67. $\lim_{x\to\infty}\frac{\sqrt{2x^2+3}}{4x+2}$

68. $\lim_{x\to-\infty}\frac{\sqrt{2x^2+3}}{4x+2}$

69. $\lim_{x\to\infty} 5^{\frac{2x}{x+3}}$

70. $\lim_{x\to-\infty}\frac{x}{\sqrt{x^2+1}}$

71. $\lim_{x\to\infty}\frac{x}{\sqrt{x^2+1}}$

72. $\lim_{x\to-\infty}\frac{3^x-3^{-x}}{3^x+3^{-x}}$

73. $\lim_{x\to\infty}\frac{3^x-3^{-x}}{3^x+3^{-x}}$

Resposta 1. 0

2. 1

3. -2

4. $\frac{2}{3}$

5. $\frac{-1}{2}$

6. $\frac{1}{2}$

7. 1

8. $\frac{1}{4}$

9. $\frac{1}{3}$

10. $\nexists$

11. $\frac{1}{2}$

12. $\frac{a-1}{3a^2}$

13. -1

14. 6

15. $\frac{n(n+1)}{2}$

16. 10

17. $\frac{n(n-1)}{2}$

18. -3

19. $\frac{mn(n-m)}{2}$

20. $\frac{2}{n+1}$

21. $\frac{n(n+1)}{2}$

22. $\frac{3^{10}}{2^{10}}$

23. $\frac{49}{24}$

24. $\frac{1}{2\sqrt{a}}$

25. $\frac{1}{2}$

26. 3

27. $\frac{4}{3}$

28. $\frac{n}{p}$

29. $\frac{-1}{56}$

30. 12

31. $\frac{3}{2}$

32. $\frac{-1}{3}$

33. 1

34. $\frac{1}{2\sqrt{x}}$

35. $\frac{1}{3x^{\frac{2}{3}}}$

36. $\frac{-1}{3}$

37. $\frac{3}{2}$

38. $\frac{1}{9}$

39. -2

40. -8

41. $\frac{4}{3}$

42. 1

43. $\frac{1}{\sqrt{2a}}$

44. 1

45. 0

46. $+\infty$

47. 0

48. 72

49. 2

50. 2

51. ∞

52. 0
53. 1
54. 1
55. 0
56. $\frac{a}{2}$
57. $\frac{-5}{2}$
58. $\frac{1}{2}$
59. 0
60. 0
61. -1
62. $\frac{2}{9}$
63. 0
64. $\frac{2}{\sqrt{3}}$
65. ∞
66. $\frac{1}{2}$
67. $\frac{\sqrt{2}}{4}$
68. $\frac{-\sqrt{2}}{4}$
69. 25
70. -1
71. 1
72. -1
73. 1

$$\lim_{x \to -1} \frac{x^3+1}{x^2+1} = \frac{0}{2} = 0$$

$\boxed{\text{Lista 6 Parte I}}$

$$\lim_{x \to 0} \frac{x^2-1}{2x^2-x+1} = \frac{-1}{-1} = 1$$

$$\lim_{x \to 1} \frac{x^2-1}{2x^2-x-1} = \frac{0}{0}$$

$$\begin{cases} x^2-1 = (x-1)(x+1) \\ 2x^2-x-1 = 2(x-1)(x+\frac{1}{2}) \end{cases}$$

$$x = 1 \pm \sqrt{1+8} \,/4$$

$$= \frac{1 \pm 3}{4} \begin{matrix} \nearrow 1 \\ \searrow -\frac{1}{2} \end{matrix}$$

$$\lim_{x \to 1} \frac{x^2-1}{2x^2-x-1} = \lim_{x \to 1} \frac{\cancel{(x-1)}(x+1)}{2\cancel{(x-1)}(x+\frac{1}{2})}$$

$$= \lim_{x \to 1} \frac{(x+1)}{2(x+\frac{1}{2})} = \frac{2}{\cancel{2}\frac{3}{\cancel{2}}}$$

$$= \cancel{\frac{4}{3}} \quad \frac{2}{3}$$

$$\lim_{x\to 3} \frac{x^2-5x+6}{x^2-8x+15} = \frac{0}{0}$$

$$x^2-5x+6 = (x-3)(x-2)$$

$$x = 5 \pm \sqrt{25-24}\,/2$$

$$= \frac{5\pm 1}{2} \begin{array}{l}\nearrow 3 \\ \searrow 2\end{array}$$

$$x^2-8x+15 = (x-3)(x-5)$$

$$x = 8 \pm \sqrt{64-60}\,/2$$

$$= \frac{8\pm 2}{2} \begin{array}{l}\nearrow 5 \\ \searrow 3\end{array}$$

$\therefore$

$$\lim_{x\to 3} \frac{x^2-5x+6}{x^2-8x+15} = \lim_{x\to 3} \frac{\cancel{(x-3)}(x-2)}{\cancel{(x-3)}(x-5)}$$

$$= \lim_{x\to 3} \frac{x-2}{x-5} = \frac{1}{-2}$$

$$= -\frac{1}{2}\ //$$

- $$\lim_{x \to -1} \frac{x^2-1}{x^2+3x+2} = \frac{0}{0}$$

$$= \lim_{x \to -1} \frac{(x-1)\cancel{(x+1)}}{\cancel{(x+1)}(x+2)} = \lim$$

$$= \lim_{x \to -1} \frac{x-1}{x+2} = \frac{-2}{1} = -2$$

$$\lim_{x \to 1} \frac{x^3-3x+2}{x^4-4x+3} = \frac{0}{0}$$

$$x^3-3x+2 = (x-1)\,P(x)$$

$$\begin{array}{l|l} x^3-3x+2 & x-1 \\ \hline \cancel{-x^3}+x^2 & x^2+x-2 = P(x) \\ \hline \cancel{x^2}-3x+2 & \\ \cancel{-x^2}+x & \\ \hline -2x+2 & \\ +2x-2 & \\ \hline 0 & \end{array}$$

$\therefore$

$$\| x^3-3x+2 = (x-1)(x^2+x-2) \|$$

$$x^4-4x+3 = (x-1)\,P(x)$$

$$\begin{array}{l|l} \cancel{x^4}-4x+3 & x-1 \\ \hline \cancel{x^4}+x^3 & x^3+x^2+x-3 \\ \hline +\cancel{x^3}-4x+3 & \\ \cancel{-x^3}+x^2 & \\ \hline \cancel{x^2}-4x+3 & \\ \cancel{-x^2}+x & \\ \hline -3x+3 & \\ +3x-3 & \\ \hline 0 & \end{array}$$

$\therefore$

$$\| x^4-4x+3 = (x-1)(x^3+x^2+x-3) \|$$

$\therefore$

$$\lim_{x\to 1} \frac{x^3-3x+2}{x^4-4x+3} = \lim_{x\to 1} \frac{\cancel{(x-1)}(x^2+x-2)}{\cancel{(x-1)}(x^3+x^2+x-3)}$$

$$= \lim_{x\to 1} \frac{x^2+x-2}{x^3+x^2+x-3} = \frac{0}{0}$$

$$\left\{ x^2+x-2 = (x-1)(x+2) \right.$$

$$x^3+x^2+x-3 = (x-1)P(x)$$

$$\begin{array}{l|l} \cancel{x^3}+x^2+x-3 & x-1 \\ \hline -\cancel{x^3}+x^2 & x^2+2x+3 \\ \hline 2\cancel{x^2}+x-3 & \\ -\cancel{2x^2}+2x & \\ \hline 3x-3 & \\ -3x+3 & \\ \hline 0 & \end{array}$$

$$\therefore \quad \| x^3+x^2+x-3 = (x-1)(x^2+2x+3) \|$$

$$\therefore \quad \lim_{x\to 1} \frac{x^2+x-2}{x^3+x^2+x-3} = \lim_{x\to 1} \frac{\cancel{(x-1)}(x+2)}{\cancel{(x-1)}(x^2+2x+3)}$$

$$= \lim_{x\to 1} \frac{x+2}{x^2+2x+3} = \frac{3}{6} = \frac{1}{2} \,//$$

$$\lim_{x\to 1} \frac{x^4-3x+2}{x^5-4x+3} = \frac{0}{0}$$

$$x^4-3x+2 = (x-1)\,P(x)$$

$$\begin{array}{l|l}
\cancel{x^4}-3x+2 & x-1 \\
\underline{-\cancel{x^4}+x^3} & x^3+x^2+x-2 \\
\cancel{x^3}-3x+2 & \\
\underline{-\cancel{x^3}+x^2} & \\
\cancel{x^2}-3x+2 & \\
\underline{-\cancel{x^2}+x} & \\
-2x+2 & \\
\underline{2x-2} & \\
0\ \ 0 &
\end{array}$$

$\therefore$

$$\|(x^4-3x+2) = (x-1)(x^3+x^2+x-2)\|$$

$$x^5-4x+3 = (x-1)\,P(x)$$

$$\begin{array}{l|l}
\cancel{x^5}-4x+3 & x-1 \\
\underline{-\cancel{x^5}+x^4} & x^4+x^3+x^2+x-3 \\
\cancel{x^4}-4x+3 & \\
\underline{-\cancel{x^4}+x^3} & \\
\cancel{x^3}-4x+3 & \\
\underline{-\cancel{x^3}+x^2} & \\
\cancel{x^2}-4x+3 & \\
\underline{-\cancel{x^2}+x} & \\
-3x+3 & \\
\underline{3x-3} & \\
0 &
\end{array}$$

$\therefore$

$$\|x^5-4x+3 = (x-1)(x^4+x^3+x^2+x-3)\|$$

$$\lim_{x \to 1} \frac{x^4 - 3x + 2}{x^5 - 4x + 3} = \lim_{x \to 1} \frac{\cancel{(x-1)}\,(x^3 + x^2 + x - 2)}{\cancel{(x-1)}\,(x^4 + x^3 + x^2 + x - 3)}$$

$$= \lim_{x \to 1} \frac{x^3 + x^2 + x - 2}{x^4 + x^3 + x^2 + x - 3}$$

$$= \frac{3 - 2}{4 - 3} = \frac{1}{1} = 1$$

$$\bullet \lim_{x \to 2} \frac{x^3-2x^2-4x+8}{x^4-8x^2+16} = \frac{8-8-8+8}{16-32+16} = \frac{0}{0}$$

$$x^3-2x^2-4x+8 = (x-2)\,P(x)$$

$$\begin{array}{l|l} \cancel{x^3} \cancel{-2x^2} -4x+8 & x-2 \\ \hline -x^3+2x^2 & x^2-4 \\ \quad -4x+8 & \\ \quad +4x-8 & \\ \hline \quad\ 0 \quad 0 & \end{array}$$

$\therefore$

$$/\!/\ x^3-2x^2-4x+8 = (x-2)(x^2-4)$$

$$= (x-2)(x-2)(x+2)$$

$$= (x-2)^2(x+2)\ /\!/$$

$$x^4-8x^2+16 = (x-2)\,P(x)$$

$$\begin{array}{l|l} \cancel{x^4} -8x^2+16 & x-2 \\ \hline -x^4+2x^3 & x^3+2x^2-4x-8 \\ \hline \cancel{2x^3}-8x^2+16 & \\ -2x^3+4x^2 & \\ \hline -4x^2+16 & \\ +4x^2-8x & \\ \hline -8x+16 & \\ 8x-16 & \\ \hline 0 & \end{array}$$

$\therefore$

$$/\!/\ x^4-8x^2+16 = (x-2)\underbrace{(x^3+2x^2-4x-8)}_{\text{fatorar}} \circledast$$

$$x^3+2x^2-4x-8 = (x-2)\;P(x)$$

$$\begin{array}{l|l}
\cancel{x^3}+2x^2-4x-8 & x-2 \\ \hline
\cancel{-x^3}+2x^2 & x^2+4x+4 \\ \hline
\cancel{4x^2}-4x-8 & \\
\cancel{-4x^2}+8x & \\ \hline
4x-8 & \\
-4x+8 & \\ \hline
0\quad 0 &
\end{array}$$

$$//\; x^3+2x^2-4x-8 = (x-2)(x^2+4x+4) \;//$$

(**)

(**) $\rightarrow$ (*) :

$$//\; x^4-8x^2+16 = (x-2)^2(x^2+4x+4) \;//$$

$$\lim_{x\to 2} \frac{\cancel{(x-2)^2}(x+2)}{\cancel{(x-2)^2}(x^2+4x+4)} = \lim_{x\to 2} \frac{x+2}{x^2+4x+4}$$

$$= \frac{2+2}{4+8+4} = \frac{4}{16} = \frac{1}{4} \;//$$

• $\lim_{x \to -1} \frac{x^3 - 2x - 1}{x^5 - 2x - 1} = \frac{-1+2-1}{-1+2-1} = \frac{0}{0}$

$x^3 - 2x - 1 = (x+1)\ P(x)$

$$
\begin{array}{l|l}
\cancel{x^3} - 2x - 1 & x+1 \\
\underline{-\cancel{x^3} - x^2} & x^2 - x - 1 \\
-\cancel{x^2} - 2x - 1 & \\
\underline{+\cancel{x^2} + x} & \\
-x - 1 & \\
\underline{+x + 1} & \\
0\ 0 &
\end{array}
$$

$\therefore$ $\| (x^3 - 2x - 1) = (x+1)(x^2 - x - 1) \|$

$x^5 - 2x - 1 = (x+1)\ P(x)$

$$
\begin{array}{l|l}
\cancel{x^5} - 2x - 1 & x+1 \\
\underline{-\cancel{x^5} - x^4} & x^4 - x^3 + x^2 - x - 1 \\
-\cancel{x^4} - 2x - 1 & \\
\underline{+\cancel{x^4} + x^3} & \\
\cancel{x^3} - 2x - 1 & \\
\underline{-\cancel{x^3} - x^2} & \\
-\cancel{x^2} - 2x - 1 & \\
\underline{+\cancel{x^2} + x} & \\
-x - 1 & \\
\underline{+x + 1} & \\
0 &
\end{array}
$$

$\therefore$ $\| x^5 - 2x - 1 = (x+1)(x^4 - x^3 + x^2 - x - 1) \|$

$$\lim_{x\to -1} \frac{x^3-2x-1}{x^5-2x-1} = \lim_{x\to -1} \frac{\cancel{(x+1)}(x^2-x-1)}{\cancel{(x+1)}(x^4-x^3+x^2-x-1)}$$

$$= \frac{(-1)^2-(-1)-1}{(-1)^4-(-1)^3+(-1)^2-(-1)-1}$$

$$= \frac{1+\cancel{1-1}}{1+1+1+\cancel{1-1}}$$

$$= \frac{1}{3}$$

- $$\lim_{x\to 2} \frac{x^2-2x}{x^2-4x+4} = \lim_{x\to 2} \frac{x\cancel{(x-2)}}{\cancel{(x-2)^2}}$$

$$= \lim_{x\to 2} \frac{x}{x-2} = \nexists$$

$$\begin{cases} \lim_{x\to 2^+} \frac{x}{x-2} = +\infty \\ \lim_{x\to 2^-} \frac{x}{x-2} = -\infty \end{cases}$$

$$\lim_{x\to 1} \frac{x^3-3x+2}{x^4-4x+3} = \frac{0}{0}$$

$$x^3-3x+2 = (x-1)\ P(x)$$

$$\begin{array}{r|l} \cancel{x^3}-3x+2 & x-1 \\ \cancel{-x^3}+x^2 & x^2+x-2 \\ \hline \cancel{x^2}-3x+2 & \\ \cancel{-x^2}+x & \\ \hline -2x+2 & \\ +2x-2 & \\ \hline 0\ 0 & \end{array}$$

$\therefore$

$$x^3-3x+2 = (x-1)\underbrace{(x^2+x-2)}$$

$$= (x-1)(x-1)(x+2)$$

$$x^4-4x+3 = (x-1)\ P(x)$$

$$\begin{array}{r|l} \cancel{x^4}-4x+3 & x-1 \\ \cancel{-x^4}+x^3 & x^3+x^2+x-3 \\ \hline \cancel{x^3}-4x+3 & \\ \cancel{-x^3}+x^2 & \\ \hline \cancel{x^2}-4x+3 & \\ \cancel{-x^2}+x & \\ \hline -3x+3 & \\ +3x-3 & \\ \hline 0\ 0 & \end{array}$$

$\therefore$

$$x^4-4x+3 = (x-1)\underbrace{(x^3+x^2+x-3)}_{\text{fatorar}} \ (*)$$

$$x^3+x^2+x-3 = (x-1)\,p(x)$$

$$\begin{array}{r|l}
x^3+x^2+x-3 & x-1 \\
\hline
-x^3+x^2 & x^2+2x+3 \\
\hline
2x^2+x-3 & \\
-2x^2+2x & \\
\hline
3x-3 & \\
-3x+3 & \\
\hline
0 &
\end{array}$$

$\therefore$ $x^3+x^2+x-3 = (x-1)(x^2+2x+3)$ (**)

(**) $\to$ (*) : $x^4-4x-3 = (x-1)^2(x^2+2x+3)$

$$\lim_{x\to1} \frac{x^3-3x+2}{x^4-4x+3} = \lim_{x\to1} \frac{\cancel{(x-1)^2}(x+2)}{\cancel{(x-1)^2}(x^2+2x+3)}$$

$$= \lim_{x\to1} \frac{(x+2)}{x^2+2x+3}$$

$$= \frac{3}{6} = \frac{1}{2}$$

$$\lim_{x\to a} \frac{x^2-(a+1)x+a}{x^3-a^3} = \frac{a^2-(a+1)a+a}{a^3-a^3} = \frac{0}{0}$$

Estratégia: fatorar um termo do tipo $(x-a)$

$$\left\{\begin{aligned} x^2-(a+1)x+a &= x^2-ax-x+a \\ &= x(x-a)-(x-a) \\ &= (x-1)(x-a) \end{aligned}\right.$$

$$\left\{\begin{aligned} &x^3-a^3 = (x-a)\,P(x) \\ &\begin{array}{l|l} \cancel{x^3}-a^3 & x-a \\ \cancel{-x^3}+ax^2 & x^2+ax+a^2 \\ \hline \cancel{ax^2}-a^3 & \\ \cancel{-ax^2}+a^2x & \\ \hline a^2x-a^3 & \\ -a^2x+a^3 & \\ \hline 0 & \end{array} \quad \therefore\ (x^3-a^3) = (x-a)(x^2+ax+a^2) \end{aligned}\right.$$

$\therefore$

$$\lim_{x\to a} \frac{x^2-(a+1)x+a}{x^3-a^3} =$$

$$= \lim_{x\to a} \frac{(x-1)\cancel{(x-a)}}{\cancel{(x-a)}(x^2+ax+a^2)}$$

$$= \lim_{x\to a} \frac{(x-1)}{x^2+ax+a^2} = \frac{a-1}{a^2+a^2+a^2}$$

$$= \frac{a-1}{3a^2}$$

$$\lim_{x \to 0} \frac{(1+x)(1+2x)(1+3x)-1}{x} = \frac{0}{0}$$

1º solução:

$(1+x)(1+2x)(1+3x) - 1 =$

$= (1+3x+2x^2)(1+3x) - 1$

$= \cancel{1} + 3x + 3x + 9x^2 + 2x^2 + 6x^3 - \cancel{1}$

$= 6x + 11x^2 + 6x^3$

$\therefore$

$$\lim_{x \to 0} \frac{(1+x)(1+2x)(1+3x)-1}{x}$$

$$= \lim_{x \to 0} \frac{6x + 11x^2 + 6x^3}{x}$$

$$= \lim_{x \to 0} 6 + 11x + 6x^2 = 6$$

$$\bullet \lim_{x\to 1}\left(\frac{1}{1-x}-\frac{3}{1-x^3}\right)=\frac{1}{0}-\frac{3}{0} \;?$$

$$=\lim_{x\to 1}\frac{1-x^3-3(1-x)}{(1-x)(1-x^3)}$$

$$=\lim_{x\to 1}\frac{1-x^3-3+3x}{(1-x)(1-x^3)}$$

$$=\lim_{x\to 1}\frac{-x^3+3x-2}{(1-x)(1-x^3)}$$

$$=\lim_{x\to 1}\frac{x^3-3x+2}{(x^3-1)(1-x)}=\frac{0}{0}$$

$x^3-3x+2=(x-1)\,P(x)$

$$\begin{array}{l|l}
x^3-3x+2 & x-1 \\ \cline{2-2}
-x^3+x^2 & x^2+x-2 \\ \cline{1-1}
x^2-3x+2 & \\
-x^2+x & \\ \cline{1-1}
-2x+2 & \\
+2x-2 & \\ \cline{1-1}
0\;0 &
\end{array}$$

$$/\!/\; x^3-3x+2=(x-1)\underbrace{(x^2+x-2)}$$

$$=(x-1)(x-1)(x+2)$$

$$=(x-1)^2(x+2)\;/\!/$$

$$\begin{cases} x^3-1 = (x-1)\ (x^2+x+1) \end{cases}$$

$\therefore$

$$\lim_{x\to 1} \frac{x^3-3x+2}{(x^3-1)(1-x)} =$$

$$= \lim_{x\to 1} \frac{\cancel{(x-1)^2}(x+2)}{\cancel{(x-1)}(x^2+x+1)\ \cancel{(1-x)}}$$

$$= \lim_{x\to 1} - \frac{x+2}{x^2+x+1} = - \frac{1+2}{1+1+1} = -1$$

$$\lim_{x \to 0} \frac{(1+x)(1+2x)(1+3x)\cdots(1+nx) - 1}{x} = \frac{0}{0}$$

Temos:

$$\left\{ \begin{aligned} (1+x)(1+2x)\cdots(1+nx) &= 1 + x + 2x + \cdots + nx \\ &\quad + \underbrace{O(x^2)}_{\substack{\text{termos de} \\ \text{ordem } \geq 2 \\ \text{em } x.}} \\ &= 1 + x\underbrace{(1+2+\cdots+n)} + O(x^2) \\ &= 1 + x\,\frac{n(n+1)}{2} + O(x^2) \end{aligned} \right.$$

$\therefore$

$$\lim_{x \to 0} \frac{(1+x)(1+2x)\cdots(1+nx) - 1}{x}$$

$$= \lim_{x \to 0} \frac{\cancel{1} + \frac{n(n+1)}{2}x + O(x^2) - \cancel{1}}{x}$$

$$= \lim_{x \to 0} \left(\frac{n(n+1)}{2} + O(x) \right) \qquad \left[\frac{O(x^2)}{x} = O(x) \right]$$

$$\lim_{x\to 0}\left(\frac{n(n+1)}{2} + \overset{\nearrow 0}{O(x)}\right)$$

$$= \frac{n(n+1)}{2} \quad //$$

• $\lim_{x\to 0} \dfrac{(1+x)^5-(1+5x)}{x^2+x^5} = \dfrac{1-1}{0} = \dfrac{0}{0}$

1.º método

$(a+b)^n = \sum_{p=0}^{n} \binom{n}{p} a^{n-p} b^p \quad ; \quad \binom{n}{p} = \dfrac{n!}{(n-p)!\,p!}$

Então,

$(1+x)^5 = 1+5x+10x^2+10x^3+5x^4+x^5$

Daí,

$\lim_{x\to 0} \dfrac{(1+x)^5-(1+5x)}{x^2(1+x^3)} =$

$= \lim_{x\to 0} \dfrac{\cancel{1}+\cancel{5x}+10x^2+10x^3+5x^4+x^5-\cancel{1}-\cancel{5x}}{x^2(1+x^3)}$

$= \lim_{x\to 0} \dfrac{\cancel{x^2}(10+\overset{0}{\cancel{10x}}+\overset{0}{\cancel{5x^2}}+\overset{0}{\cancel{x^3}})}{\cancel{x^2}(1+\overset{0}{\cancel{x^3}})}$

$= \underline{\underline{10}}$

• $\lim\limits_{x \to 0} \dfrac{(1+x)^n - (1+nx)}{x^2 + x^n}$

Temos

$$(1+x)^n = \sum_{p=0}^{n} \binom{n}{p} 1^{n-p} x^p = \sum_{p=0}^{n} \binom{n}{p} x^p$$

$$= \binom{n}{0} + \binom{n}{1} x + \binom{n}{2} x^2 + \dots + \binom{n}{n} x^n$$

Então:

$$\left\{ \begin{aligned} (1+x)^n - (1+nx) &= \cancel{\binom{n}{0}} + \cancel{\binom{n}{1} x} + \binom{n}{2} x^2 + \dots \\ &\quad + \binom{n}{n} x^n \cancel{- 1} \cancel{- nx} \\ &= \binom{n}{2} x^2 + \binom{n}{3} x^3 + \dots + \binom{n}{n} x^n \\ &= x^2 \left(\binom{n}{2} + \binom{n}{3} x + \dots + \binom{n}{n} x^{n-2} \right) \end{aligned} \right.$$

$$\lim_{x\to 0} \frac{(1+x)^n - (1+nx)}{x^2 + x^n}$$

$$= \lim_{x\to 0} \frac{\cancel{x^2}\left(\binom{n}{2} + \binom{n}{3}x + \dots + \binom{n}{n}x^{n-2}\right)}{\cancel{x^2}(1+x^{n-2})}$$

$$= \lim_{x\to 0} \frac{\binom{n}{2} + \binom{n}{3}\cancel{x}^{\to 0} + \dots + \binom{n}{n}\cancel{x^{n-2}}^{\to 0}}{1 + \underbrace{x^{n-2}}_{\to 0}}$$

$$= \binom{n}{2} = \frac{n(n-1)}{2}$$

• $\lim\limits_{x \to 0} \dfrac{(1+3x)^2 - (1+2x)^3}{x^2} = \dfrac{0}{0}$

Solução

$\begin{cases} (1+3x)^2 = 1 + 6x + 9x^2 \\ (1+2x)^3 = 1 + 3 \cdot 2x + 3(2x)^2 + 8x^3 \\ \qquad = 1+6x + 12x^2 + 8x^3 \end{cases}$

$\therefore$

$\lim\limits_{x \to 0} \dfrac{(1+3x)^2 - (1+2x)^3}{x^2}$

$= \lim\limits_{x \to 0} \dfrac{\cancel{1}+\cancel{6x}+9x^2 - (\cancel{1}+\cancel{6x}+12x^2+8x^3)}{x^2}$

$= \lim\limits_{x \to 0} \dfrac{9x^2 - 12x^2 - 8x^3}{x^2}$

$= \lim\limits_{x \to 0} \dfrac{-3\cancel{x^2} - 8x^3}{\cancel{x^2}} = \lim\limits_{x \to 0} -3-8x$

$= -3$

$$\lim_{x\to 0} \frac{(1+mx)^n - (1+nx)^m}{x^2}$$

Temos

$$(1+mx)^n = \sum_{p=0}^{n} \binom{n}{p} (mx)^p$$

$$= \binom{n}{0} + \binom{n}{1} mx + \binom{n}{2} m^2x^2 + O(x^3)$$

$$(1+nx)^m = \sum_{p=0}^{m} \binom{m}{p} (nx)^p$$

$$= \binom{m}{0} + \binom{m}{1} nx + \binom{m}{2} n^2x^2 + O(x^3)$$

$\therefore$

$$(1+mx)^n - (1+nx)^m =$$

$$= \overbrace{\binom{n}{0}}^{1} + \overbrace{\binom{n}{1}}^{n} mx + \overbrace{\binom{n}{2}}^{\frac{n(n-1)}{2}} m^2x^2 + O(x^3)$$

$$- \underbrace{\binom{m}{0}}_{1} - \underbrace{\binom{m}{1}}_{m} nx - \underbrace{\binom{m}{2}}_{\frac{m(m-1)}{2}} n^2x^2 + O(x^3)$$

$$= \cancel{1} + \cancel{n m x} + \frac{n(n-1)}{2} m^2 x^2$$

$$- \cancel{1} - \cancel{m n x} - \frac{m(m-1)}{2} n^2 x^2 + O(x^3)$$

$$= \left[\frac{n(n-1)}{2} m^2 - \frac{m(m-1)}{2} n^2\right] x^2 + O(x^3)$$

$$\lim_{x \to 0} \frac{(1+mx)^n - (1+nx)^m}{x^2} =$$

$$= \lim_{x \to 0} \frac{\cancel{x^2}\left[\frac{n(n-1)}{2} m^2 - \frac{m(m-1)}{2} n^2\right] + O(x^3)}{\cancel{x^2}}$$

$$= \lim_{x \to 0} \left[\frac{n(n-1)}{2} m^2 - \frac{m(m-1)}{2} n^2 + \cancelto{0}{O(x)}\right]$$

$$= \frac{n(n-1)}{2} m^2 - \frac{m(m-1)}{2} n^2$$

$$= \frac{\cancel{n^2 m^2} - n m^2 - \cancel{m^2 n^2} + m n^2}{2}$$

$$= \frac{mn(n-m)}{2}$$

$$\lim_{x\to 1} \frac{x^{n+1} - x^n + x - 1}{2x^n - x^{n-1} - 1} = \frac{0}{0}$$

$$\begin{cases} x^{n+1} - x^n + x - 1 = x^n(x-1) + (x-1) \\ \qquad = (x-1)(x^n+1) \end{cases}$$

$$2x^n - x^{n-1} - 1 = (x-1)\,P(x)$$

$$\begin{array}{l|l} \cancel{2x^n} - x^{n-1} - 1 & x-1 \\ \cline{2-2} -\cancel{2x^n} + 2x^{n-1} & 2x^{n-1} + \underbrace{x^{n-2}} + \dots + \underbrace{x}+1 \\ \hline \quad \cancel{x^{n-1}} - 1 & \\ \quad -\cancel{x^{n-1}} + x^{n-2} & \\ \hline \quad x^{n-2} - 1 & \\ \quad \dots & \\ \hline \quad x - 1 & \\ \quad -x + 1 & \\ \hline \quad 0 & \end{array}$$

$\therefore$

$// \; 2x^n - x^{n-1} - 1 =$

$= (x-1)(2x^{n-1} + x^{n-2} + \dots + x + 1) //$

$\therefore$

$$\lim_{x\to 1} \frac{x^{n+1} - x^n + x - 1}{2x^n - x^{n-1} - 1} =$$

$$= \lim_{x\to 1} \frac{\cancel{(x-1)}(x^n+1)}{\cancel{(x-1)}(2x^{n-1} + \underset{1}{x^{n-2}} + \dots + \underset{1}{x} + 1)}$$

$$= \frac{1+1}{2 + \underbrace{1 + \dots + 1}_{n-2} + 1}$$

$$= \frac{2}{2 + n - \cancel{2} + 1} = \frac{2}{n+1}$$

$$\lim_{x \to 1} \frac{x + x^2 + \cdots + x^n - n}{x - 1} = \frac{0}{0}$$

$$\begin{cases} x + x^2 + \cdots + x^n - \underbrace{n} = \\ = x - \underbrace{1} + x^2 - \underbrace{1} + \cdots + x^n - \underbrace{1} \\ = (x-1) + \underbrace{(x-1)(x+1)}_{2\text{ terms}} + \cdots + (x-1)\underbrace{(x^{n-1} + x^{n-2} + \cdots + x + 1)}_{n\text{-terms}} \end{cases}$$

$\therefore$

$$\lim_{x \to 1} \frac{x + x^2 + \cdots + x^n - n}{x - 1} =$$

$$= \lim_{x \to 1} \left\{ \frac{\cancel{(x-1)} \left[1 + (x+1) + (x^2 + x + 1) + \cdots + (x^{n-1} + x^{n-2} + \cdots + x + 1) \right]}{\cancel{(x-1)}} \right\}$$

$$= \lim_{x \to 1} 1 + (x+1) + \cdots + \underbrace{(x^{n-1} + \cdots + x + 1)}$$

$$= 1 + 2 + \cdots + n$$

$$= \frac{n(n+1)}{2} \quad //$$

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$$\lim_{x\to 2} \frac{(x^2-x-2)^{20}}{(x^3-12x+16)^{10}} = \frac{0}{0}$$

$x^3-12x+16 = (x-2)\ P(x)$

$$\begin{array}{l|l} x^3-12x+16 & x-2 \\ -x^3+2x^2 & x^2+2x-8 \\ \hline 2x^2-12x+16 & \\ -2x^2+4x & \\ \hline -8x+16 & \\ +8x-16 & \\ \hline 0\ 0 & \end{array}$$

$x^2+2x-8 = (x-2)(x+4)$

$\therefore$ $//x^3-12x+16 = (x-2)(x^2+2x-8)$

$= (x-2)^2(x+4)//$

$x^2-x-2=0$

$x = 1 \pm \sqrt{1+8}\ /2 = \frac{1\pm 3}{2}$ $\nearrow 2$ $\searrow -1$

$\therefore$ $//x^2-x-2 = (x-2)(x+1)//$

$$\therefore \lim_{x \to 2} \frac{(x^2 - x - 2)^{20}}{(x^3 - 12x + 16)^{10}}$$

$$= \lim_{x \to 2} \frac{\cancel{(x-2)^{20}}(x+1)^{20}}{\cancel{(x-2)^{20}}(x+4)^{10}}$$

$$= \frac{3^{20}}{6^{10}} = \frac{3^{20}}{3^{10} \cdot 2^{10}} = \frac{3^{10}}{2^{10}}$$

$$\lim_{x\to 1} \frac{x^{100}-2x+1}{x^{50}-2x+1} =$$

$$\left\{ \begin{array}{l|l}
\cancel{x^{100}} - 2x + 1 & x-1 \\
\cancel{-x^{100}} + x^{99} & x^{99} + \underbrace{x^{98}} + \dots + x^2 + x - 1 \\
\hline
\cancel{x^{99}} - 2x + 1 & \\
\cancel{-x^{99}} + x^{98} & \\
\hline
\underbrace{x^{98} - 2x + 1} & \vdots \\
\dots & \\
\hline
\cancel{x^2} - 2x + 1 & \\
\cancel{-x^2} + x & \\
\hline
-x + 1 &
\end{array} \right.$$

$$// \ x^{100} - 2x + 1 = (x-1)(x^{99} + \dots + x - 1)$$

$$\left\{ \begin{array}{l|l}
\cancel{x^{50}} - 2x + 1 & x-1 \\
\cancel{-x^{50}} + x^{49} & x^{49} + x^{48} + \underbrace{x^{47}} + \dots + x^2 + \\
\hline
\cancel{x^{49}} - 2x + 1 & + x - 1 \\
\cancel{-x^{49}} + x^{48} & \\
\hline
\cancel{x^{48}} - 2x + 1 & \vdots \\
\cancel{-x^{48}} + x^{47} & \\
\hline
\underbrace{x^{47} - 2x + 1} & \\
\dots & \\
\hline
\cancel{x^2} - 2x + 1 & \\
\cancel{-x^2} + x & \\
\hline
-x + 1 & \\
+x - 1 & \\
\hline
0 &
\end{array} \right.$$

$$// \ (x^{50} - 2x + 1) =$$

$$= (x-1)(x^{49} + \dots + x - 1) \ //$$

$$\lim_{x\to 1} \frac{\cancel{(x-1)}(\overbrace{x^{99}+x^{98}+\cdots+x}^{99\text{ terms}}-1)}{\cancel{(x-1)}(x^{49}+x^{48}+\cdots+x-1)} =$$

$$= \frac{\overbrace{1+1+\cdots+1}^{99\text{ terms}}-1}{\underbrace{1+1+\cdots+1}_{49\text{ terms}}-1}$$

$$= \frac{98}{48} = \frac{49}{24}$$

• $\lim_{x \to a} \frac{\sqrt{x} - \sqrt{a}}{x - a} = \frac{0}{0}$

$\frac{\sqrt{x} - \sqrt{a}}{x-a} = \frac{\sqrt{x} - \sqrt{a}}{x-a} \cdot \frac{\sqrt{x} + \sqrt{a}}{\sqrt{x} + \sqrt{a}}$

$= \frac{\cancel{x-a}}{\cancel{(x-a)}(\sqrt{x} + \sqrt{a})} = \frac{1}{\sqrt{x} + \sqrt{a}}$

$\therefore$

$\lim_{x \to a} \frac{\sqrt{x} - \sqrt{a}}{x-a} = \lim_{x \to a} \frac{1}{\sqrt{x} + \sqrt{a}} = \frac{1}{2\sqrt{a}}$

•

$$\lim_{x\to 7} \frac{2-\sqrt{x-3}}{x^2-49} = \frac{0}{0}$$

$$= \lim_{x\to 7} \frac{2-\sqrt{x-3}}{x^2-49} \cdot \frac{2+\sqrt{x-3}}{2+\sqrt{x-3}}$$

$$= \lim_{x\to 7} \frac{4-(x-3)}{(x-7)(x+7)(2+\sqrt{x-3})}$$

$$= \lim_{x\to 7} \frac{-\cancel{(7-x)}}{\cancel{(x-7)}(x+7)(2+\sqrt{x-3})}$$

$$= \lim_{x\to 7} -\frac{1}{(x+7)(2+\sqrt{x-3})}$$

$$= -\frac{1}{14(2+2)} =$$

$$= -\frac{1}{56}$$

$$\lim_{x\to 1} \frac{\sqrt{x}-1}{x-1} = \frac{0}{0}$$

$$= \lim_{x\to 1} \frac{\sqrt{x}-1}{x-1} \frac{\sqrt{x}+1}{\sqrt{x}+1}$$

$$= \lim_{x\to 1} \frac{\cancel{(x-1)}}{\cancel{(x-1)}(\sqrt{x}+1)}$$

$$= \lim_{x\to 1} \frac{1}{\sqrt{x}+1} = \frac{1}{2} //$$

$$\lim_{x \to 64} \frac{\sqrt{x} - 8}{\sqrt[3]{x} - 4}$$

Seja

$$\begin{cases} y^6 = x & ; \quad x \to 64 \,,\; y \to 2 \\ y^3 = x^{1/2} \\ y^2 = x^{1/3} \end{cases}$$

$\therefore$

$$\lim_{x \to 64} \frac{\sqrt{x} - 8}{\sqrt[3]{x} - 4} =$$

$$= \lim_{y \to 2} \frac{y^3 - 8}{y^2 - 4}$$

$$= \lim_{y \to 2} \frac{\cancel{(y-2)}(y^2+2y+4)}{\cancel{(y-2)}(y+2)}$$

$$= \frac{4+4+4}{4} = \frac{12}{4} = 3 \;//$$

Obs. : $y^3-8 = (y-2)\,P(x)$

$$
\begin{array}{l|l}
\cancel{y^3}-8 & y-2 \\ \cline{2-2}
-\cancel{y^3}+2y^2 & y^2+2y+4 \\ \cline{1-1}
\cancel{2y^2}-8 & \\
-\cancel{2y^2}+4y & \\ \cline{1-1}
4y-8 & \\
-4y+8 & \\ \cline{1-1}
0 &
\end{array}
$$

$\therefore$ $y^3-8=(y-2)(y^2+2y+4)$

$$\lim_{x\to 1} \frac{\sqrt[3]{x}-1}{\sqrt[4]{x}-1} = \frac{0}{0}$$

Seja

$$\begin{cases} y^{12} = x \quad ; \quad x\to 1 \Leftrightarrow y\to 1 \\ y^4 = x^{1/3} \\ y^3 = x^{1/4} \end{cases}$$

$\therefore$

$$\lim_{x\to 1} \frac{\sqrt[3]{x}-1}{\sqrt[4]{x}-1} = \lim_{y\to 1} \frac{y^4-1}{y^3-1}$$

$$= \lim_{y\to 1} \frac{\cancel{(y-1)}(y^3+y^2+y+1)}{\cancel{(y-1)}(y^2+y+1)}$$

$$= \frac{1+1+1+1}{1+1+1} = \frac{4}{3}$$

$$
\begin{array}{l|l}
y^4 - 1 & y - 1 \\
\hline
-y^4 + y^3 & y^3 + y^2 + y + 1 \\
\hline
y^3 - 1 & \\
-y^3 + y^2 & \\
\hline
y^2 - 1 & \\
-y^2 + y & \\
\hline
y - 1 & \\
-y + 1 & \\
\hline
0 \quad 0 &
\end{array}
$$

$$\bullet \lim_{x\to 1} \frac{\sqrt[5]{x}-1}{\sqrt[3]{x}-1} = \frac{0}{0}$$

Seja
$$\begin{cases} y^{15} = x \quad ; \quad x \to 1 \Leftrightarrow y \to 1 \\ y^3 = x^{1/5} \\ y^5 = x^{1/3} \end{cases}$$

$\therefore$

$$\lim_{x\to 1} \frac{\sqrt[5]{x}-1}{\sqrt[3]{x}-1} = \lim_{y\to 1} \frac{y^3-1}{y^5-1}$$

$$= \lim_{y\to 1} \frac{\cancel{(y-1)}(y^2+y+1)}{\cancel{(y-1)}(y^4+y^3+y^2+y+1)}$$

$$= \frac{1+1+1}{5} = \frac{3}{5}$$

$$\lim_{x \to 1} \frac{\sqrt[p]{x} - 1}{\sqrt[n]{x} - 1}$$

$$\begin{cases} y^{np} = x & , \quad x \to 1 \Leftrightarrow y \to 1 \\ y^{n} = x^{\frac{1}{p}} \\ y^{p} = x^{1/n} \end{cases}$$

$$\lim_{x \to 1} \frac{\sqrt[p]{x} - 1}{\sqrt[n]{x} - 1} = \lim_{y \to 1} \frac{y^{n} - 1}{y^{p} - 1}$$

$$= \lim_{y \to 1} \frac{\cancel{(y-1)} \overbrace{(y^{n-1} + y^{n-2} + \cdots + y + 1)}^{n\text{-times}}}{\cancel{(y-1)} \underbrace{(y^{p-1} + \cdots + y + 1)}_{p\text{ times}}}$$

$$= \frac{n}{p}$$

$$\lim_{x\to 7} \frac{2-\sqrt{x-3}}{x^2-49} = \frac{0}{0}$$

$$= \lim_{x\to 7} \frac{(2-\sqrt{x-3})(2+\sqrt{x-3})}{(x^2-49)(2+\sqrt{x-3})}$$

$$= \lim_{x\to 7} \frac{4-(x-3)}{(x-7)(x+7)(2+\sqrt{x-3})}$$

$$= \lim_{x\to 7} \frac{-\cancel{(x-7)}}{\cancel{(x-7)}(x+7)(2+\sqrt{x-3})}$$

$$= \lim_{x\to 7} \frac{-1}{(x+7)(2+\sqrt{x-3})}$$

$$= \frac{-1}{14 \cdot 4} = \frac{-1}{56}$$

• $\lim\limits_{x \to 8} \dfrac{x-8}{\sqrt[3]{x}-2}$

Sejá

$$\sqrt[3]{x} = y \Leftrightarrow \begin{cases} x = y^3 \\ x \to 8 \Leftrightarrow y \to 2 \end{cases}$$

∴

$$\lim_{x \to 8} \frac{x-8}{\sqrt[3]{x}-2} = \lim_{y \to 2} \frac{y^3-8}{y-2} = \frac{0}{0}$$

$$= \lim_{y \to 2} \frac{\cancel{(y-2)}(y^2+2y+4)}{\cancel{(y-2)}}$$

$$\begin{array}{l|l} y^3 - 8 & y-2 \\ \hline -y^3 + 2y^2 & y^2 + 2y + 4 \\ \hline 2y^2 - 8 & \\ -2y^2 + 4y & \\ \hline 4y - 8 & \\ -4y + 8 & \\ \hline 0 & \end{array}$$

$$= \lim_{y \to 2} (y^2 + 2y + 4)$$

$$= 4 + 4 + 4$$

$$= 12$$

• $\lim\limits_{x \to 1} \dfrac{\sqrt{x} - 1}{\sqrt[3]{x} - 1}$

Seja

$$y = \sqrt[6]{x} \Leftrightarrow \begin{cases} y^6 = x \\ x \to 1 \Leftrightarrow y \to 1 \end{cases}$$

$$\lim_{x \to 1} \frac{\sqrt{x} - 1}{\sqrt[3]{x} - 1} = \lim_{y \to 1} \frac{y^3 - 1}{y^2 - 1} = \frac{0}{0}$$

$$= \lim_{y \to 1} \frac{\cancel{(y-1)}(y^2 + y + 1)}{\cancel{(y-1)}(y+1)}$$

$$= \frac{1+1+1}{1+1} = \frac{3}{2}$$

$$\begin{array}{l|l} y^3 - 1 & y - 1 \\ \cline{2-2} -\cancel{y^3} + y^2 & y^2 + y + 1 \\ \hline \cancel{y^2} - 1 & \\ -\cancel{y^2} + y & \\ \hline y - 1 & \end{array}$$

$$\cdot \lim_{x\to 4} \frac{3-\sqrt{5+x}}{1-\sqrt{5-x}} = \frac{0}{0}$$

$$= \lim_{x\to 4} \frac{\left(3-\sqrt{5+x}\right)\cdot\left(\frac{3+\sqrt{5+x}}{3+\sqrt{5+x}}\right)}{\left(1-\sqrt{5-x}\right)\cdot\left(\frac{1+\sqrt{5-x}}{1+\sqrt{5-x}}\right)}$$

$$= \lim_{x\to 4} \frac{\frac{9-(5+x)}{3+\sqrt{5+x}}}{\frac{1-(5-x)}{1-\sqrt{5-x}}}$$

$$= \lim_{x\to 4} \frac{\frac{4+x}{3+\sqrt{5+x}}}{\frac{-4+x}{1+\sqrt{5-x}}} =$$

$$= \lim_{x\to 4} \frac{\cancel{(4+x)}}{3+\sqrt{5+x}} \cdot \frac{1+\sqrt{5-x}}{\cancel{(-4+x)}} \quad -$$

$$= \lim_{x\to 4} (-)\frac{1+\sqrt{5-x}}{3+\sqrt{5+x}} = -\frac{2}{6} = \frac{-1}{3}$$

$$\lim_{x\to 0} \frac{\sqrt{1+x} - \sqrt{1-x}}{x} = \frac{0}{0}$$

$$= \lim_{x\to 0} \frac{\sqrt{1+x} - \sqrt{1-x}}{x} \cdot \frac{\sqrt{1+x} + \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}}$$

$$= \lim_{x\to 0} \frac{\cancel{1}+x - (\cancel{1}-x)}{x\left(\sqrt{1+x} + \sqrt{1-x}\right)}$$

$$= \lim_{x\to 0} \frac{2\cancel{x}}{\cancel{x}\left(\sqrt{1+x} + \sqrt{1-x}\right)}$$

$$= \lim_{x\to 0} \frac{2}{\sqrt{1+x} + \sqrt{1-x}}$$

$$= \frac{2}{1+1} = 1$$

• $$\lim_{h\to 0} \frac{\sqrt{x+h}-\sqrt{x}}{h} \quad (x>0) = \frac{0}{0}$$

$$= \lim_{h\to 0} \frac{\sqrt{x+h}-\sqrt{x}}{h} \cdot \left(\frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}\right)$$

$$= \lim_{h\to 0} \frac{\cancel{x}+h-\cancel{x}}{h\left(\sqrt{x+h}+\sqrt{x}\right)}$$

$$= \lim_{h\to 0} \frac{\cancel{h}}{\cancel{h}\left(\sqrt{x+h}+\sqrt{x}\right)}$$

$$= \frac{1}{2\sqrt{x}}$$

- $\lim_{h\to 0} \dfrac{\sqrt[3]{x+h} - \sqrt[3]{x}}{h}$ $(x \neq 0)$

Sej:

$$\begin{cases} y = \sqrt[3]{x} \Leftrightarrow y^3 = x \\ z = \sqrt[3]{x+h} \Leftrightarrow z^3 = x + h = y^3 + h \\ h \to 0 \Leftrightarrow z \to \sqrt[3]{x} \quad ; \quad h = z^3 - y^3 \end{cases}$$

$\therefore$

$$\lim_{h\to 0} \frac{\sqrt[3]{x+h} - \sqrt[3]{x}}{h} =$$

$$= \lim_{z \to x^{1/3}} \frac{z - y}{z^3 - y^3}$$

$$= \lim_{z \to x^{1/3}} \frac{\cancel{(z-y)}}{\cancel{(z-y)}(z^2 + zy + y^2)}$$

$$= \lim_{z \to x^{1/3}} \frac{1}{z^2 + \underbrace{zy}_{x^{1/3}} + \underbrace{y^2}_{x^{\frac{2}{3}}}} = \frac{1}{x^{\frac{2}{3}} + x^{\frac{1}{3}} x^{\frac{1}{3}} + x^{\frac{2}{3}}}$$

$$= \frac{1}{3x^{2/3}} \quad //$$

Obs.:

$$
\begin{array}{l|l}
\cancel{z^3} - y^3 & z - y \\
\cline{2-2}
-\cancel{z^3} + z^2 y & z^2 + zy + y^2 \\
\hline
\cancel{z^2 y} - y^3 & \\
-\cancel{z^2 y} + zy^2 & \\
\hline
zy^2 - y^3 & \\
-zy^2 + y^3 & \\
\hline
0 \quad 0 &
\end{array}
$$

$\therefore$

$$z^3 - y^3 = (z-y)(z^2 + zy + y^2)$$

$$\lim_{x\to 3} \frac{\sqrt{x^2-2x+6} - \sqrt{x^2+2x-6}}{x^2-4x+3}$$

$$= \lim_{x\to 3} \frac{\sqrt{x^2-2x+6} - \sqrt{x^2+2x-6}}{x^2-4x+3} \cdot \frac{\sqrt{x^2-2x+6}+\sqrt{x^2+2x-6}}{\sqrt{x^2-2x+6}+\sqrt{x^2+2x-6}}$$

$$= \lim_{x\to 3} \frac{\cancel{x^2}-2x+6-(\cancel{x^2}+2x-6)}{(x^2-4x+3)\left(\sqrt{x^2-2x+6}+\sqrt{x^2+2x-6}\right)}$$

$$= \lim_{x\to 3} \frac{-4x+12}{\underline{(x^2-4x+3)}\left(\sqrt{x^2-2x+6}+\sqrt{x^2+2x-6}\right)}$$

$$= \lim_{x\to 3} \frac{-4\cancel{(x-3)}}{\cancel{(x-3)}(x-1)\left(\sqrt{x^2-2x+6}+\sqrt{x^2+2x-6}\right)}$$

$$= \lim_{x\to 3} \frac{-4}{(x-1)\left(\sqrt{x^2-2x+6}+\sqrt{x^2+2x-6}\right)}$$

$$= \frac{-\cancel{4}^{\,2}}{\cancel{2}(3+3)} = -\frac{2}{6} = -\frac{1}{3}$$

$x^2-4x+3=0$

$x = 4 \pm \sqrt{16-12}/2$

$= \frac{4\pm 2}{2} \to 3,\ 1$

- $\lim\limits_{x\to 0} \dfrac{\sqrt{1+x} - 1}{\sqrt[3]{1+x} - 1} = \dfrac{0}{0}$

Seja

$$\begin{cases} y^6 = (1+x) \quad ; \quad x \to 0 \Leftrightarrow y \to 1 \\ y^3 = (1+x)^{1/2} \\ y^2 = (1+x)^{1/3} \end{cases}$$

$\therefore$

$$\lim_{x\to 0} \frac{\sqrt{1+x} - 1}{\sqrt[3]{1+x} - 1} =$$

$$= \lim_{y\to 1} \frac{y^3 - 1}{y^2 - 1}$$

$$= \lim_{y\to 1} \frac{\cancel{(y-1)}(y^2+y+1)}{\cancel{(y-1)}(y+1)}$$

$$= \frac{3}{2}$$

$$\lim_{x\to 1} \frac{\sqrt[3]{x^2} - 2\sqrt[3]{x} + 1}{(x-1)^2} = \frac{0}{0}$$

$y^3 = x \;\; \Leftarrow \;\; x \to 1 \Rightarrow y \to 1$

$$\lim_{x\to 1} \frac{x^{\frac{2}{3}} - 2x^{\frac{1}{3}} + 1}{(x-1)^2} =$$

$$= \lim_{y\to 1} \frac{y^2 - 2y + 1}{(y^3 - 1)^2}$$

$$= \lim_{y\to 1} \frac{(y-1)^2}{[(y-1)(y^2+y+1)]^2}$$

$$= \lim_{y\to 1} \frac{\cancel{(y-1)^2}}{\cancel{(y-1)^2}(y^2+y+1)^2}$$

$$\begin{array}{r|l} y^3-1 & y-1 \\ \cline{2-2} -y^3+y^2 & y^2+y+1 \\ \cline{1-1} y^2-1 & \\ -y^2+y & \\ \cline{1-1} y-1 & \end{array}$$

$$= \lim_{y\to 1} \frac{1}{(y^2+y+1)^2} = \frac{1}{(3)^2}$$

$$= \frac{1}{9}$$

$$\lim_{x \to 3} \frac{x^2 - 5x + 6}{\sqrt{x+1} - \sqrt{3x-5}} = \frac{0}{0}$$

$$= \lim_{x \to 3} \frac{(x^2-5x+6)}{(\sqrt{x+1} - \sqrt{3x-5})} \frac{(\sqrt{x+1} + \sqrt{3x-5})}{(\sqrt{x+1} + \sqrt{3x-5})}$$

$$= \lim_{x \to 3} \frac{(x^2-5x+6)(\sqrt{x+1} + \sqrt{3x-5})}{x+1-(3x-5)}$$

$$= \lim_{x \to 3} \frac{(x^2-5x+6)(\sqrt{x+1} + \sqrt{3x-5})}{-2x+6}$$

$$= \lim_{x \to 3} \frac{(x^2-5x+6)(\sqrt{x+1} + \sqrt{3x-5})}{-2(x-3)}$$

$$= \lim_{x \to 3} \frac{\cancel{(x-3)}(x-2)(\sqrt{x+1} + \sqrt{3x-5})}{-2\cancel{(x-3)}}$$

$$= \frac{1 \cdot (2+2)}{-2} = -2$$

$$(x^2-5x+6 = (x-3)(x-2))$$

$$\bullet \lim_{x\to -3} \frac{x^2+2x-3}{x+1+\sqrt[3]{x^2-1}} = \frac{0}{0}$$

Seja

$$\begin{cases} z = \sqrt[3]{x^2-1} \\ a = x+1 \end{cases}$$

$$z^3+a^3 = (z+a)(z^2-az+a^2)$$

$$\Leftrightarrow$$

$$z+a = \frac{z^3+a^3}{z^2-az+a^2}$$

$$\sqrt[3]{x^2-1} + x+1 = \frac{x^2-1+(x+1)^3}{(x^2-1)^{\frac{2}{3}} - (x+1)(x^2-1)^{\frac{1}{3}} + (x+1)^2}$$

$$= \frac{\underbrace{x^2}-\cancel{1}+x^3+3x+\underbrace{3x^2}+\cancel{1}}{}$$

$$= (x^3+3x+4x^2)/(\dots)$$

$$= x(\underbrace{x^2+4x+3})/(\dots)$$

$$= \frac{x(x+3)(x+1)}{(x^2-1)^{2/3}-(x+1)(x^2-1)^{1/3}+(x+1)^2}$$

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$$x^2+2x-3 = (x+3)(x-1)$$

$\therefore$

$$\lim_{x\to -3} \frac{x^2+2x-3}{x+1+\sqrt[3]{x^2-1}} =$$

$$= \lim_{x\to -3} \frac{\cancel{(x+3)}(x-1)}{\dfrac{x\cancel{(x+3)}(x+1)}{(x^2-1)^{2/3}-(x+1)(x^2-1)^{1/3}+(x+1)^2}}$$

$$= \lim_{x\to -3} \frac{\left[(x^2-1)^{2/3}-(x+1)(x^2-1)^{1/3}+(x+1)^2\right](x-1)}{x(x+1)}$$

$$= \frac{\left[(8)^{2/3}-(-2)(8)^{1/3}+(-2)^2\right](-4)}{(-3)(-2)}$$

$$= \frac{(4+4+4)(-4)}{6}$$

$$= -8$$

$$\lim_{x\to 1} \frac{\sqrt[3]{4x^5-3} - 1}{3x^2 - x - 2}$$

$$\begin{cases} y = \sqrt[3]{4x^5-3} \\ a = 1 \end{cases}$$

$$y^3 - a^3 = (y-a)(y^2 + ay + a^2)$$

$$\underbrace{4x^5 - 3 - 1}_{4x^5-4} = \left(\sqrt[3]{4x^5-3} - 1\right)\left((4x^5-3)^{\frac{2}{3}} + (4x^5-3)^{1/3} + 1\right)$$

$\therefore$

$$\sqrt[3]{4x^5-3} - 1 = \frac{4x^5-4}{(4x^5-3)^{\frac{2}{3}} + (4x^5-3)^{\frac{1}{3}} + 1}$$

$$3x^2 - x - 2 = 3(x-1)\left(x + \frac{2}{3}\right) = (x-1)(3x+2)$$

$\therefore$

$$\lim_{x\to 1} \frac{\sqrt[3]{4x^5-3}-1}{3x^2-x-2}$$

$$= \lim_{x\to 1} \frac{\dfrac{4(x^5-1)}{(4x^5-3)^{\frac{2}{3}}+(4x^5-3)^{\frac{1}{3}}+1}}{(x-1)(3x+2)}$$

$$= \lim_{x\to 1} \frac{4\cancel{(x-1)}(x^4+x^3+x^2+x+1)}{\cancel{(x-1)}(3x+2)\left[(4x^5-3)^{\frac{2}{3}}+(4x^5-3)^{\frac{1}{3}}+1\right]}$$

$$= \frac{4\cdot(\cancel{5})}{\cancel{5}\cdot[1+1+1]} = \frac{4}{3}$$

• $$\lim_{x \to 1} \frac{x^2 - 1}{\sqrt[5]{2x^5 - 1} - 1}$$

Seja

$$y = \sqrt[5]{2x^5 - 1}$$

Então

$$y^5 - 1 = (y-1)(y^4 + y^3 + y^2 + y + 1)$$
$$\Leftrightarrow$$
$$y - 1 = \frac{y^5 - 1}{y^4 + y^3 + y^2 + y + 1}$$

$\therefore$

$$\left\{ \begin{aligned} \sqrt[5]{2x^5 - 1} - 1 &= \frac{2x^5 - 1 - 1}{(2x^5-1)^{\frac{4}{5}} + (2x^5-1)^{\frac{3}{5}} + \dots + 1} \\ &= \frac{2(x^5 - 1)}{(\dots)} \\ &= \frac{2(x-1)(x^4 + x^3 + \dots + 1)}{(2x^5-1)^{4/5} + \dots + 1} \end{aligned} \right.$$

$$\left\{ x^2 - 1 = (x+1)(x-1) \right.$$

$\therefore$

$$\lim_{x \to 1} \frac{x^2 - 1}{\sqrt[5]{2x^5 - 1} - 1} =$$

$$= \lim_{x \to 1} \frac{(x+1)\cancel{(x-1)}}{\dfrac{2\cancel{(x-1)}(x^4 + x^3 + \dots + x + 1)}{(2x^5-1)^{\frac{4}{5}} + (2x^5-1)^{\frac{3}{5}} + (2x^5-1)^{\frac{2}{5}} + (2x^5-1)^{\frac{1}{5}} + 1}}$$

$$= \lim_{x \to 1} \frac{(x+1)\left[(2x^5-1)^{\frac{4}{5}} + (2x^5-1)^{\frac{3}{5}} + \dots + 1\right]}{2(x^4 + x^3 + \dots + x + 1)}$$

$$= \frac{2\left[1 + 1 + 1 + 1 + 1\right]}{2(4+1)}$$

$$= \cancel{2}\ 1$$

$$\bullet \lim_{x\to a} \frac{\sqrt{a-x} + \sqrt{a} - \sqrt{x}}{\sqrt{a^2-x^2}} = \frac{0}{0}$$

$$= \lim_{x\to a} \frac{\cancel{\sqrt{a-x}}\left[1 + \frac{\sqrt{a}-\sqrt{x}}{\sqrt{a-x}}\right]}{\cancel{\sqrt{a-x}}\sqrt{a+x}}$$

$$= \lim_{x\to a} \frac{\left(1 + \frac{\sqrt{a}-\sqrt{x}}{\sqrt{a-x}}\right)}{\sqrt{a+x}}$$

$$= \lim_{x\to a} \frac{1 + \frac{\sqrt{a}-\sqrt{x}}{\sqrt{a-x}} \cdot \frac{\sqrt{a}+\sqrt{x}}{\sqrt{a}+\sqrt{x}}}{\sqrt{a+x}}$$

$$= \lim_{x\to a} \frac{1 + \frac{a-x}{\sqrt{a-x}(\sqrt{a}+\sqrt{x})}}{\sqrt{a+x}}$$

$$= \lim_{x\to a} \frac{1 + \frac{\cancel{\sqrt{a-x}}^{\,0}}{\sqrt{a}+\sqrt{x}}}{\sqrt{a+x}} = \frac{1}{\sqrt{a+a}}$$

$$= \frac{1}{\sqrt{2a}}$$

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Limites quando $x \to \infty$

44. $\lim\limits_{x \to +\infty} \dfrac{(x+1)^2}{x^2+1} = \lim\limits_{x \to +\infty} \dfrac{x^2+2x+1}{x^2+1}$

$= \lim\limits_{x \to +\infty} \dfrac{\cancel{x^2}\left(1+\frac{2}{x}+\frac{1}{x^2}\right)}{\cancel{x^2}\left(1+\frac{1}{x^2}\right)}$

$= 1$ //

45. $\lim\limits_{x \to +\infty} \dfrac{1000x}{x^2-1} = \lim\limits_{x \to +\infty} \dfrac{1000}{x-\frac{1}{x}} = \dfrac{1000}{\infty}$

$= 0$ //

46. $\lim\limits_{x \to +\infty} \dfrac{x^2-5x+1}{3x+7} = \lim\limits_{x \to +\infty} \dfrac{x^2\left(1-\frac{5}{x}+\frac{1}{x^2}\right)}{x\left(3+\frac{7}{x}\right)}$

$= \lim\limits_{x \to +\infty} x \dfrac{\left(1-\cancel{\frac{5}{x}}^{\to 0}+\cancel{\frac{1}{x^2}}^{\to 0}\right)}{3+\cancel{\frac{7}{x}}^{\to 0}}$

$= \lim\limits_{x \to +\infty} \dfrac{x}{3} = +\infty$ //

47. $\lim\limits_{x\to+\infty} \dfrac{2x^2-x+3}{x^3-8x+5} = \lim\limits_{x\to+\infty} \dfrac{x^2\left(2-\frac{1}{x}+\frac{3}{x^2}\right)}{x^3\left(1-\frac{8}{x^2}+\frac{5}{x^3}\right)}$

$= \lim\limits_{x\to+\infty} \dfrac{1}{x} \dfrac{\left(2-\cancel{\frac{1}{x}}^{\to 0}+\cancel{\frac{3}{x^2}}^{\to 0}\right)}{1-\cancel{\frac{8}{x^2}}^{\to 0}+\cancel{\frac{5}{x^3}}^{\to 0}}$

$= 0$ //

48. $\lim\limits_{x\to+\infty} \dfrac{(2x+3)^3(3x-2)^2}{x^5+5} = \lim\limits_{x\to+\infty} \dfrac{\left[(2x)\left(1+\frac{3}{2x}\right)\right]^3\left[3x\left(1-\frac{2}{3x}\right)\right]^2}{x^5+5}$

$= \lim\limits_{x\to+\infty} \dfrac{8x^3\, 9x^2 \left(1+\frac{3}{2x}\right)^3\left(1-\frac{2}{3x}\right)^2}{x^5+5}$

$= \lim\limits_{x\to+\infty} \dfrac{72x^5 \cdot \left(1+\frac{3}{2x}\right)^3\left(1-\frac{2}{3x}\right)^2}{x^5\left(1+\frac{5}{x^5}\right)}$

$= \lim\limits_{x\to+\infty} \dfrac{72\cancel{x^5}}{\cancel{x^5}} = 72$ //

49. $\lim\limits_{x\to+\infty} \dfrac{2x^2-3x-4}{\sqrt{x^4+1}} = \lim\limits_{x\to+\infty} \dfrac{2x^2\left(1-\frac{3}{2x}-\frac{2}{x^2}\right)}{\sqrt{x^4\left(1+\frac{1}{x^4}\right)}}$

$= \lim\limits_{x\to+\infty} \dfrac{2\cancel{x^2}}{\cancel{x^2}} \dfrac{\left(1-\frac{3}{2x}-\frac{2}{x^2}\right)}{\sqrt{1+\frac{1}{x^4}}}$

$= 2$ //

50. $\lim_{x \to +\infty} \dfrac{2x+3}{x+\sqrt[3]{x}} =$

$\dfrac{x^{1/3}}{x} = x^{1/3-1} = x^{-2/3} = \dfrac{1}{x^{2/3}}$

$= \lim_{x \to +\infty} \dfrac{\cancel{x}(2+\frac{3}{x}\to 0)}{\cancel{x}(1+\frac{1}{x^{2/3}}\to 0)}$

$= 2$

51. $\lim_{x \to \infty} \dfrac{x^2}{10+x\sqrt{x}} = \lim_{x \to +\infty} \dfrac{x^2}{x^{3/2}+10}$

$= \quad = \lim_{x \to +\infty} \dfrac{1}{\frac{x^{3/2}}{x^2}+\frac{10}{x^2}}$

$\frac{3}{2} - 2 = \frac{3-4}{2} = -\frac{1}{2}$

$= \lim_{x \to +\infty} \dfrac{1}{x^{-\frac{1}{2}}+\frac{10}{x^2}}$

$= \lim_{x \to +\infty} \dfrac{1}{\frac{1}{\sqrt{x}}+\frac{10}{x^2}} = +\infty$

52. $\lim_{x \to +\infty} \dfrac{\sqrt[3]{x^2+1}}{x+1} = \lim_{x \to +\infty} \dfrac{\sqrt[3]{x^2(1+\frac{1}{x^2})}}{x+1}$

$= \lim_{x \to +\infty} \dfrac{x^{\frac{2}{3}}\sqrt[3]{1+\frac{1}{x^2}}}{x(1+\frac{1}{x})}$ $\qquad \frac{2}{3}-1 = -\frac{1}{3}$

$= \lim_{x \to +\infty} x^{-\frac{1}{3}} \dfrac{\sqrt[3]{1+\frac{1}{x^2}}}{1+\frac{1}{x}}$

$= \lim_{x \to +\infty} \overset{\to 0}{\frac{1}{\sqrt[3]{x}}} \dfrac{\sqrt[3]{1+\frac{1}{x^2}\to 0}}{1+\frac{1}{x}\to 0} = 0$

tilibra

53. $\lim_{x\to+\infty} \frac{\sqrt{x}}{\sqrt{x+\sqrt{x+\sqrt{x}}}}$

$= \lim_{x\to+\infty} \sqrt{\frac{x}{x+\sqrt{x+\sqrt{x}}}}$

$= \lim_{x\to+\infty} \sqrt{\frac{1}{1+\frac{\sqrt{x+\sqrt{x}}}{x}}}$

$= \lim_{x\to+\infty} \sqrt{\frac{1}{1+\sqrt{\frac{x+\sqrt{x}}{x^2}}}}$

$= \lim_{x\to+\infty} \sqrt{\frac{1}{1+\sqrt{\frac{1}{x}+\frac{\sqrt{x}}{x^2}}}}$ $\qquad \frac{\sqrt{x}}{x^2} = \frac{x^{1/2}}{x^2} = x^{\frac{1}{2}-2} = x^{-\frac{3}{2}}$

$= \lim_{x\to+\infty} \sqrt{\frac{1}{1+\sqrt{\underbrace{\frac{1}{x}}_{\to 0}+\underbrace{\frac{1}{x^{3/2}}}_{\to 0}}}}$

$= 1$ //

54. $\lim_{x\to\infty} \frac{x}{\sqrt[3]{x^3+10}} = \lim_{x\to+\infty} \frac{1}{\frac{\sqrt[3]{x^3+10}}{x}}$

$= \lim_{x\to+\infty} \frac{1}{\sqrt[3]{\frac{x^3}{x^3}+\frac{10}{x^3}}} = \lim_{x\to+\infty} \frac{1}{\sqrt[3]{1+\underbrace{\frac{10}{x^3}}_{\to 0}}}$

$= 1$ //

tilibra

55. $\lim_{x \to +\infty} \left(\sqrt{x+a} - \sqrt{x} \right) = \infty - \infty$

$= \lim_{x \to +\infty} \left(\sqrt{x+a} - \sqrt{x} \right) \frac{\left(\sqrt{x+a} + \sqrt{x} \right)}{\sqrt{x+a} + \sqrt{x}}$

$= \lim_{x \to +\infty} \frac{\cancel{x} + a - \cancel{x}}{\sqrt{x+a} + \sqrt{x}}$

$= \lim_{x \to +\infty} \frac{a}{\sqrt{x+a} + \sqrt{x}} = 0 //$

56. $\lim_{x \to +\infty} \left(\sqrt{x(x+a)} - x \right) = \infty - \infty$

Não!

$= \lim_{x \to +\infty} \left(\sqrt{x^2 \left(1 + \frac{a}{x}\right)} - x \right)$

$= \lim_{x \to +\infty} \left(x \sqrt{1 + \frac{a}{x}} - x \right)$

$= \lim_{x \to +\infty} x \left(\sqrt{1 + \frac{a}{x}} - 1 \right) = \infty . 0 \; ?$

$\lim_{x \to +\infty} \left(\sqrt{x(x+a)} - x \right) \frac{\left(\sqrt{x(x+a)} + x \right)}{\sqrt{x(x+a)} + x} =$

$= \lim_{x \to +\infty} \frac{x(x+a) - x^2}{\sqrt{x(x+a)} + x}$

$= \lim_{x \to +\infty} \frac{\cancel{x^2} + ax - \cancel{x^2}}{\sqrt{x(x+a)} + x} =$

$$= \lim_{x \to +\infty} \frac{ax}{\sqrt{x(x+a)} + x}$$

$$= \lim_{x \to +\infty} \frac{a}{\sqrt{\frac{x(x+a)}{x^2}} + 1}$$

$$= \lim_{x \to +\infty} \frac{a}{\sqrt{1 + \frac{a}{x}} + 1}$$

$$= \frac{a}{1+1} = \frac{a}{2}$$

57. $\lim_{x \to +\infty} \left(\sqrt{x^2 - 5x + 6} - x \right) = \infty - \infty$

$$= \lim_{x \to +\infty} \frac{\left(\sqrt{x^2-5x+6} - x \right) \left(\sqrt{x^2-5x+6} + x \right)}{\sqrt{x^2-5x+6} + x}$$

$$= \lim_{x \to +\infty} \frac{x^2 - 5x + 6 - x^2}{\sqrt{x^2-5x+6} + x}$$

$$= \lim_{x \to +\infty} \frac{-5x + 6}{\sqrt{x^2-5x+6} + x}$$

$$= \lim_{x \to +\infty} \frac{\cancel{x}(-5 + \frac{6}{x})}{\cancel{x}\sqrt{1 - \frac{5}{x} + \frac{6}{x^2}} + \cancel{x}} = \lim_{x \to +\infty} \frac{(-5 + \cancel{\frac{6}{x}}^{0})}{\sqrt{1 - \cancel{\frac{5}{x}}^{0} + \cancel{\frac{6}{x^2}}^{0}} + 1}$$

$$= \frac{-5}{2}$$

58. $\lim\limits_{x\to+\infty} x\left(\sqrt{x^2+1}-x\right) = \infty\cdot 0$

$$= \lim_{x\to+\infty} \frac{x\left(\sqrt{x^2+1}-x\right)\left(\sqrt{x^2+1}+x\right)}{\left(\sqrt{x^2+1}+x\right)}$$

$$= \lim_{x\to+\infty} \frac{x\left(\cancel{x^2}+1-\cancel{x^2}\right)}{\left(\sqrt{x^2+1}+x\right)}$$

$$= \lim_{x\to+\infty} \frac{x}{\sqrt{x^2+1}+x}$$

$$= \lim_{x\to+\infty} \frac{\cancel{x}}{\cancel{x}\sqrt{1+\frac{1}{x^2}}+\cancel{x}}$$

$$= \lim_{x\to+\infty} \frac{1}{\sqrt{1+\frac{1}{x^2}\xrightarrow{}0}+1} = \frac{1}{2}$$

59. $\lim\limits_{x\to+\infty} \left(x+\sqrt[3]{1-x^3}\right) = \infty-\infty$

Temos

$$(a^3+b^3) = (a+b)\,(a^2-ab+b^2)$$

$$\Leftrightarrow$$

$$a+b = \frac{a^3+b^3}{a^2-ab+b^2}$$

$$\downarrow \quad \downarrow$$

$$x+\sqrt[3]{1-x^3} = \frac{x^3+1-x^3}{x^2-x\sqrt[3]{1-x^3}+(1-x^3)^{\frac{2}{3}}} = \frac{1}{x^2-x\sqrt[3]{1-x^3}+(1-x^3)^{2/3}}$$

$\Longrightarrow$

$$\lim_{x\to+\infty}\left(x+\sqrt[3]{1-x^3}\right)=$$

$$=\lim_{x\to+\infty}\frac{1}{x^2-x\sqrt[3]{1-x^3}+(1-x^3)^{2/3}}$$

$$=\lim_{x\to+\infty}\frac{1}{x^2-x(1-x^3)^{1/3}+(1-x^3)^{2/3}}$$

$$=\lim_{x\to+\infty}\frac{1}{x^2-x\left[x^3\left(\frac{1}{x^3}-1\right)\right]^{1/3}+\left[x^3\left(\frac{1}{x^3}-1\right)\right]^{2/3}}$$

$$=\lim_{x\to+\infty}\frac{1}{x^2-x\,x\left[\frac{1}{x^3}-1\right]^{1/3}+x^2\left[\frac{1}{x^3}-1\right]^{2/3}}$$

$$=\lim_{x\to+\infty}\frac{1}{x^2\left(\underbrace{1-\left(\frac{1}{x^3}-1\right)^{1/3}+\left(\frac{1}{x^3}-1\right)^{2/3}}_{3}\right)}=0$$

$$=\lim_{x\to+\infty}\frac{1}{\underbrace{\underset{\infty}{x^2}\left(\underbrace{1-\left(\underset{0}{\frac{1}{x^3}}-1\right)^{1/3}+\left(\underset{0}{\frac{1}{x^3}}-1\right)^{2/3}}_{3}\right)}_{\infty}}$$

$$=0$$

60. $\lim_{x\to+\infty} \left(\sqrt{x^2+1} - \sqrt{x^2-1}\right) = \infty - \infty$

$= \lim_{x\to+\infty} \left(\sqrt{x^2+1} - \sqrt{x^2-1}\right) \dfrac{\left(\sqrt{x^2+1} + \sqrt{x^2-1}\right)}{\sqrt{x^2+1} + \sqrt{x^2-1}}$

$= \lim_{x\to+\infty} \dfrac{\cancel{x^2}+1 - (\cancel{x^2}-1)}{\sqrt{x^2+1} + \sqrt{x^2-1}}$

$= \lim_{x\to+\infty} \dfrac{2}{\sqrt{x^2+1} + \sqrt{x^2-1}} = \dfrac{2}{\infty+\infty} = \dfrac{2}{\infty} = 0$

$= 0$ //

61. $\lim_{x\to\infty} \left(\dfrac{\overset{\infty}{1-2x}}{\underset{\infty}{\sqrt[3]{1+8x^3}}} + \overset{\nearrow 0}{2^{-x^2}} \right)$ $(*)$

Aqui vemos que

$\lim_{x\to\infty} 2^{-x^2} = 2^{-\infty} = 0.$

Assim, o limite dado em $(*)$ existirá se existir

$\lim_{x\to\infty} \dfrac{1-2x}{\sqrt[3]{1+8x^3}}$.

(Neste caso temos $(*) = \lim_{x\to\infty} \dfrac{1-2x}{\sqrt[3]{1+8x^3}}$).

→

Temos

$$\lim_{x\to\infty} \frac{1-2x}{\sqrt[3]{1+8x^3}}$$

$$= \lim_{x\to\infty} \frac{\cancel{x}\left(\frac{1}{x}-2\right)}{\cancel{x}\sqrt[3]{\frac{1}{x^3}+8}}$$

$$= \lim_{x\to\infty} \frac{\left(\cancel{\frac{1}{x}}^{0}-2\right)}{\sqrt[3]{\cancel{\frac{1}{x^3}}^{0}+8}} = \frac{-2}{\sqrt[3]{8}} = \frac{-2}{2} = -1$$

---

62. $$\lim_{x\to+\infty} \left(\frac{x^3}{3x^2-4} - \frac{x^2}{3x+2}\right) = ?$$

$$= \lim_{x\to+\infty} \frac{x^3(3x+2) - x^2(3x^2-4)}{(3x^2-4)(3x+2)}$$

$$= \lim_{x\to+\infty} \frac{\cancel{3x^4}+2x^3-\cancel{3x^4}+4x^2}{(3x^2-4)(3x+2)}$$

$$= \lim_{x\to+\infty} \frac{2x^3+4x^2}{x^2\left(3-\frac{4}{x^2}\right)x\left(3+\frac{2}{x}\right)}$$

$$= \lim_{x\to+\infty} \frac{2\cancel{x^3}\left(1+\frac{2}{x}\right)}{\cancel{x^3}\left(3-\frac{4}{x^2}\right)\left(3+\frac{2}{x}\right)}$$

$$= \lim_{x\to+\infty} \frac{2\left(1+\frac{2}{x}\right)}{\left(3-\frac{4}{x^2}\right)\left(3+\frac{2}{x}\right)} = \frac{2}{9}$$

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63. $\displaystyle \lim_{x\to+\infty}\left(\overset{\infty}{\sqrt{9x^2+1}}\ \overset{-\infty}{-\,3x}\right)$

$\displaystyle = \lim_{x\to+\infty}\frac{\left(\sqrt{9x^2+1}-3x\right)\left(\sqrt{9x^2+1}+3x\right)}{\sqrt{9x^2+1}+3x}$

$\displaystyle = \lim_{x\to+\infty}\frac{\cancel{9x^2}+1-\cancel{9x^2}}{\sqrt{9x^2+1}+3x}$

$\displaystyle = \lim_{x\to+\infty}\frac{1}{\sqrt{9x^2+1}+3x} = \frac{1}{\infty} = 0$ //

64. $\displaystyle \lim_{x\to+\infty}\frac{2\sqrt{x}+3\sqrt[3]{x}+5\sqrt[5]{x}}{\sqrt{3x-2}+\sqrt[3]{2x-3}}$

$\displaystyle = \lim_{x\to+\infty}\frac{2x^{1/2}+3x^{1/3}+5x^{1/5}}{x^{1/2}\sqrt{3-\frac{2}{x}}+x^{1/3}\sqrt{2-\frac{3}{x}}}$

$\frac{1}{3}-\frac{1}{2}=\frac{-1}{6}$

$\displaystyle = \lim_{x\to+\infty}\frac{\cancel{x^{1/2}}\left(2+3\frac{x^{1/3}}{x^{1/2}}+5\frac{x^{1/5}}{x^{1/2}}\right)}{\cancel{x^{1/2}}\left(\sqrt{3-\frac{2}{x}}+\frac{x^{1/3}}{x^{1/2}}\sqrt{2-\frac{3}{x}}\right)}$

$\frac{1}{5}-\frac{1}{2}=\frac{-3}{10}$

$\displaystyle = \lim_{x\to+\infty}\frac{2+\frac{3}{x^{1/6}}\to 0+\frac{5}{x^{3/10}}\to 0}{\sqrt{3-\frac{2}{x}\,0}+\frac{1}{x^{1/6}}\,0\sqrt{2-\frac{3}{x}}}$

$\displaystyle = \frac{2}{\sqrt{3}}$ //

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65. $\lim\limits_{x \to -\infty} \left(\sqrt{2x^2 - 3} - 5x \right) = +\infty + \infty = +\infty$

66. $\lim\limits_{x \to +\infty} x \left(\overset{\infty \; - \infty}{\sqrt{x^2+1} - x} \right) =$

$= \lim\limits_{x \to +\infty} x \left(\sqrt{x^2+1} - x \right) \dfrac{\left(\sqrt{x^2+1} + x \right)}{\sqrt{x^2+1} + x} =$

$= \lim\limits_{x \to +\infty} x \dfrac{\left(x^2+1 - x^2 \right)}{\sqrt{x^2+1} + x}$

$= \lim\limits_{x \to +\infty} \dfrac{x}{\sqrt{x^2+1} + x} = \lim\limits_{x \to +\infty} \dfrac{\cancel{x}}{\cancel{x}\sqrt{1+\frac{1}{x^2}} + \cancel{x}}$

$= \lim\limits_{x \to +\infty} \dfrac{1}{\sqrt{1+\frac{1}{x^2}^{\to 0}} + 1} = \dfrac{1}{2}$

67. $\lim\limits_{x \to +\infty} \dfrac{\sqrt{2x^2+3}}{4x+2} = \dfrac{+\infty}{+\infty}$

$= \lim\limits_{x \to \infty} \dfrac{\sqrt{x^2\left(2+\frac{3}{x^2}\right)}}{2x\left(2+\frac{1}{x}\right)}$ $\downarrow$ $x > 0 \Rightarrow \sqrt{x^2} = x$

$= \lim\limits_{x \to +\infty} \dfrac{\cancel{x}\sqrt{2+\frac{3}{x^2}}}{2\cancel{x}\left(2+\frac{1}{x}\right)}$

$= \lim\limits_{x \to +\infty} \dfrac{1}{2} \dfrac{\sqrt{2+\frac{3}{x^2}^{\to 0}}}{2+\frac{1}{x}^{\to 0}} = \dfrac{1}{2} \dfrac{\sqrt{2}}{2} = \dfrac{\sqrt{2}}{4}$

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68. $\lim\limits_{x \to -\infty} \dfrac{\sqrt{2x^2+3}}{4x+2} = \dfrac{\infty}{-\infty}$

$= \lim\limits_{x \to -\infty} \dfrac{\sqrt{x^2\left(2+\frac{3}{x^2}\right)}}{2x\left(2+\frac{1}{x}\right)}$ $\quad x<0,\ \sqrt{x^2}=-x$

$= \lim\limits_{x \to -\infty} \dfrac{-\cancel{x}\sqrt{2+\frac{3}{x^2}}}{2\cancel{x}\left(2+\frac{1}{x}\right)}$

$= \lim\limits_{x \to -\infty} \dfrac{-1}{2} \dfrac{\sqrt{2+\frac{3}{x^2}}\to 0}{2+\frac{1}{x}\to 0}$

$= \dfrac{-1}{2}\dfrac{\sqrt{2}}{2} = -\dfrac{\sqrt{2}}{4}$ //

69. $\lim\limits_{x \to +\infty} 5^{2x/x+3} =$

$= \lim\limits_{x \to +\infty} 5^{\frac{2}{1+\frac{3}{x}}} = 5^{\lim\limits_{x\to+\infty}\frac{2}{1+\frac{3}{x}\to 0}}$

$= 5^2 = 25$ //

70. $\lim\limits_{x \to -\infty} \dfrac{x}{\sqrt{x^2+1}} = \dfrac{-\infty}{+\infty}$

$= \lim\limits_{x \to -\infty} \dfrac{x}{\sqrt{x^2\left(1+\frac{1}{x^2}\right)}} = \lim\limits_{x \to -\infty} \dfrac{x}{-x\sqrt{1+\frac{1}{x^2}}}$

$= \lim\limits_{x \to -\infty} -\dfrac{1}{\sqrt{1+\frac{1}{x^2}\to 0}} = -1$ //

71. $\lim_{x\to+\infty} \frac{x}{\sqrt{x^2+1}} = \lim_{x\to+\infty} \frac{\cancel{x}}{\cancel{x}\sqrt{1+\frac{1}{x^2}}}$

$= \lim_{x\to+\infty} \frac{1}{\sqrt{1+\frac{1}{x^2}}} = 1$ //

72. $\lim_{x\to-\infty} \frac{3^x-3^{-x}}{3^x+3^{-x}} = \lim_{x\to-\infty} \frac{\cancel{3^{-x}}(3^{2x}-1)}{\cancel{3^{-x}}(3^{2x}+1)}$

$= \lim_{x\to-\infty} \left(\frac{0-1}{0+1}\right) = -1$ //

73. $\lim_{x\to+\infty} \frac{3^x-3^{-x}}{3^x+3^{-x}} = \lim_{x\to+\infty} \frac{\cancel{3^x}(1-3^{-2x})}{\cancel{3^x}(1+3^{-2x})}$

$= \lim_{x\to+\infty} \frac{1-3^{-2x}}{1+3^{-2x}} = \frac{1}{1} = 1$ //