

## Cálculo 3 - Lista 4

### Curvas definidas parametricamente

Esboce o gráfico da curva representada pelas funções vetoriais a seguir

1.  $\vec{r}(t) = t\vec{i}$ ,  $-1 \leq t \leq \frac{1}{2}$
2.  $\vec{r}(t) = \cos \pi t \vec{k}$ ,  $-1 \leq t \leq \frac{1}{3}$
3.  $\vec{r}(t) = t\vec{i} + t\vec{j} + t\vec{k}$
4.  $\vec{r}(t) = 2t\vec{i} - 3t\vec{j} + \vec{k}$
5.  $\vec{r}(t) = (2t + 1)\vec{i} + (t - 1)\vec{j} + 3t\vec{k}$
6.  $\vec{r}(t) = -16t^2\vec{k}$ ,  $t \geq 0$
7.  $\vec{r}(t) = t\vec{j} + t^2\vec{k}$
8.  $\vec{r}(t) = (t^4 + 1)\vec{i} + t\vec{j}$
9.  $\vec{r}(t) = t^3\vec{i} + t^2\vec{j}$
10.  $\vec{r}(t) = \cos t\vec{i} + \sin t\vec{j}$ ,  $0 \leq t \leq \frac{\pi}{2}$
11.  $\vec{r}(t) = \cos 3t\vec{i} + \sin 3t\vec{j}$ ,  $0 \leq t \leq \frac{\pi}{2}$
12.  $\vec{r}(t) = 2 \cos t\vec{i} - \sin t\vec{j} - 3\vec{k}$ ,  $-\pi \leq t \leq 0$
13.  $\vec{r}(t) = \cos t\vec{i} + \sin t\vec{j} + t^2\vec{k}$
14.  $\vec{r}(t) = 3 \sin t\vec{i} + 3 \sin t\vec{j} - 3\sqrt{2} \cos t\vec{k}$
15.  $\vec{r}(t) = t\vec{i} + \cos 2t\vec{j} + \sin 2t\vec{k}$

Encontre a função vetorial que representa a curva obtida pela interseção das superfícies dadas a seguir. [Sugestão: Identifique geometricamente a forma da curva pela análise da interseção das superfícies. Uma vez reconhecida a curva, parametrize-a de modo a respeitar a orientação dada. Note que mais de uma parametrização é possível.]

16.  $x + 2y + 3z = 6$  e  $y - 2z = 3$  orientada de modo que  $z$  aumenta ao longo da curva.
17.  $x^2 + y^2 = 2$  e  $z = 4$  orientada de modo que  $y$  aumenta no primeiro octante.
18.  $z = x^2 + y^2$  e  $x^2 + y^2 = 5$  orientada de modo que  $x$  aumenta no primeiro octante.
19.  $z = \sqrt{x^2 + y^2}$  e  $y = x$  orientada de modo que ao se parametrizar  $(x(t), y(t))$  tem-se  $(x, y)$  se afastando da origem  $(0, 0)$  para valores crescentes de  $t$ .

Determine quais das parametrizações a seguir é suave, suave por parte ou nenhuma das duas.

20.  $\vec{r}(t) = t\vec{i} + t^2\vec{j} + t^3\vec{k}$
21.  $\vec{r}(t) = |t|\vec{i} + t\vec{j} + t\vec{k}$
22.  $\vec{r}(t) = (1 + t)^{\frac{3}{2}}\vec{i} + (1 - t)^{\frac{3}{2}}\vec{j} + \frac{3t}{2}\vec{k}$
23.  $\vec{r}(t) = \cos^2 t\vec{i} + \sin^2 t\vec{j} + t^2\vec{k}$

24.  $\vec{r}(t) = (e^t - t)\vec{i} + t^2\vec{j} + t^3\vec{k}$

Encontre parametrizações suaves para as curvas a seguir

25. A linha reta passando por  $(-3, 2, 1)$  e  $(4, 0, 5)$ .

26. O círculo no plano  $xy$  centrado na origem e com raio 6.

Encontre parametrizações suaves por partes para as curvas a seguir

27. O quadrado no plano  $xy$  cujos vértices são  $(3, 0)$ ,  $(3, 3)$ ,  $(0, 3)$  e  $(0, 0)$ .

28. O triângulo no plano  $xy$  cujos vértices são  $(0, 0)$ ,  $(2, 0)$ ,  $(0, 2)$ .

Determine o comprimento das curvas a seguir

29.  $\vec{r}(t) = \cos^3 t \vec{i} + \sin^3 t \vec{j}$ ,  $0 \leq t \leq 2\pi$ ,

30.  $\vec{r}(t) = 2t \vec{i} + t^2 \vec{j} + \ln t \vec{k}$ ,  $1 \leq t \leq 2$

31.  $\vec{r}(t) = \frac{1}{3}(1+t)^{3/2}\vec{i} + \frac{1}{3}(1-t)^{3/2}\vec{j} + \frac{1}{2}t\vec{k}$ ,  $-1 \leq t \leq 1$

32.  $\vec{r}(t) = e^t \vec{i} + e^{-t}\vec{j} + \sqrt{2}t\vec{k}$ ,  $0 \leq t \leq 1$

33.  $\vec{r}(t) = 2(t^2 - 1)^{3/2}\vec{i} + 3t^2\vec{j} + 3t^2\vec{k}$ ,  $0 \leq t \leq \sqrt{8}$

Encontre para cada uma das curvas o vetor tangente, o vetor normal e a curvatura.

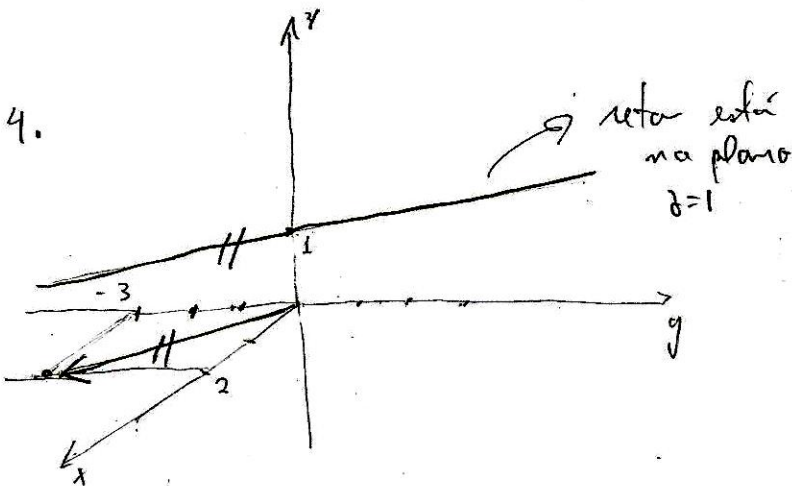
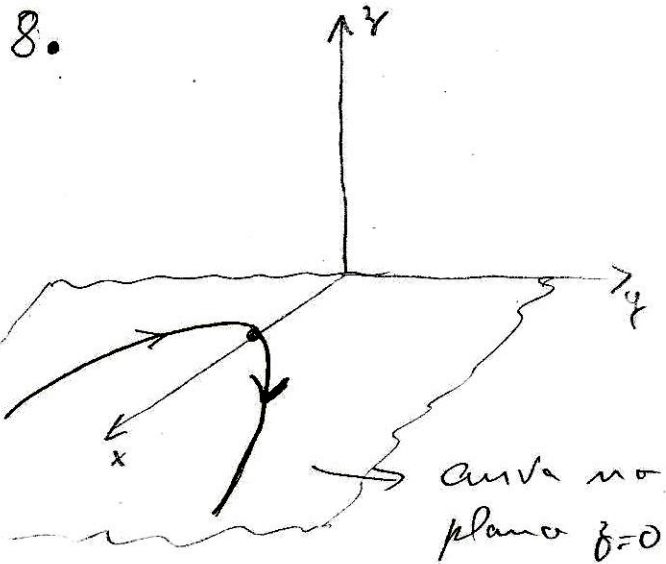
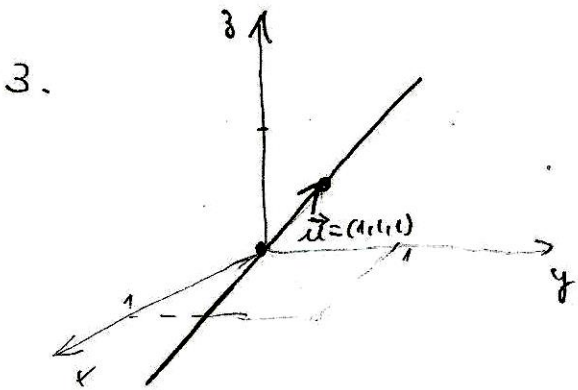
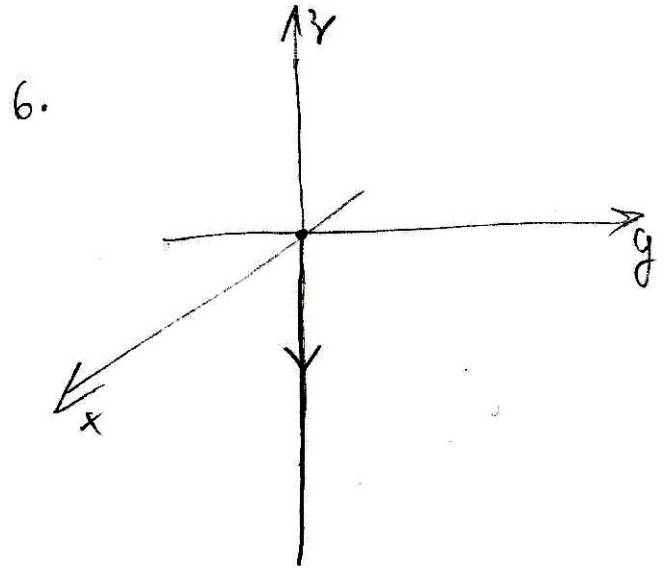
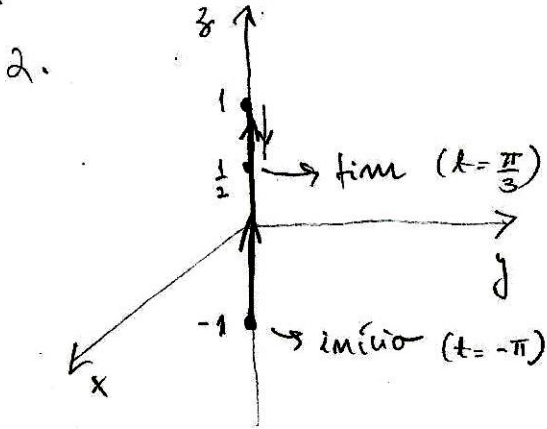
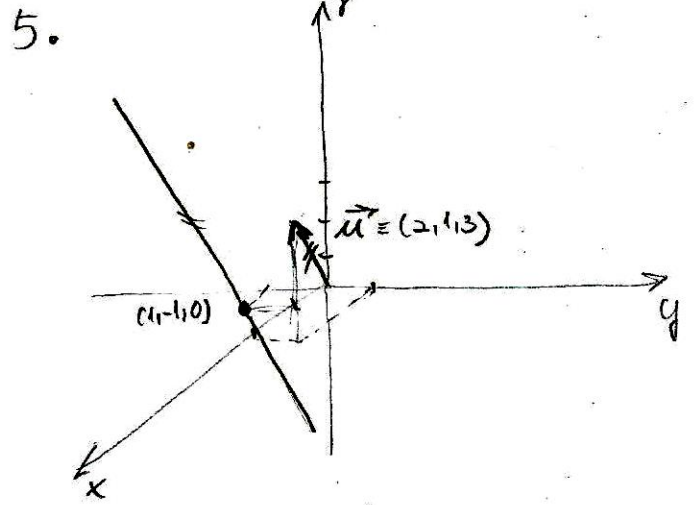
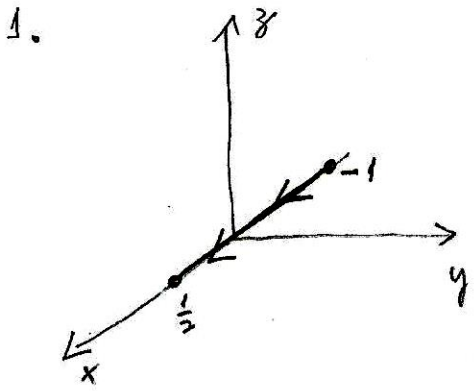
34.  $\vec{r}(t) = (t^2 + 4)\vec{i} + 2t\vec{j}$

35.  $\vec{r}(t) = \cos t \vec{i} + \cos t \vec{j} + \sqrt{2} \sin t \vec{k}$

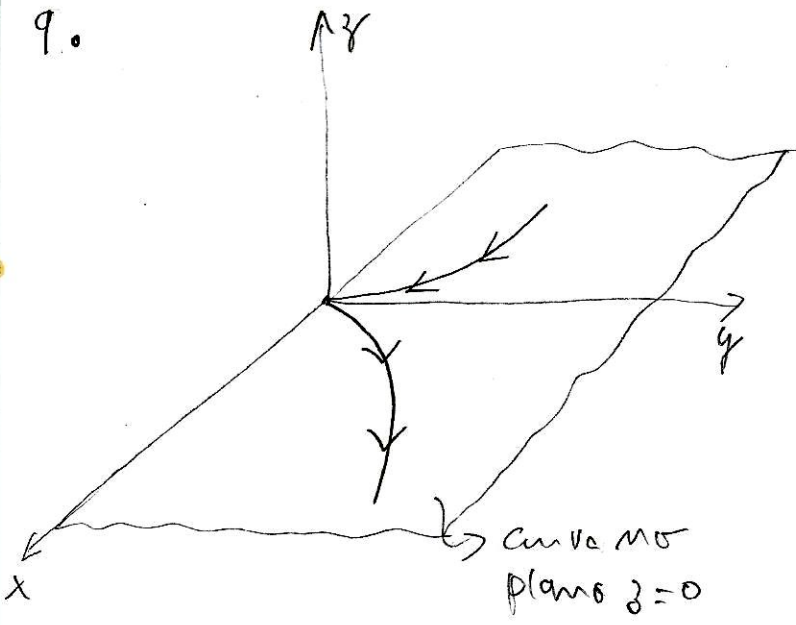
36.  $\vec{r}(t) = 2t \vec{i} + t^2 \vec{j} + \frac{1}{3}t^3 \vec{k}$

37.  $\vec{r}(t) = e^t \vec{i} + e^{-t}\vec{j} + \sqrt{2}t\vec{k}$

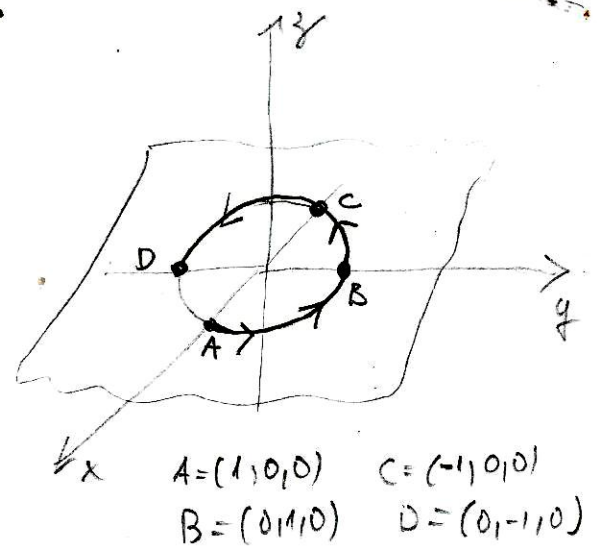
38.  $\vec{r}(t) = 2t\vec{i} + t^2\vec{j} + \ln t\vec{k}$



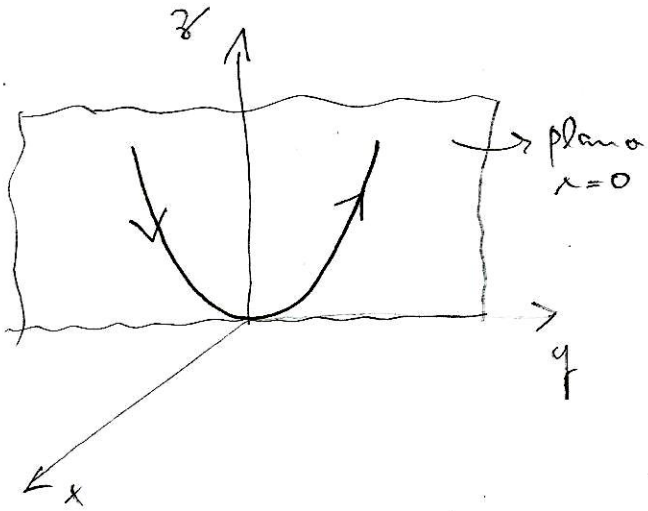
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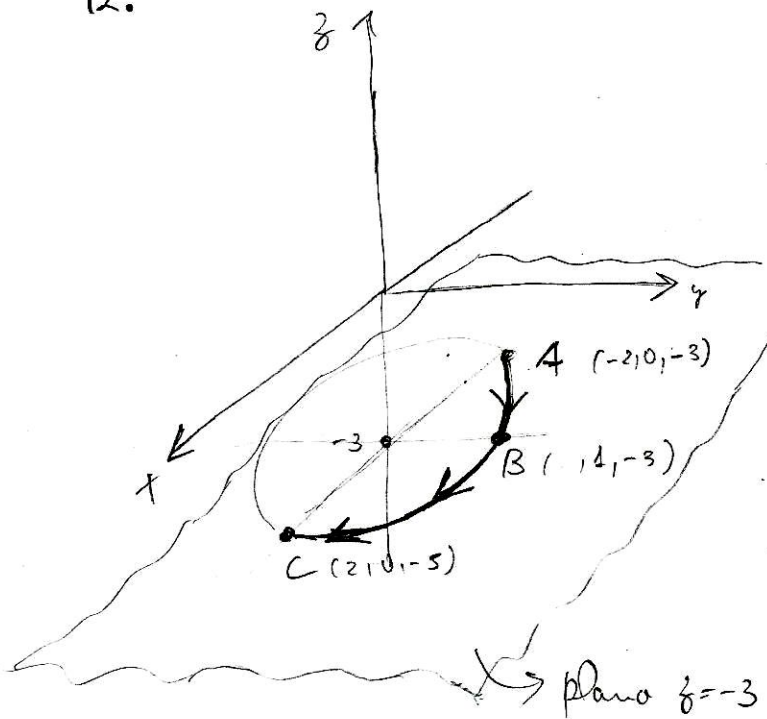
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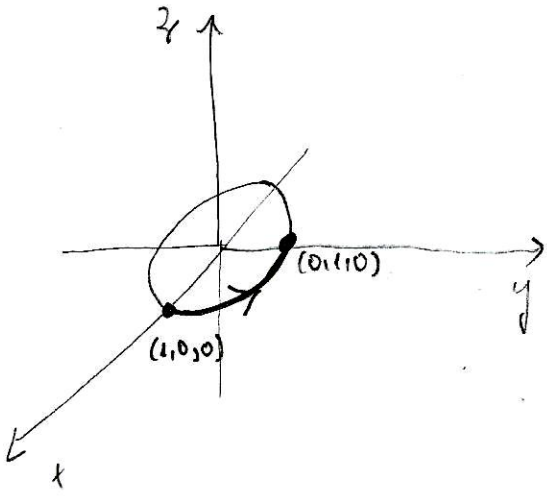
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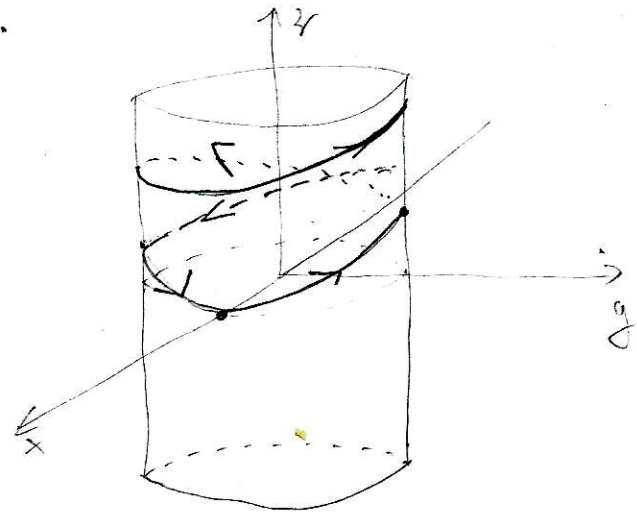
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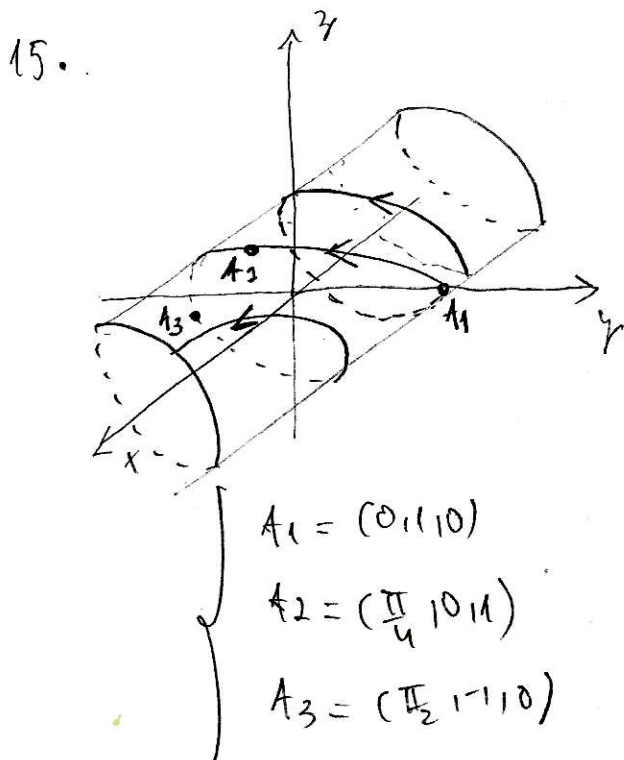
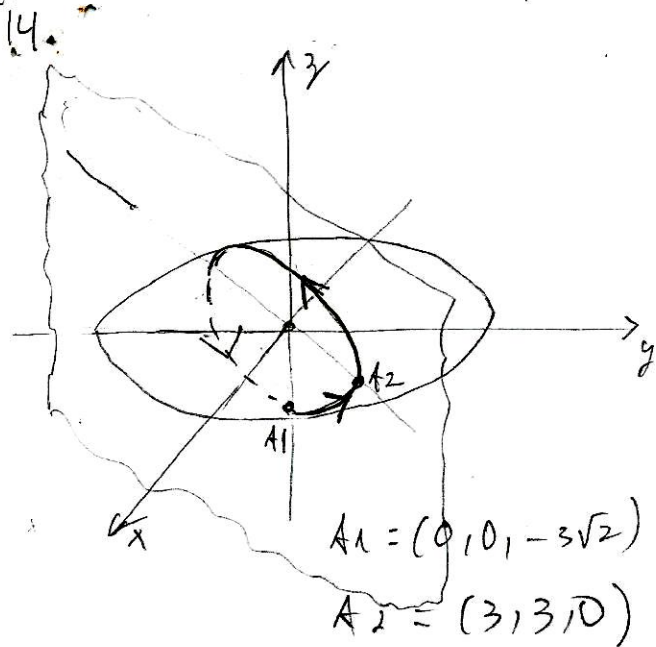


10.



13.





16.

$$\vec{r}(t) = -7t \hat{i} + (3+2t) \hat{j} + t \hat{k}$$

$$\left\{ \begin{array}{l} t > 0 \\ \text{(reta)} \end{array} \right.$$

17.

$$\left\{ \begin{array}{l} \vec{r}(t) = \sqrt{2} \cos t \hat{i} + \sqrt{2} \sin t \hat{j} + 4 \hat{k} \\ 0 \leq t \leq 2\pi \end{array} \right.$$

(círculo no plano  $z=4$ )

18.

$$\vec{r}(t) = \sqrt{5} \sin t \hat{i} + \sqrt{5} \cos t \hat{j} + 5 \hat{k}$$

$$0 \leq t \leq 2\pi$$

19.

$$\vec{r}(t) = t \hat{i} + t \hat{j} + \sqrt{2}|t| \hat{k}$$

$$t \geq 0$$

20. Suave

21. Suave por partes  
( $I = (-\infty, 0] \cup [0, +\infty)$ )

22. Suave

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25.

$$\left\{ \begin{array}{l} \vec{r} = (-3+7t) \hat{i} + (2-2t) \hat{j} + (1+4t) \hat{k} \\ t \in \mathbb{R} \end{array} \right.$$

26.

$$\vec{r}(t) = 6 \cos t \hat{i} + 6 \sin t \hat{j}$$

$$0 \leq t \leq 2\pi$$

27.

$$\vec{r}(t) = \begin{cases} t \hat{i}, & 0 \leq t \leq 3 \\ 2\hat{i} + (t-3)\hat{j}, & 3 \leq t \leq 6 \\ (9-t)\hat{i} + 3\hat{j}, & 6 \leq t \leq 9 \\ (12-t)\hat{j}, & 9 \leq t \leq 12 \end{cases}$$

28.

$$\vec{r}(t) = \begin{cases} t \hat{i}, & 0 \leq t \leq 2 \\ (4-t)\hat{i} + (t-2)\hat{j}, & 2 \leq t \leq 4 \\ (6-t)\hat{j}, & 4 \leq t \leq 6 \end{cases}$$

35.

$$\vec{r}(t) = -\frac{\sin t}{\sqrt{2}} \hat{i} - \frac{\sin t}{\sqrt{2}} \hat{j} + \cos t \hat{k}$$

$$\vec{N}(t) = -\frac{\cos t}{\sqrt{2}} \hat{i} - \frac{\cos t}{\sqrt{2}} \hat{j} - \sin t \hat{k}$$

$$k(t) = \frac{1}{\sqrt{2}}$$

$$36. \vec{r}(t) = \frac{2}{t^2+2} \hat{i} + \frac{2t}{t^2+2} \hat{j} + \frac{t^2}{t^2+2} \hat{k}$$

$$\vec{N}(t) = \frac{-2t}{(t^2+2)} \hat{i} + \frac{2-t^2}{(t^2+2)} \hat{j} + \frac{2t}{(t^2+2)} \hat{k}$$

$$k(t) = \frac{2}{(t^2+2)^2}$$

29. 6

30.  $3 + \ln 2$ 31.  $\sqrt{3}$ 32.  $e - \frac{1}{e}$ 

33. 52

38.

$$\vec{r}(t) = \frac{2t}{2t^2+1} \hat{i} + \frac{2t^2}{2t^2+1} \hat{j} + \frac{1}{2t^2+1} \hat{k}$$

$$\vec{N}(t) = \frac{1-2t^2}{2t^2+1} \hat{i} + \frac{2t}{2t^2+1} \hat{j} - \frac{2t}{2t^2+1} \hat{k}$$

37.

$$\vec{r}(t) = \frac{e^t \hat{i} - e^{-t} \hat{j} + \sqrt{2} \hat{k}}{e^t + e^{-t}}$$

$$\vec{N}(t) = \frac{\sqrt{2}}{(e^t + e^{-t})} \hat{i} + \frac{\sqrt{2}}{e^t + e^{-t}} \hat{j} - \frac{e^t - e^{-t}}{e^t + e^{-t}} \hat{k}$$

$$k(t) = \frac{\sqrt{2}}{(e^t + e^{-t})^2}$$

$$34. \vec{r}(t) = \frac{t}{\sqrt{t^2+1}} \hat{i} + \frac{\hat{j}}{\sqrt{t^2+1}}$$

$$\vec{N}(t) = \frac{\hat{i} - t \hat{j}}{\sqrt{t^2+1}}, \quad k(t) = \frac{1}{2(t^2+1)^{3/2}}$$

$$k(t) = \frac{2t}{(2t^2+1)^2}$$