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1. Calcule $\int_D xy \, dA$ onde D é a região limitada pelas curvas

$$\begin{cases} x-y=2 \\ x-y=-1 \\ 2x+3y=1 \\ 2x+3y=0 \end{cases}$$

Faça a mudança de variáveis

$(x,y) \longrightarrow (u,v)$ onde $\begin{cases} x=\frac{1}{5}(3u+v) \\ y=\frac{1}{5}(v-2u). \end{cases}$

2. Calcule $\int_B \frac{1}{x} \, dV$ onde B é a região limitada por $\begin{cases} x+y+z=4,\ y=x,\ x=1,\ x=2, \\ y=0,\ z=0 \end{cases}$

3. Determine o volume do sólido no interior da esfera $x^2+y^2+z^2=4$ e do cilindro $x^2+y^2=1$. Use coordenadas cilíndricas

4. Calcule $\int_B (x^2+y^2+z^2)^2 \, dV$ onde B é a região limitada superiormente pela esfera $x^2+y^2+z^2=1$ e inferiormente por $z=\sqrt{x^2+y^2}$. Use coordenadas esféricas.

$$\int xy \, dA$$

$$D: \begin{cases} x - y = 2 \\ x - y = -1 \\ 2x + 3y = 1 \\ 2x + 3y = 0 \end{cases} \qquad \begin{aligned} x &= \frac{1}{5}(3u + v) \\ y &= \frac{1}{5}(v - 2u) \end{aligned}$$

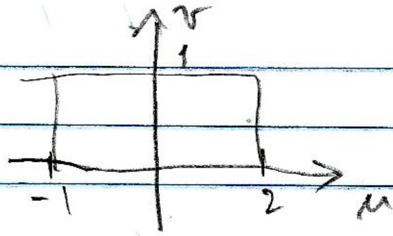
$$\begin{cases} x - y = 2 \\ \therefore \\ \frac{1}{5}(3u + \cancel{v}) - \frac{1}{5}(\cancel{v} - 2u) = 2 \\ \quad \frac{3}{5}u + \frac{2}{5}u = 2 \quad \therefore \quad \underline{\underline{u = 2}} \end{cases}$$

$$\begin{cases} x - y = -1 \\ \frac{1}{5}(3u + \cancel{v}) - \frac{1}{5}(\cancel{v} - 2u) = -1 \\ \quad \frac{3}{5}u + \frac{2}{5}u = -1 \quad \therefore \quad \underline{\underline{u = -1}} \end{cases}$$

0.78

$$\begin{cases} 2x + 3y = 1 \\ \frac{2}{5}(\cancel{3u} + v) + \frac{3}{5}(v - \cancel{2u}) = 1 \\ \quad \frac{2}{5}v + \frac{3}{5}v = 1 \quad \therefore \quad \underline{\underline{v = 1}} \end{cases}$$

$$\begin{cases} 2x + 3y = 0 \\ \frac{2}{5}(\cancel{3u} + v) + \frac{3}{5}(v - \cancel{2u}) = 0 \\ \quad \frac{2}{5}v + \frac{3}{5}v = 0 \quad \therefore \quad \underline{\underline{v = 0}} \end{cases}$$



$$\det\begin{pmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{pmatrix} = \det\begin{pmatrix} \frac{3}{5} & \frac{1}{5} \\ -\frac{2}{5} & \frac{1}{5} \end{pmatrix} = \frac{3}{25} + \frac{2}{25} = \frac{5}{25}$$

$$= \frac{1}{5} \quad 0.2$$

$$\therefore \int_{D(x,y)} xy \, dA = \int_{D(u,v)} \frac{1}{5}(3u+v)\,\frac{1}{5}(v-2u) \cdot \frac{1}{5}\, du\, dv$$

$$= \frac{1}{125}\int_{v=0}^{1}\left(\int_{u=-1}^{2} (3u+v)(v-2u)\, du\right) dv$$

$$= \frac{1}{125}\int_{v=0}^{1}\int_{u=-1}^{2} \left(3uv - 6u^2 + v^2 - 2uv\right) du\, dv$$

$$= \frac{1}{125}\int_{v=0}^{1}\int_{u=-1}^{2} \left(uv - 6u^2 + v^2\right) du\, dv$$

$$= \frac{1}{125}\int_{v=0}^{1}\left[\frac{u^2 v}{2} - \frac{6u^3}{3} + v^2 u\right]_{u=-1}^{2} dv$$

$$= \frac{1}{125}\int_{v=0}^{1}\left(\frac{4v}{2} - 2\cdot 8 + 2v^2 - \frac{v}{2} - 2 + v^2\right) dv$$

$$= \frac{1}{125}\int_{v=0}^{1}\left(\frac{3v}{2} - 18 + 3v^2\right) dv$$

$$= \frac{1}{125}\left(\frac{3v^2}{4} - 18v + \frac{3v^3}{3}\right)_{v=0}^{1} =$$

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$$= \frac{1}{125}\left(\frac{3}{4} - 18 + 1\right)$$

$$= \frac{1}{125}\left(\frac{3}{4} - 17\right)$$

$$= \frac{1}{125}\left(\frac{3-68}{4}\right) = \frac{1}{125}\,(-)\,\frac{65}{4}$$

$$= -\frac{13}{25 \cdot 4}$$

$$= -\frac{13}{100}$$ //

$$\begin{array}{r|l} 65 & 5 \\ 15 & 13 \\ 0 & \end{array}$$

$$\begin{array}{r|l} 125 & 5 \\ 25 & 25 \\ 0 & \end{array}$$

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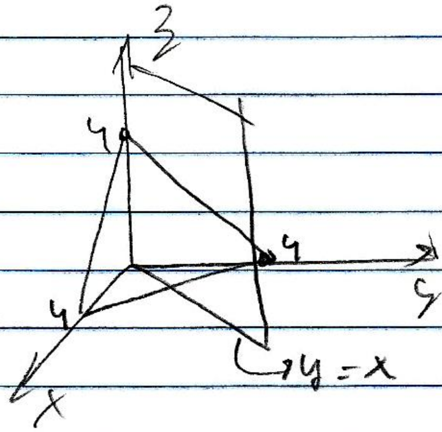
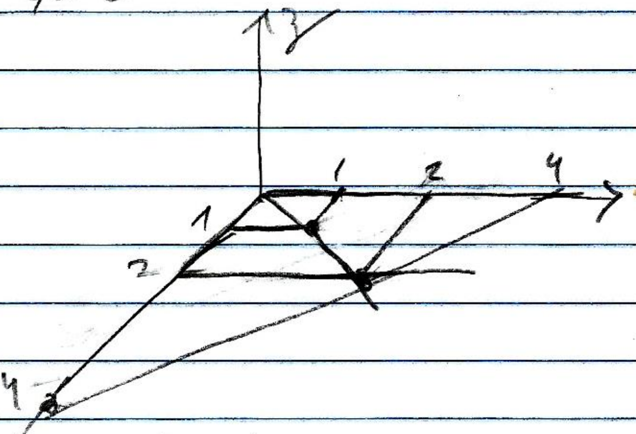
$$\int_B \frac{1}{x}\, dV$$

$$\begin{cases} x+y+z=4 \\ y=x \\ x=1 \\ x=2 \\ z=0 \\ y=0 \end{cases}$$



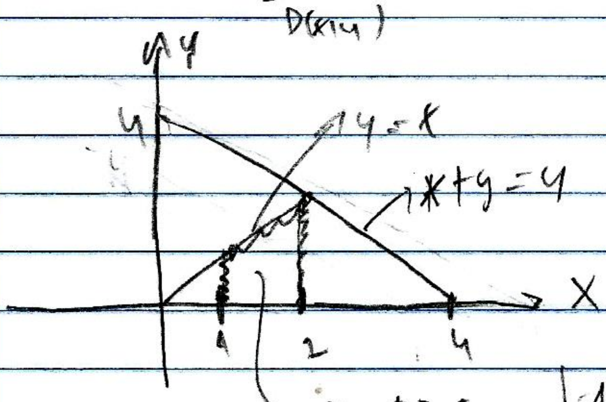


$$B: \begin{cases} 0 \le z \le 4-x-y \\ (x,y) \in D \end{cases}$$

0.5

$$\int_{D(x,y)} \int_{z=0}^{4-x-y} \frac{1}{x}\, dz \; dx\,dy = \int_{D(x,y)} \frac{1}{x} z \Big]_{z=0}^{4-x-y} dx\,dy$$

$$= \int_{D(x,y)} \frac{1}{x}(4-x-y)\, dx\,dy$$



$$\begin{cases} y = x \\ x+y=4 \end{cases} \rightarrow 2x = 4$$

$$x = 2$$

$$D: \begin{cases} 1 \le x \le 2 & 0.25 \\ 0 \le y \le x & 0.25 \end{cases}$$

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$$\int_B \frac{1}{x}\, dV = \int_{D(x,y)} \left(\int_{z=0}^{4-x-y} \frac{1}{x}\, dz \right) dx\,dy$$

$$= \int_{D(x,y)} \frac{1}{x} z \Big]_{z=0}^{4-x-y} dx\,dy$$

$$= \int_{D(x,y)} \frac{4-x-y}{x}\, dx\,dy$$

$$= \int_{x=1}^{2} \left(\int_{y=0}^{x} \frac{4-x-y}{x}\, dy \right) dx$$

$$= \int_{x=1}^{2} \left(\frac{4-x}{x}\right) y - \frac{1}{x}\frac{y^2}{2} \Big]_{y=0}^{x} dx$$

$$= \int_{x=1}^{2} \left\{ (4-x) - \frac{1}{2}x \right\} dx$$

$$= \int_{x=1}^{2} \left(4 - \frac{3x}{2}\right) dx = \left(4x - \frac{3}{2}\frac{x^2}{2}\right)\Big|_{x=1}^{2}$$

$$= 8 - \frac{3}{4}\cdot 4 - \left(4 - \frac{3}{4}\right)$$

$$= 5 - 4 + \frac{3}{4} = 1 + \frac{3}{4} = \frac{7}{4}$$

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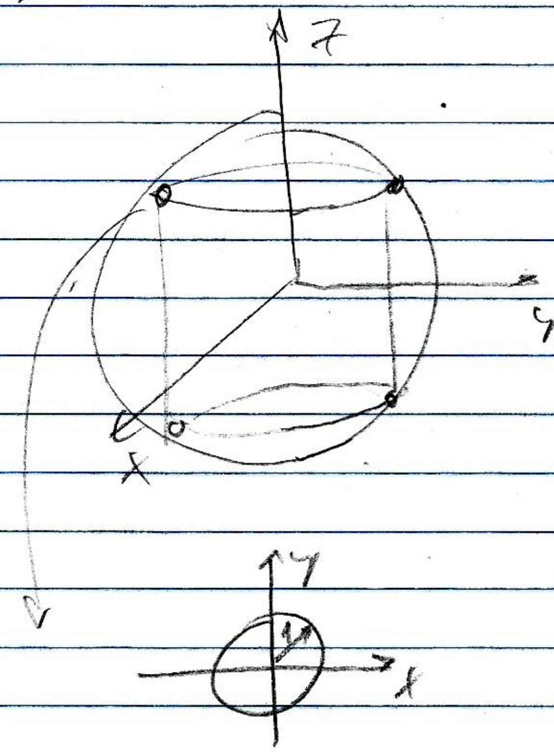
Obs: Não considerar a integral se a integral de z não estiver correta.

3.



$$B : \begin{cases} -\sqrt{4-\rho^2} \le z \le \sqrt{4-\rho^2} \\ 0 \le \theta \le 2\pi \\ 0 \le \rho \le 1 \end{cases}$$

1.0

$$\text{Volume} = \int_B 1\, dV = \int_{\theta=0}^{2\pi} \left(\int_{\rho=0}^{1} \left(\int_{z=-\sqrt{4-\rho^2}}^{\sqrt{4-\rho^2}} \rho\, dz \right) d\rho \right) d\theta$$

$$= \int_{\theta=0}^{2\pi} \int_{\rho=0}^{1} \rho z \Big|_{z=-\sqrt{4-\rho^2}}^{\sqrt{4-\rho^2}} d\rho\, d\theta$$

$$= \int_{\theta=0}^{2\pi} \int_{\rho=0}^{1} 2\rho\sqrt{4-\rho^2}\, d\rho\, d\theta$$

$$= \int_{\theta=0}^{2\pi} -\frac{2}{3}(4-\rho^2)^{3/2} \Big]_{\rho=0}^{1} d\theta$$

$$= \int_{\theta=0}^{2\pi} \left(-\frac{2}{3}\, 3^{3/2} + \frac{2}{3}\cdot 8 \right) d\theta =$$

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$$= \int_{\theta=0}^{2\pi} \left(-2\sqrt{3} + \frac{16}{3}\right) d\theta$$

$$= \left(-2\sqrt{3} + \frac{16}{3}\right) \theta \Big]_{\theta=0}^{2\pi}$$

$$= 2\pi\left(\frac{16}{3} - 2\sqrt{3}\right) //$$

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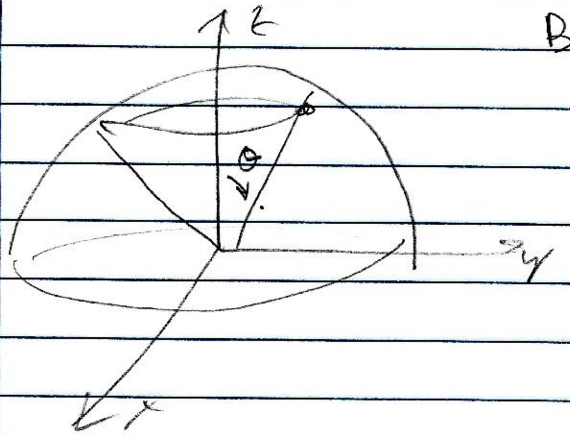
4. $\int_B (x^2+y^2+z^2)^2 \, dV$



$$B: \begin{cases} 0 \le r \le 1 \\ 0 \le \theta \le \frac{\pi}{4} \\ 0 \le \varphi \le 2\pi \end{cases}$$

1.0

$$\int_B (x^2+y^2+z^2)^2 \, dV = \int_{\varphi=0}^{2\pi} \int_{\theta=0}^{\frac{\pi}{4}} \int_{r=0}^{1} r^4 r^2 \sin\theta \, dr \, d\theta \, d\varphi$$

$$= \int_{\varphi=0}^{2\pi} \int_{\theta=0}^{\frac{\pi}{4}} \int_{r=0}^{1} r^6 \sin\theta \, dr \, d\theta \, d\varphi$$

$$= \int_{\varphi=0}^{2\pi} \int_{\theta=0}^{\frac{\pi}{4}} \frac{r^7}{7} \sin\theta \Big]_{r=0}^{1} d\theta \, d\varphi$$

$$= \int_{\varphi=0}^{2\pi} \int_{\theta=0}^{\frac{\pi}{4}} \frac{1}{7} \sin\theta \, d\theta \, d\varphi$$

$$\frac{1}{7}\left(\frac{2-\sqrt{2}}{2}\right) 2\pi =$$

$$= \int_{\varphi=0}^{2\pi} -\frac{1}{7} \cos\theta \Big]_0^{\pi/4} d\varphi$$

$$= \frac{\pi}{7}(2-\sqrt{2})$$

$$= \int_{\varphi=0}^{2\pi} \left(-\frac{1}{7}\frac{\sqrt{2}}{2} + \frac{1}{7}\right) d\varphi$$

$$= \frac{1}{7}\left(1 - \frac{\sqrt{2}}{2}\right) 2\pi \, //$$

1.0

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