Cálculo B - Lista 1

1. $\int \sin^4 x \cos x \, dx$

Substituição : $\begin{cases} u = \sin x \\ du = \cos x \, dx \end{cases}$

$\therefore$

$$\int \underbrace{\sin^4 x}_{u^4} \underbrace{\cos x \, dx}_{du} = \int u^4 \, du = \frac{1}{5} u^5 + C$$

$$= \frac{1}{5} \sin^5 x + C$$

2. $\int \sin^5 x \cos x \, dx = \frac{1}{6} \sin^6 x + C$

(Análoga ao caso dado em 1)

3. $\int \cos^3 4x \sin 4x \, dx$

Seja $u = 4x \;\therefore\; du = 4dx \;\therefore\; dx = \frac{1}{4} du$

Daí

$$\int \cos^3 4x \sin 4x \, dx = \int \cos^3 u \sin u \, \frac{1}{4} du$$

$$= \frac{1}{4} \underbrace{\int \cos^3 u \sin u \, du}$$

$$= \frac{1}{4} (-1) \frac{\cos^4 u}{4} = -\frac{1}{16} \cos^4 4x + C$$

4. $\int \cos^6 \frac{1}{2}x \ \sin \frac{1}{2}x \ dx$

$u = \frac{1}{2}x \longrightarrow du = \frac{1}{2} dx$

$\therefore \ dx = 2\,du$

Daí

$$\int \cos^6 \frac{1}{2}x \ \sin\frac{1}{2}x \ dx = \int \cos^6 u \ \sin u \ 2\,du$$

$$= 2\int \cos^6 u \ \sin u \ du$$

$$= 2(-1)\frac{\cos^7 u}{7}$$

$$= -\frac{2}{7}\cos^7 u = -\frac{2}{7}\cos^7\frac{1}{2}x + C$$

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5. $\int \sin^3 x \ dx = \int \underbrace{\sin^2 x}\ \sin x \ dx$

$$= \int (\underbrace{1-\cos^2 x}) \sin x \ dx$$

$$= \int \sin x \ dx - \int \cos^2 x \sin x \ dx$$

$$= -\cos x - (-1)\frac{\cos^3 x}{3} + C$$

$$= -\cos x + \frac{1}{3}\cos^3 x + C$$

6. $\int \sin^2 3x \, dx$

$u = 3x \rightarrow du = 3\,dx \; \therefore \; dx = \frac{1}{3} du$

Daí

$$\int \sin^2 3x \, dx = \int \sin^2 u \, \frac{1}{3} du = \frac{1}{3} \int \underbrace{\sin^2 u}_{(*)} \, du$$

$$= \frac{1}{3} \int \left(\frac{1 - \cos 2u}{2} \right) du$$

$$= \frac{1}{6} \int (1 - \cos 2u) \, du$$

$$= \frac{1}{6} \int 1 \, du - \frac{1}{6} \int \cos 2u \, du$$

$$= \frac{1}{6} u - \frac{1}{6} \frac{1}{2} \sin 2u + C$$

$$= \frac{1}{6} u - \frac{1}{12} \sin 2u + C$$

$$= \frac{1}{6} 3x - \frac{1}{12} \sin 6x + C$$

$$= \frac{1}{2} x - \frac{1}{12} \sin 6x + C$$

(*) : Usou-se que $\sin^2 u = \dfrac{1 - \cos 2u}{2}$

7. $\int \sin^4 z \, dz$

Duas soluções:

i. $\int \sin^4 z \, dz = \int \sin^2 z \, \sin^2 z \, dz$

$$= \int \frac{(1-\cos 2z)}{2} \cdot \frac{(1-\cos 2z)}{2} \, dz$$

$$= \int \frac{1}{4}\left(1 - 2\cos 2z + \cos^2 2z\right) dz$$

$$= \int \left(\frac{1}{4} - \frac{1}{2}\cos 2z + \frac{1}{4}\cos^2 2z\right) dz$$

$$= \frac{1}{4}\int 1 \, dz - \frac{1}{2}\int \cos 2z \, dz + \frac{1}{4}\int \cos^2 2z \, dz$$

$$= \frac{1}{4} z - \frac{1}{2}\,\frac{\sin 2z}{2} + \frac{1}{4}\int \frac{1+\cos 4z}{2} \, dz$$

$$= \frac{1}{4} z - \frac{1}{4}\sin 2z + \frac{1}{8}\int 1 \, dz + \frac{1}{8}\int \cos 4z \, dz$$

$$= \frac{1}{4} z - \frac{1}{4}\sin 2z + \frac{1}{8} z + \frac{1}{8}\,\frac{1}{4}\sin 4z + C$$

$$= \frac{3}{8} z - \frac{1}{4}\sin 2z + \frac{1}{32}\sin 4z + C$$

11º

Usa-se a fórmula:

$$\int \sin^n z\,dz = -\frac{1}{n}\sin^{n-1} z\cos z + \frac{(n-1)}{n}\int \sin^{n-2} z\,dz$$

Daí

$$\int \sin^4 z\,dz = -\frac{1}{4}\sin^3 z\cos z + \frac{3}{4}\int \sin^2 z\,dz$$

$$= -\frac{1}{4}\sin^3 z\cos z + \frac{3}{4}\int \frac{(1-\cos 2z)}{2}\,dz$$

$$= -\frac{1}{4}\sin^3 z\cos z + \frac{3}{8}\int 1\,dz - \frac{3}{8}\int \cos 2z\,dz$$

$$= -\frac{1}{4}\sin^3 z\cos z + \frac{3}{8}z - \frac{3}{8}\frac{1}{2}\sin 2z + C$$

$$= -\frac{1}{4}\sin^3 z\cos z + \frac{3}{8}z - \frac{3}{16}\underbrace{\sin 2z}_{2\sin z\cos z} + C$$

$$= -\frac{1}{4}\sin^3 z\cos z + \frac{3}{8}z - \frac{3}{8}\sin z\cos z + C$$

8. $\int \cos^5 x \, dx = \int \cos^4 x \cos x \, dx = \int (\cos^2 x)^2 \cos x \, dx$

$= \int (1 - \operatorname{sen}^2 x)^2 \cos x \, dx$

$= \int (1 - 2\operatorname{sen}^2 x + \operatorname{sen}^4 x) \cos x \, dx$

$= \int \cos x \, dx - 2 \int \operatorname{sen}^2 x \cos x \, dx + \int \operatorname{sen}^4 x \cos x \, dx$

$= \operatorname{sen} x - 2 \frac{\operatorname{sen}^3 x}{3} + \frac{\operatorname{sen}^5 x}{5} + C$

$= \operatorname{sen} x - \frac{2}{3} \operatorname{sen}^3 x + \frac{1}{5} \operatorname{sen}^5 x + C$

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9. $\int \cos^2 \frac{x}{2} \, dx$

$u = \frac{x}{2} \rightarrow du = \frac{1}{2} dx \rightarrow dx = 2du$

Daí

$\int \cos^2 \frac{x}{2} \, dx = \int \cos^2 u \; 2du = 2 \int \cos^2 u \, du =$

$= \cancel{2} \int \frac{1 + \cos 2u}{\cancel{2}} \, du = \int 1 du + \int \cos 2u \, du$

$= u + \frac{1}{2} \operatorname{sen} 2u + C$

$= \frac{1}{2} x + \frac{1}{2} \operatorname{sen} x + C$

10. $\int \sin^3 x \cos^3 x \, dx =$

$$= \int \sin^3 x \cos^2 x \cos x \, dx$$

$$= \int \sin^3 x \, (1 - \sin^2 x) \cos x \, dx$$

$$= \int (\sin^3 x - \sin^5 x) \cos x \, dx$$

$$= \int (\sin^3 x \cos x - \sin^5 x \cos x) \, dx$$

$$= \int \sin^3 x \cos x \, dx - \int \sin^5 x \cos x \, dx$$

$$= \frac{1}{4} \sin^4 x - \frac{1}{6} \sin^6 x + C$$

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11. $\int \sin^2 x \cos^3 x \, dx = \int \sin^2 x \cos^2 x \cos x \, dx$

$$= \int \sin^2 x \, (1 - \sin^2 x) \cos x \, dx$$

$$= \int \sin^2 x \cos x \, dx - \int \sin^3 x \cos x \, dx$$

$$= \frac{1}{3} \sin^3 x - \frac{1}{4} \sin^4 x + C$$

12. $\int \cos^6 x \, dx$

Temos

$$(*): \int \cos^n x \, dx = \frac{1}{n} \cos^{n-1} x \, \sin x + \frac{(n-1)}{n} \int \cos^{n-2} x \, dx$$

Mas

$$\int \cos^2 x \, dx = \int \frac{(1+\cos 2x)}{2} dx = \frac{1}{2}\int 1 \, dx + \frac{1}{2}\int \cos 2x \, dx$$

$$= \frac{1}{2} x + \frac{1}{2} \frac{1}{2} \sin 2x$$

$$= \frac{1}{2} x + \frac{1}{4} \sin 2x$$

Tomando $n=4$ em $(*)$ obtem-se:

$$\int \cos^4 x \, dx = \frac{1}{4} \cos^3 x \sin x + \frac{3}{4} \int \cos^2 x \, dx$$

$$= \frac{1}{4} \cos^3 x \sin x + \frac{3}{4} \left(\frac{1}{2} x + \frac{1}{4} \sin 2x \right)$$

$$= \frac{1}{4} \cos^3 x \sin x + \frac{3}{8} x + \frac{3}{16} \sin 2x$$

Daí

$$\int \cos^6 x \, dx = \frac{1}{6} \cos^5 x \sin x + \frac{5}{6} \int \cos^4 x \, dx$$

$$= \frac{1}{6} \cos^5 x \sin x + \frac{5}{6} \left(\frac{1}{4} \cos^3 x \sin x + \frac{3}{8} x + \frac{3}{16} \sin 2x \right)$$

$$= \frac{1}{6} \cos^5 x \sin x + \frac{5}{24} \cos^3 x \sin x + \frac{5}{16} x + \frac{5}{32} \sin 2x$$

$$= \frac{1}{6}\cos^5 x \operatorname{sen} x + \frac{5}{24}\cos^3 x \operatorname{sen} x + \frac{5}{16}x +$$

$$+ \frac{5}{\cancel{32}\ 16}\cancel{2}\operatorname{sen} x \cos x$$

$\therefore$

$$\int \cos^6 x \, dx = \frac{1}{6}\cos^5 x \operatorname{sen} x + \frac{5}{24}\cos^3 x \operatorname{sen} x +$$

$$+ \frac{5}{16}\operatorname{sen} x \cos x + \frac{5}{16}x + C$$

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13. $\int \operatorname{sen}^5 x \cos^2 x \, dx =$

$$= \int \operatorname{sen} x \operatorname{sen}^4 x \cos^2 x \, dx$$

$$= \int \operatorname{sen} x (\operatorname{sen}^2 x)^2 \cos^2 x \, dx$$

$$= \int \operatorname{sen} x (1-\cos^2 x)^2 \cos^2 x \, dx$$

$$= \int \operatorname{sen} x (1 - 2\cos^2 x + \cos^4 x) \cos^2 x \, dx$$

$$= \int (\operatorname{sen} x - 2\operatorname{sen} x \cos^2 x + \operatorname{sen} x \cos^4 x) \cos^2 x \, dx$$

$$= \int (\operatorname{sen} x \cos^2 x - 2\operatorname{sen} x \cos^4 x + \operatorname{sen} x \cos^6 x) \, dx$$

$$= \int \cos^2 x \operatorname{sen} x \, dx - 2\int \cos^4 x \operatorname{sen} x \, dx + \int \cos^6 x \operatorname{sen} x \, dx$$

$$= -\frac{1}{3}\cos^3 x + 2\,\frac{1}{5}\cos^5 x - \frac{1}{7}\cos^7 x + C$$

14. $\int \sin^2 2t \cos^4 2t \, dt$

$$u = 2t \rightarrow du = 2dt \rightarrow dt = \frac{1}{2} du$$

Dar

$$\int \sin^2 2t \cos^4 2t \, dt = \int \sin^2 u \cos^4 u \, \frac{1}{2} du$$

$$= \frac{1}{2} \int \sin^2 u \cos^4 u \, du$$

$$= \frac{1}{2} \int (1-\cos^2 u) \cos^4 u \, du$$

$$= \frac{1}{2} \int \cos^4 u \, du - \frac{1}{2} \int \cos^6 u \, du \quad (*)$$

$$= \frac{1}{2} \int \cos^4 u \, du - \frac{1}{2} \left[\frac{1}{6} \cos^5 u \sin u + \frac{5}{6} \int \cos^4 u \, du \right]$$

$$= \frac{1}{2} \int \cos^4 u \, du - \frac{1}{12} \cos^5 u \sin u - \frac{5}{12} \int \cos^4 u \, du$$

$$= \frac{1}{12} \int \cos^4 u \, du - \frac{1}{12} \cos^5 u \sin u$$

$$= \frac{1}{12} \left[\frac{1}{4} \cos^3 u \sin u + \frac{3}{4} \int \cos^2 u \, du \right] - \frac{1}{12} \cos^5 u \sin u$$

$$= \frac{1}{48} \cos^3 u \sin u + \frac{1}{16} \int \cos^2 u \, du - \frac{1}{12} \cos^5 u \sin u$$

$$= \frac{1}{48}\cos^3 u \sin u + \frac{1}{16}\int \frac{(1+\cos 2u)}{2}\,du +$$

$$- \frac{1}{12}\cos^5 u \sin u$$

$$= \frac{1}{48}\cos^3 u \sin u + \frac{1}{32}\int 1\,du + \frac{1}{32}\int \cos 2u\,du$$

$$- \frac{1}{12}\cos^5 u \sin u$$

$$= \frac{1}{48}\cos^3 u \sin u + \frac{1}{32}u + \frac{1}{32}\frac{1}{2}\sin 2u$$

$$- \frac{1}{12}\cos^5 u \sin u$$

$$= \frac{1}{48}\cos^3 2t \sin 2t + \frac{1}{\cancel{32}}\cancel{2}t + \frac{1}{64}\sin 4t$$

$$- \frac{1}{12}\cos^5 2t \sin 2t$$

$$= -\frac{1}{12}\cos^5 2t \sin 2t + \frac{1}{48}\cos^3 2t \sin 2t +$$

$$+ \frac{1}{64}\sin 4t + \frac{1}{16}t + C$$

15.

$$\int \sin^2 3t \cos^2 3t \, dt =$$

$$= \int (\underbrace{\sin 3t \cos 3t}) ^2 \, dt$$

usou-se que

$$\begin{cases} \sin 6t = \sin(2.3t) \\ = 2\sin 3t \cos 3t \end{cases}$$

$$= \int \left(\underbrace{\tfrac{1}{2}\sin 6t}\right)^2 dt$$

$$= \int \frac{1}{4} \sin^2 6t \, dt$$

$$= \frac{1}{4}\int \underbrace{\sin^2 6t} \, dt$$

Usou-se que

$$\begin{cases} \sin^2 6t = \dfrac{1-\cos(2.6t)}{2} \\ = \dfrac{1-\cos 12t}{2} \end{cases}$$

$$= \frac{1}{4}\int \left(\frac{1-\cos 12t}{2}\right) dt$$

$$= \frac{1}{8}\int 1 \, dt - \frac{1}{8}\int \cos 12t \, dt$$

$$= \frac{1}{8} t - \frac{1}{8}\frac{\sin 12t}{12} + C$$

$$= \frac{1}{8} t - \frac{1}{96}\sin 12t + C$$

16) $\int \sqrt{\cos z}\ \sin^3 z\, dz =$

$= \int \sqrt{\cos z}\ \sin^2 z\ \sin z\, dz$

$= \int \sqrt{\cos z}\ (1-\cos^2 z)\sin z\, dz$

$= \int \sqrt{\cos z}\ \sin z\, dz - \int \sqrt{\cos z}\ \cos^2 z\ \sin z\, dz$

$= \int \cos^{1/2} z\ \sin z\, dz - \int \cos^{5/2} z\ \sin z\, dz$

$= \frac{2}{3(-)} \cos^{3/2} z - \frac{2}{7(-)} \cos^{7/2} z + C$

$= -\frac{2}{3}\cos^{3/2} z + \frac{2}{7}\cos^{7/2} z + C$

17) $\int \frac{\cos^3 3x}{\sqrt[3]{\sin 3x}}\, dx =$

$= \int \frac{\cos 3x\ \cos^2 3x}{\sqrt[3]{\sin 3x}}\, dx$

$= \int \frac{\cos 3x\ (1-\sin^2 3x)}{\sqrt[3]{\sin 3x}}\, dx$

$= \int \frac{\cos 3x}{\sqrt[3]{\sin 3x}}\, dx - \int \frac{\cos 3x\ \sin^2 3x}{\sqrt[3]{\sin 3x}}\, dx$

$= \int \cos 3x\, (\sin 3x)^{-\frac{1}{3}}\, dx - \int \cos 3x\ \sin^2 3x\ (\sin 3x)^{-\frac{1}{3}}\, dx$

$= \frac{3}{2(3)} (\sin 3x)^{\frac{2}{3}} - \int \cos 3x\ (\sin 3x)^{\frac{5}{3}}\, dx$

$= \frac{1}{2}(\sin 3x)^{2/3} - \frac{1}{8}(\sin 3x)^{\frac{8}{3}} + C$

18) $\int \sin^{3}\frac{1}{2}y \cos^2\frac{1}{2}y \, dy =$

$= \int \sin^2\frac{y}{2} \sin\frac{y}{2} \cos^2\frac{y}{2} \, dy$

$= \int \left(1-\cos^2\frac{y}{2}\right) \sin\frac{y}{2} \cos^2\frac{y}{2} \, dy$

$= \int \sin\frac{y}{2} \cos^2\frac{y}{2} \, dy - \int \sin\frac{y}{2} \cos^4\frac{y}{2} \, dy$

$= \frac{1(2)}{3(-)} \cos^3\frac{y}{2} - \frac{1(2)}{5(-)} \cos^5\frac{y}{2} + C$

$= -\frac{2}{3}\cos^3\frac{y}{2} + \frac{2}{5}\cos^5\frac{y}{2} + C$

19) $\int \cos \underset{m}{4x} \cos \underset{n}{3x} \, dx =$

$\left(\cos mx \cos nx = \frac{1}{2}\cos(m+n)x + \frac{1}{2}\cos(m-n)x\right)$

$= \frac{1}{2}\int (\cos 7x + \cos x) \, dx$

$= \frac{1}{2}\int \cos 7x \, dx + \frac{1}{2}\int \cos x \, dx$

$= \frac{\sin 7x}{14} + \frac{\sin x}{2} + C$

20) $\int \sin \underset{m}{2x} \cos \underset{n}{4x}\, dx =$

$$\left(\sin mx \cos nx = \frac{1}{2} \sin(m-n)x + \frac{1}{2} \sin(m+n)x \right)$$

$$= \int \left(\frac{1}{2} \sin(-2x) + \frac{1}{2} \sin 6x \right) dx$$

$$= -\frac{1}{2} \int \sin 2x\, dx + \frac{1}{2} \int \sin 6x\, dx$$

$$= -\frac{1}{2} \frac{\cos 2x}{(-2)} + \frac{1}{2} \frac{\cos 6x}{(-6)} + C$$

$$= \frac{1}{4} \cos 2x - \frac{1}{12} \cos 6x + C$$

21) $\int \sin \underset{m}{3y} \cos \underset{n}{5y}\, dy =$

$$= \int \left(\frac{1}{2} \sin(-2y) + \frac{1}{2} \sin 8y \right) dy$$

$$= -\frac{1}{2} \int \sin 2y\, dy + \frac{1}{2} \int \sin 8y\, dy$$

$$= -\frac{1}{2} \frac{\cos 2y}{-2} + \frac{1}{2} \frac{\cos 8y}{-8} + C$$

$$= \frac{1}{4} \cos 2y - \frac{1}{16} \cos 8y + C$$

$\cos mx \cos nx = \frac{1}{2}\cos(m+n)x + \frac{1}{2}\cos(m-$

22) $\int \underset{m}{\cos t} \, \underset{n}{\cos 3t} \, dt =$

$= \frac{1}{2}\int \cos 4t \, dt + \frac{1}{2}\int \cos(-2t) \, dt$

$= \frac{1}{2} \frac{\sin 4t}{4} + \frac{1}{2}\int \cos 2t \, dt$

$= \frac{\sin 4t}{8} + \frac{1}{2} \frac{\sin 2t}{2} + C$

$= \frac{\sin 4t}{8} + \frac{\sin 2t}{4} + C$ //

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23) $\int (\sin 3t - \sin 2t)^2 \, dt =$

$= \int (\sin^2 3t - 2\sin 3t \sin 2t + \sin^2 2t) \, dt$

$= \underbrace{\int \sin^2 3t \, dt}_{1} - 2\underbrace{\int \sin 3t \sin 2t \, dt}_{2} + \underbrace{\int \sin^2 2t \, dt}_{3}$

1 $= \int \sin^2 3t \, dt = \int \frac{1 - \cos 6t}{2} \, dt$

$= \frac{1}{2}t - \frac{1}{2} \frac{\sin 6t}{6}$

$= \frac{1}{2}t - \frac{1}{12}\sin 6t$ ✓

$\sin^2 3t = \frac{1 - \cos 6t}{2}$

2 $= -2\int \sin 3t \sin 2t \, dt$

$\sin \underset{3}{m}x \sin \underset{2}{n}x = \frac{1}{2}\cos(m-n)x - \frac{1}{2}\cos(m+n)x$

$= -2\int \left(\frac{1}{2}\cos t - \frac{1}{2}\cos 5t\right) dt$

$= -\int \cos t \, dt + \int \cos 5t \, dt = -\sin t + \frac{\sin 5t}{5}$ ✓

③ $= \int \sin^2 2t \, dt$

$= \int \frac{1-\cos 4t}{2} \, dt$

$= \frac{1}{2} t - \frac{1}{2} \int \cos 4t \, dt$

$= \frac{t}{2} - \frac{1}{2} \frac{\sin 4t}{4}$ ✓

Daí,

$$\int (\sin 3t - \sin 2t)^2 \, dt = \frac{1}{2} t - \frac{1}{12} \sin 6t$$

$$- \sin t + \frac{\sin 5t}{5}$$

$$+ \frac{t}{2} - \frac{\sin 4t}{8}$$

$$\boxed{\int (\sin 3t - \sin 2t)^2 \, dt = t - \sin t - \frac{\sin 4t}{8} + \frac{\sin 5t}{5} - \frac{\sin 6t}{12}}$$

24) $\int \sin x \, \sin 3x \, \sin 5x \, dx$

Temos:

$$\sin mx \sin nx = \frac{1}{2} \cos (m-n) x - \frac{1}{2} \cos (m+n) x$$

$$\Leftarrow$$

$$// \sin \underset{m}{x} \sin \underset{n}{3x} = \frac{1}{2} \cos (-2x) - \frac{1}{2} \cos (4x)$$

$$= \frac{1}{2} \cos 2x - \frac{1}{2} \cos 4x \; //$$

→

Daí:

$\sin x \sin 3x \sin 5x =$

$= \left(\frac{1}{2}\cos 2x - \frac{1}{2}\cos 4x\right)\sin 5x$

(★) $= \frac{1}{2}\cos 2x \sin 5x - \frac{1}{2}\cos 4x \sin 5x$

Mas

$\sin mx \cos nx = \frac{1}{2}\sin(m-n)x + \frac{1}{2}\sin(m+n)x$

$\Leftarrow$

(★★) $\begin{cases} \cos 2x \sin 5x = \frac{1}{2}\sin 3x + \frac{1}{2}\sin 7x \\ \cos 4x \sin 5x = \frac{1}{2}\sin x + \frac{1}{2}\sin 9x \end{cases}$

$\Rightarrow$ subst. (★★) em (★):

$\| \sin x \sin 3x \sin 5x = \frac{1}{4}\sin 3x + \frac{1}{4}\sin 7x - \frac{1}{4}\sin x - \frac{1}{4}\sin 9x \|$

$\therefore \int \sin x \sin 3x \sin 5x \, dx =$

$= \int \left(\frac{1}{4}\sin 3x + \frac{1}{4}\sin 7x - \frac{1}{4}\sin x - \frac{1}{4}\sin 9x\right) dx$

$$\boxed{\int \sin x \sin 3x \sin 5x \, dx = -\frac{1}{12}\cos 3x - \frac{1}{28}\cos 7x + \frac{1}{4}\cos x + \frac{1}{36}\cos 9x + C}$$

25) $\int_0^{\pi/2} \cos^3 x \, dx =$

$= \int_0^{\pi/2} \cos^2 x \cos x \, dx$

$= \int_0^{\pi/2} (1 - \sin^2 x) \cos x \, dx$

$= \int_0^{\pi/2} (\cos x - \sin^2 x \cos x) \, dx$

$= \sin x \Big|_0^{\pi/2} - \frac{\sin^3 x}{3} \Big|_0^{\pi/2}$

$= 1 - \frac{1}{3} = \frac{2}{3}$ //

26) $\int_0^1 \sin^{3} \frac{\pi}{2} t \, dt =$

$= \int_0^1 \sin^2 \frac{\pi}{2} t \sin \frac{\pi}{2} t \, dt$

$= \int_0^1 (1 - \cos^2 \frac{\pi}{2} t) \sin \frac{\pi}{2} t \, dt$

$= \int_0^1 \sin \frac{\pi}{2} t \, dt - \int_0^1 \cos^2 \frac{\pi}{2} t \sin \frac{\pi}{2} t \, dt$

$= -\frac{2}{\pi} \cos \frac{\pi}{2} t \Big|_0^1 - \frac{2}{\pi} \frac{1}{3(-)} \cos^3 \frac{\pi}{2} t \Big|_0^1$

$= \frac{-2}{\pi} \left[-1 \right] + \frac{2}{3\pi} \left[-1 \right]$

$= \frac{2}{\pi} - \frac{2}{3\pi} = \frac{4}{3\pi}$ //

27) $\int_0^1 \sin^{4}\left(\frac{1}{2}\pi x\right) \, dx$

$= \int_0^1 \left[\sin^2\left(\frac{\pi x}{2}\right)\right]^2 dx$

$= \int_0^1 \left(\frac{1-\cos \pi x}{2}\right)^2 dx$

$= \frac{1}{4}\int_0^1 (1 - 2\cos\pi x + \cos^2 \pi x) \, dx$

$= \underbrace{\frac{1}{4}\int_0^1 dx}_{} - \frac{1}{2}\int_0^1 \cos\pi x \, dx + \frac{1}{4}\int_0^1 \underbrace{\cos^2 \pi x}_{} \, dx$

$= \frac{1}{4} - \frac{1}{2} \cancel{\frac{\sin \pi x}{\pi}}^{\,0} \Big|_0^1 + \frac{1}{4}\int_0^1 \frac{1+\cos 2\pi x}{2} \, dx$

$= \frac{1}{4} + \frac{1}{8}\left[x + \frac{\sin 2\pi x}{2\pi}\right]_0^1$

$= \frac{1}{4} + \frac{1}{8}\left[1 + 0\right]$

$= \frac{1}{4/2} + \frac{1}{8} = \frac{3}{8}$

28) $\int_0^{\pi/3} \sin^3 t \cos^2 t \, dt$

$= \int_0^{\pi/3} \underbrace{\sin^2 t} \cos^2 t \sin t \, dt$

$= \int_0^{\pi/3} (1-\cos^2 t) \cos^2 t \sin t \, dt$

$= \int_0^{\pi/3} \cos^2 t \sin t \, dt - \int_0^{\pi/3} \cos^4 t \sin t \, dt$

$= -\frac{\cos^3 t}{3} \Big|_0^{\pi/3} + \frac{\cos^5 t}{5} \Big|_0^{\pi/3}$

$\cos\frac{\pi}{3} = \cos 60° = \frac{1}{2}$

$= -\left[\frac{\cos^3\frac{\pi}{3} - \cos^3 0}{3}\right] + \frac{1}{5}\left(\cos^5\frac{\pi}{3} - \cos^5 0\right)$

$= -\frac{1}{3}\left(\left(\frac{1}{2}\right)^3 - 1\right) + \frac{1}{5}\left(\left(\frac{1}{2}\right)^5 - 1\right)$

$= -\frac{1}{3}\left(\frac{1}{8} - 1\right) + \frac{1}{5}\left(\frac{1}{32} - 1\right)$

$= -\frac{1}{3}\left(-\frac{7}{8}\right) + \frac{1}{5}\left(-\frac{31}{32}\right)$

$= \frac{7}{24} - \frac{31}{160} = \frac{7 \times 160 - 31 \times 24}{24 \times 160}$

$= \frac{1120 - 744}{3840} = \frac{376}{3840} = \frac{\cancel{94}}{\cancel{960}} = \frac{47}{480}$

29) $\int_0^1 \sin^2 \pi t \cos^2 \pi t \, dt =$

$(\sin 2x = 2\sin x \cos x)$

$= \int_0^1 (\sin \pi t \cos \pi t)^2 \, dt$

$= \int_0^1 \left(\frac{1}{2} \sin 2\pi t\right)^2 dt$

$= \int_0^1 \frac{1}{4} \sin^2 2\pi t \, dt$

$= \frac{1}{4} \int_0^1 \sin^2 2\pi t \, dt$

$= \frac{1}{4} \int_0^1 \frac{1 - \cos 4\pi t}{2} \, dt$

$= \frac{1}{8} \left[t - \frac{\sin 4\pi t}{4\pi}\right]_0^1$

$= \frac{1}{8} [1 - 0] = \frac{1}{8}$ //

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30) $\int_0^{\pi/6} \sin 2x \cos 4x \, dx =$

de (20)

$= \left[\frac{1}{4} \cos 2x - \frac{1}{12} \cos 6x\right]_0^{\pi/6}$

$= \frac{1}{4}\left(\cos \frac{\pi}{3} - 1\right) - \frac{1}{12}(\cos \pi - \cos 0)$

$= \frac{1}{4}\left(\frac{1}{2} - 1\right) - \frac{1}{12}(-1 - 1)$

$= \frac{1}{4}\left(-\frac{1}{2}\right) - \frac{1}{12}(-2)$

$= -\frac{1}{8} + \frac{1}{6} = \frac{-6+8}{48} = \frac{2}{48} = \frac{1}{24}$ //

33) $\int 2\sin x \cos x \, dx$

a) $u = \sin x \longrightarrow du = \cos x \, dx$

$\int 2 \sin x \cos x \, dx = 2\int u \, du$

$= \cancel{2}\dfrac{u^2}{\cancel{2}} + C$

$\therefore \int 2 \sin x \cos x \, dx = \sin^2 x + C$

b) $u = \cos x \longrightarrow du = -\sin x \, dx$

$\int 2 \sin x \underline{\cos x} \, dx = -2\int u \, du$

$= -2\dfrac{u^2}{2} + C' = -u^2 + C'$

$\therefore \int 2 \sin x \cos x \, dx = -\cos^2 x + C'$

c) $2\sin x \cos x = \sin 2x$

$\int 2 \sin x \cos x \, dx = \int \sin 2x \, dx$

$= \dfrac{\cos 2x}{-2} + C''$

$\int 2 \sin x \cos x \, dx = -\dfrac{1}{2}\cos 2x + C''$

→ isto é temos aqui :

29) $\int 2\sin x \cos x \, dx = \begin{cases} \sin^2 x + C & (\star) \\ -\cos^2 x + C' & (\star\star) \\ -\frac{1}{2}\cos 2x + C'' & (\star\star\star) \end{cases}$

que são equivalentes.

De facto:

$$\star\star = \underbrace{-\cos^2 x}_{} + C' = -(1-\sin^2 x) + C'$$

$$= \sin^2 x \underbrace{-1 + C'}_{}$$

$$= \sin^2 x + C = (\star)$$

$$(\star\star\star) = -\frac{1}{2}\underbrace{\cos 2x}_{} + C'' = -\frac{1}{2}(2\cos^2 x - 1) + C''$$

$$= \underbrace{-\cos^2 x}_{} + \frac{1}{2} + C''$$

$$= \sin^2 x - 1 + \frac{1}{2} + C''$$

$$= \sin^2 x \underbrace{-\frac{1}{2} + C''}_{}$$

$$= \sin^2 x + C = \star$$

34) $\int_0^{\pi} \sin^2 nx \, dx = \frac{\pi}{2}$ (n inteiro positivo)

$$\int_0^{\pi} \sin^2 nx \, dx = \int_0^{\pi} \frac{1-\cos 2nx}{2} \, dx$$

$$= \left[\frac{1}{2}x - \frac{1}{2} \frac{\sin 2nx}{2n} \right]_0^{\pi}$$

$$= \frac{\pi}{2} - \frac{1}{4n} \sin \cancelto{0}{2n\pi}$$

$$= \frac{\pi}{2}$$

35) $\int_0^{\pi} \cos^n x \, dx = 0$ (n inteiro positivo ímpar)

$n = 2k+1$

$$\int_0^{\pi} \cos^{2k+1} x \, dx = \int_0^{\pi} \cos^{2k} x \cos x \, dx$$

$$(a+b)^n = \sum_{p=0}^{n} \binom{n}{p} a^{n-p} b^p \qquad = \int_0^{\pi} \underbrace{(\cos^2 x)}^{k} \cos x \, dx$$

$a = 1$, $b = -\sin^2 x$

$$= \int_0^{\pi} (1-\sin^2 x)^k \cos x \, dx$$

$$= \int_0^{\pi} \sum_{r=0}^{k} \binom{k}{r} (-1)^r \sin^{2r} x \cos x \, dx$$

$$= \sum_{r=0}^{k} \binom{k}{r} (-1)^r \underbrace{\int_0^{\pi} \sin^{2r} x \cos x \, dx}$$

$$= \sum_{r=0}^{k} \binom{k}{r} (-1)^r \left. \frac{\sin^{2r+1} x}{2r+1} \right|_0^{\pi} = 0$$

36) $\int_{-1}^{1} \cos n\pi x \cos m\pi x \, dx = \begin{cases} 0, & m \neq n \\ 1, & m = n \end{cases}$

$\cos n\pi x \cos m\pi x = \frac{1}{2} \cos (n+m)\pi x + \frac{1}{2} \cos (n-m)\pi x$

$\Leftrightarrow$

(A) $\left\{ \int_{-1}^{1} \cos n\pi x \cos m\pi x \, dx = \frac{1}{2} \int_{-1}^{1} \cos (n+m)\pi x \, dx + \frac{1}{2} \int_{-1}^{1} \cos (n-m)\pi x \, dx \right.$

Se $m \neq n$:

$\frac{1}{2} \int_{-1}^{1} \cos (n+m)\pi x \, dx = \frac{1}{2} \left. \frac{\sin (n+m)\pi x}{(n+m)\pi} \right|_{-1}^{1} = 0$

$\frac{1}{2} \int_{-1}^{1} \cos (n-m)\pi x \, dx = \frac{1}{2} \left. \frac{\sin (n-m)\pi x}{(n-m)\pi} \right|_{-1}^{1} = 0$

$\therefore \int_{-1}^{1} \cos n\pi x \cos m\pi x \, dx = 0$ se $m \neq n$

Se $m = n$:

$\frac{1}{2} \int_{-1}^{1} \cos (n+m)\pi x \, dx = \frac{1}{2} \int_{-1}^{1} \cos 2m\pi x \, dx$

$= \frac{1}{2} \left. \frac{\sin 2m\pi x}{2m\pi} \right|_{-1}^{1} = 0$

$\frac{1}{2} \int_{-1}^{1} \underbrace{\cos (n-m)\pi x}_{\cos 0\pi x} \, dx = \frac{1}{2} \int_{-1}^{1} dx = \frac{1}{2} \, 2 = 1$

$\boxed{\int_{-1}^{1} \cos n\pi x \cos m\pi x \, dx = 1 \text{ se } m = n}$