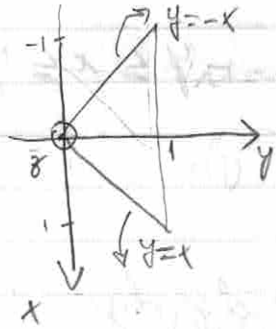
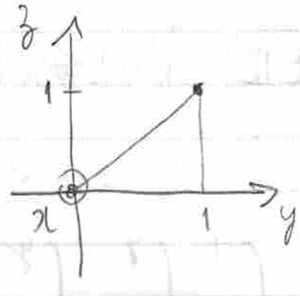
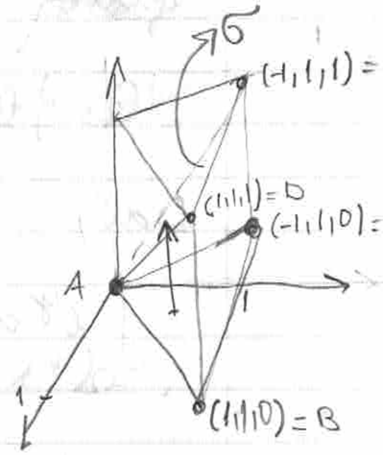
(11) Gullick

1. $\int_{\Omega} e^{z}\, dV$







Ω = Região do $\mathbb{R}^3$ cujos vértices são os pontos
$A=(0,0,0)$, $B=(1,1,0)$, $C=(-1,1,0)$, $D=(1,1,1)$, $E=(-1,$

precisaremos da eq. do plano σ gerado pelos pontos A, D, E.

Aqui

$\vec{AD} = (1,1,1)$
$\vec{AE} = (-1,1,1)$

e então

$$\vec{n} = \vec{AD} \times \vec{AE} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ -1 & 1 & 1 \end{vmatrix}$$

$$= \cancel{\hat{i}} - \hat{j} + \hat{k} + \hat{k} - \cancel{\hat{i}} - \hat{j}$$

$$= -2\hat{j} + 2\hat{k} = (0,-2,2)$$

Daí temos a eq. do plano σ como:

$$-2y + 2z = 0$$

Podemos então parametrizar a região Ω como

$$\Omega = \{(x,y,z) \in \mathbb{R}^3 \mid 0 \leq z \leq y, \ -y \leq x \leq y, \ 0 \leq$$

Daí,

$$\int_{\Omega} e^y \, dV = \int_{y=0}^{1} \int_{z=0}^{y} \int_{x=-y}^{y} e^y \, dx \, dz \, dy$$

$$= \int_{y=0}^{1} \int_{z=0}^{y} e^y x \Big]_{x=-y}^{y} dz \, dy$$

$$= \int_{y=0}^{1} \int_{z=0}^{y} 2e^y y \, dz \, dy$$

$$= \int_{y=0}^{1} 2e^y y z \Big]_{z=0}^{y} dy$$

$$= \int_{y=0}^{1} 2e^y y^2 \, dy \quad (*)$$

Mas

$$\int e^y y^2 \, dy = e^y y^2 - \int 2e^y y \, dy = e^y y^2 - 2\int$$

$u = y^2 \rightarrow du = 2y\,dy$
$dv = e^y dy \rightarrow v = e^y$

1. cont.

Mas

$$(**) = \int e^y y\,dy = ye^y - \int e^y dy$$

$$u = y \to du = dy \qquad = ye^y - e^y$$

$$dv = e^y dy \to v = e^y$$

$$\begin{cases} \int e^y y^2 dy = e^y y^2 - 2(ye^y - e^y) \\ \qquad = e^y y^2 - 2ye^y + 2e^y \\ \qquad = e^y(y^2 - 2y + 2) \end{cases} \quad (***)$$

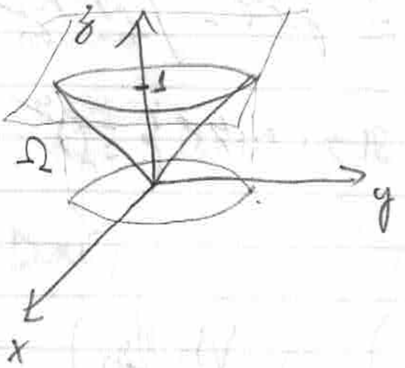
$(***) \to (*)$:

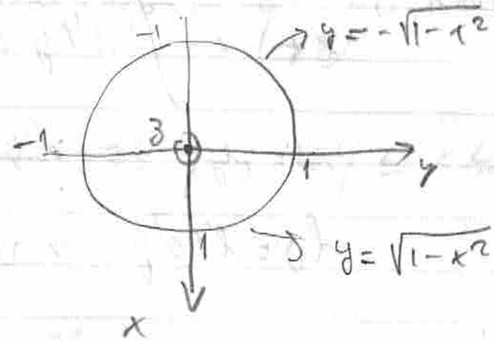
$$\int_\Omega e^y dV = 2\int_0^1 e^y y^2 dy = 2e^y(y^2-2y+2)\Big|_0^1$$

$$= 2e(1-\cancel{2}+\cancel{2}) - 2\cdot 2$$

$$= 2e - 4$$

15 2. $\int_\Omega zy \, dV$





$$\Omega = \{ (x,y,z) \in \mathbb{R}^3 \mid -1 \leq x \leq 1 \, , \, -\sqrt{1-x^2} \leq y \leq \sqrt{1-x^2} \, , \, \sqrt{x^2+y^2} \leq z \leq 1 \}$$

$$\int_\Omega zy \, dV = \int_{x=-1}^{1} \int_{y=-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{\sqrt{x^2+y^2}}^{1} zy \, dz \, dy \, dx$$

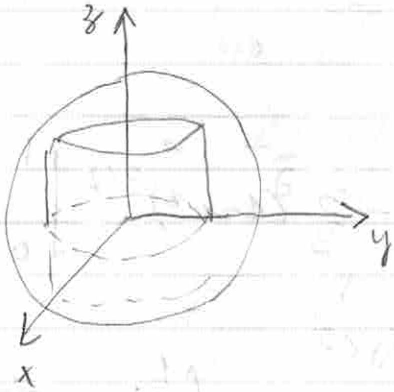
$$= \int_{x=-1}^{1} \int_{y=-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \frac{z^2 y}{2} \Big]_{\sqrt{x^2+y^2}}^{1} dy \, dx$$

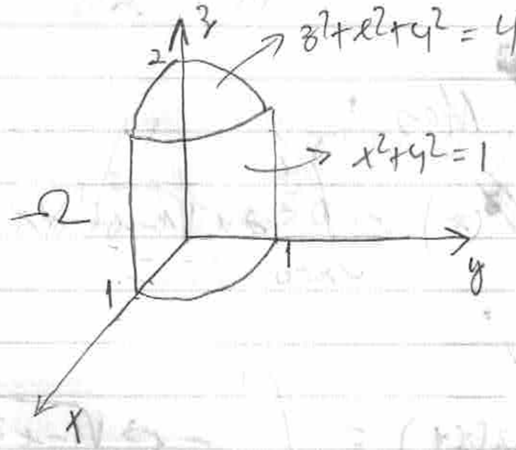
$$= \int_{x=-1}^{1} \int_{y=-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \frac{y}{2} (1 - x^2 - y^2) \, dy \, dx =$$

$$= \int_{x=-1}^{1} \frac{y^2}{4} - \frac{y^2}{4} x^2 - \frac{y^4}{8} \Big]_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}$$

$$= \int_{x=-1}^{1} \frac{1}{4} (1-x^2) - \frac{(1-x^2)}{4} x^2 - \frac{(1-x^2)^2}{8} - [\ldots] = 0$$

17 3. $\int_{\Omega} xz \, dV$





$$\Omega = \{(x,y,z) \in \mathbb{R}^3 \mid 0 \le x \le 1,\ 0 \le y \le \sqrt{1-x^2},\ 0 \le z \le \sqrt{4-x^2-y^2}$$

$$\int_{\Omega} xz \, dV = \int_{x=0}^{1} \int_{y=0}^{\sqrt{1-x^2}} \int_{z=0}^{\sqrt{4-x^2-y^2}} xz \, dz \, dy \, dx$$

$$= \int_{x=0}^{1} \int_{y=0}^{\sqrt{1-x^2}} \frac{xz^2}{2} \Big|_{z=0}^{\sqrt{4-x^2-y^2}} dy \, dx$$

$$= \int_{x=0}^{1} \int_{y=0}^{\sqrt{1-x^2}} \frac{x(4-x^2-y^2)}{2} \, dy \, dx$$

$$= \int_{x=0}^{1} \int_{y=0}^{\sqrt{1-x^2}} \left(2x - \frac{x^3}{2} - \frac{xy^2}{2}\right) dy \, dx$$

$$= \int_{x=0}^{1} 2xy - \frac{x^3}{2}y - \frac{xy^3}{6} \Big]_{y=0}^{\sqrt{1-x^2}} dx$$

$$= \int_{x=0}^{1} \Big(\underset{(*)}{2x\sqrt{1-x^2}} \ \underset{(**)}{- \frac{x^3}{2}\sqrt{1-x^2}} \ \underset{(***)}{- \frac{x}{6}(1-x^2)^{3/2}} \Big) dx$$

Mas

$$(*) = \int_{x=0}^{1} 2x\sqrt{1-x^2}\, dx = -\frac{2}{3}(1-x^2)^{3/2}\Big]_0^1 = \frac{2}{3}$$

$$(**) = \int_{x=0}^{1} -\frac{x^3}{2}\sqrt{1-x^2}\, dx = -\frac{1}{2}\underbrace{\int_{x=0}^{1} x^3\sqrt{1-x^2}\, dx}$$

$$\left\{\begin{array}{l} u = x^2 \longrightarrow du = 2x\,dx \\ dv = x\sqrt{1-x^2}\,dx \longrightarrow v = -\frac{1}{3}(1-x^2)^{3/2} \\ \therefore \int x^3\sqrt{1-x^2}\,dx = -\frac{x^2}{3}(1-x^2)^{3/2} + \frac{2}{3}\int x(1-x^2)^{3/2} \\ \qquad = -\frac{x^2}{3}(1-x^2)^{3/2} + \frac{2}{3}\frac{-1}{5}(1-x^2)^{5/2} \\ \qquad = -\frac{x^2}{3}(1-x^2)^{3/2} - \frac{2}{15}(1-x^2)^{5/2} \end{array}\right.$$

$$\Leftarrow$$

$$\int_{x=0}^{1} x^3\sqrt{1-x^2}\, dx = -\frac{x^2}{3}(1-x^2)^{3/2} - \frac{2}{15}(1-x^2)^{5/2}\Big|$$

$$= \frac{2}{15}$$

$\therefore$

$$(**) = -\frac{1}{2}\frac{2}{15} = \frac{-1}{15}$$

3. Cont.

$$/\!/ (***) = \int_{x=0}^{1} -\frac{x}{6}(1-x^2)^{3/2}\,dx$$

$$= -\frac{1}{6}\,\frac{-1}{5}(1-x^2)^{5/2}\Big|_0^1 = \frac{1}{30}(1-x^2)^{5/2}\Big|_0^1$$

$$= -\frac{1}{30}$$

$\therefore$

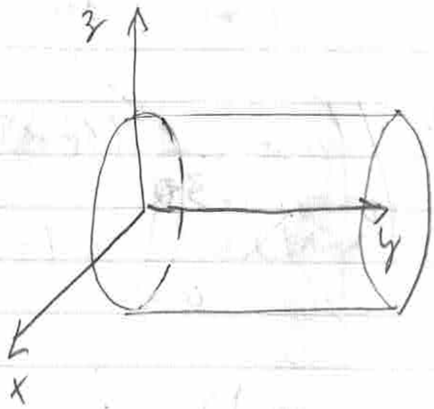
$$\int_\Omega xz\,dV = (*) + (**) + (***)$$

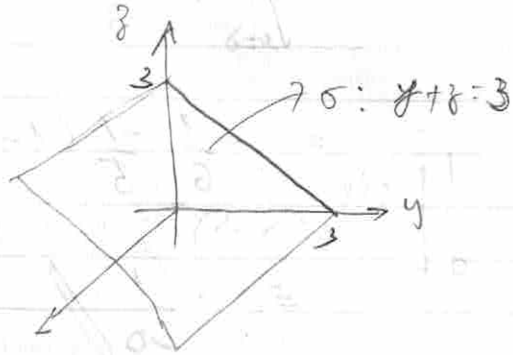
$$= \frac{2}{3} - \frac{1}{15} - \frac{1}{30}.$$

$$= \frac{20-2-1}{30} = \frac{17}{30}$$

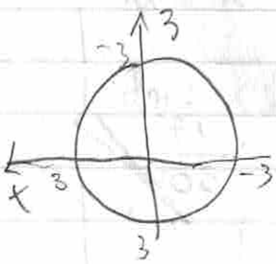
Barkley
pg. 1016
Ex. 17

4. $\int_{\Omega} (3x + xz)\, dV$





$\Omega = \{ (x,y,z) \in \mathbb{R}^3 \mid 0 \le y \le 3 - z,\ -3 \le x \le 3,\ -\sqrt{9-x^2} \le z$



$$\int_{\Omega} (3x + xz)\, dV = \int_{x=-3}^{3} \int_{z=-\sqrt{9-x^2}}^{\sqrt{9-x^2}} \int_{y=0}^{3-z} (3x + xz)\, dy\, dz\, dx$$

$$= \int_{x=-3}^{3} \int_{z=-\sqrt{9-x^2}}^{\sqrt{9-x^2}} (3x + xz)\, y \Big|_{y=0}^{3-z} dz\, dx.$$

$$= \int_{x=-3}^{3} \int_{z=-\sqrt{9-x^2}}^{\sqrt{9-x^2}} \underbrace{(3x + xz)(3 - z)}_{9x + \cancel{3xz} - \cancel{3xz} - xz^2} dz\, dx$$

4. cont.

$$= \int_{x=-3}^{3} \int_{z=-\sqrt{9-x^2}}^{\sqrt{9-x^2}} (9x - xz^2)\, dz\, dx$$

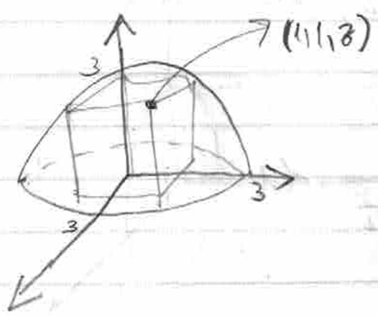
$$= \int_{x=-3}^{3} 9xz - \frac{xz^3}{3} \Big]_{z=-\sqrt{9-x^2}}^{\sqrt{9-x^2}} dx$$

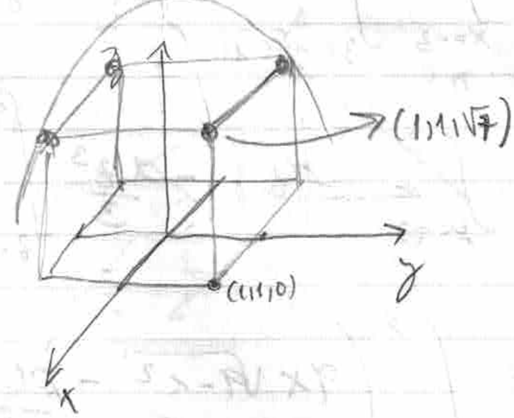
$$= \int_{x=-3}^{3} \Big(9x\sqrt{9-x^2} - \frac{x}{3}(9-x^2)^{3/2} + 9x\sqrt{9-x^2} - \frac{x}{3}(9$$

$$= \int_{x=-3}^{3} \left(18x\sqrt{9-x^2} - \frac{2x}{3}(9-x^2)^{3/2} \right) dx$$

$$= 18\frac{-1}{3}(9-x^2)^{3/2} - \frac{2}{3}\frac{-1}{5}(9-x^2)^{5/2} \Big]_{-3}^{3} = 0$$

5. $\int_\Omega z\,dV$





$(1,1,z) \in x^2+y^2+z^2=9$

$1+1+z^2=9$

$z = \pm\sqrt{7}$

$\Omega = \{(x,y,z) \in \mathbb{R}^2 \mid -1 \le x \le 1,\ -1 \le y \le 1,\ 0 \le z \le \sqrt{9-x^2-y^2}\}$

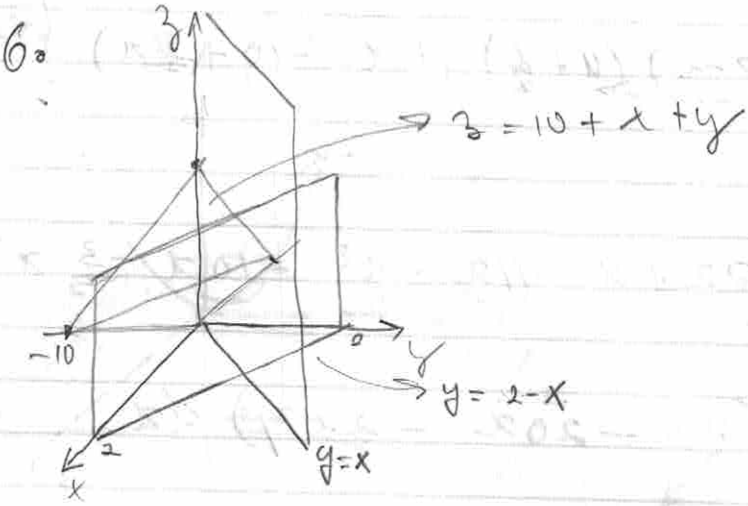
$$\int_\Omega z\,dV = \int_{x=-1}^{1}\int_{y=-1}^{1}\int_0^{\sqrt{9-x^2-y^2}} z\,dz\,dx\,dy$$

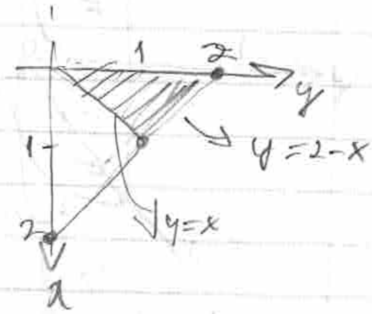
$$= \int_{x=-1}^{1}\int_{y=-1}^{1} \frac{z^2}{2}\Big]_0^{\sqrt{9-x^2-y^2}} dx\,dy$$

$$= \int_{x=-1}^{1}\int_{y=-1}^{1} \frac{9-x^2-y^2}{2}\,dx\,dy$$

$$= \int_{x=-1}^{1} \frac{9}{2}y - \frac{x^2y}{2} - \frac{y^3}{6}\Big]_{y=-1}^{1} dx$$

6.





$$\Omega = \{(x,y,z) \in \mathbb{R}^3 \mid 0 \le x \le 1 \;,\; x \le y \le 2-x$$
$$0 \le z \le 10 + x + y\}$$

$$V = \int_\Omega dx\,dy\,dz = \int_{x=0}^{1}\int_{y=x}^{2-x}\int_{z=0}^{10+x+y} dz\,dy\,dx$$

$$= \int_{x=0}^{1}\int_{y=x}^{2-x} (10+x+y)\,dy\,dx$$

$$= \int_{x=0}^{1} (10+x)y + \frac{y^2}{2}\Big]_{y=x}^{2-x} dx$$

$$= \int_{x=0}^{1} \Big((10+x)(2-x) + \frac{(2-x)^2}{2}$$
$$- (10+x)x - \frac{x^2}{2}\Big) dx$$

$$= \int_{x=0}^{1} \Big\{ (2-x)\Big(10+x + \frac{2-x}{2}\Big) +$$
$$+ x\Big(-10-x-\frac{x}{2}\Big)\Big\} dx$$

$$= \int_0^1 \left\{ (2-x)\left(11+\frac{x}{2}\right) + x\left(-10-\frac{3}{2}x\right) \right\} dx$$

$$= \int_0^1 \left(22 + x - 11x - \frac{x^2}{2} - 10x - \frac{3}{2}x^2 \right) dx$$

$$= \int_0^1 \left(22 - 20x - 2x^2 \right) dx$$

$$= 22x - \overset{10}{\cancel{20}}\frac{x^2}{\cancel{2}} - \frac{2x^3}{3} \Big]_0^1$$

$$= 22 - 10 - \frac{2}{3}$$

$$= 12 - \frac{2}{3} = \frac{36-2}{3} = \frac{34}{3}$$

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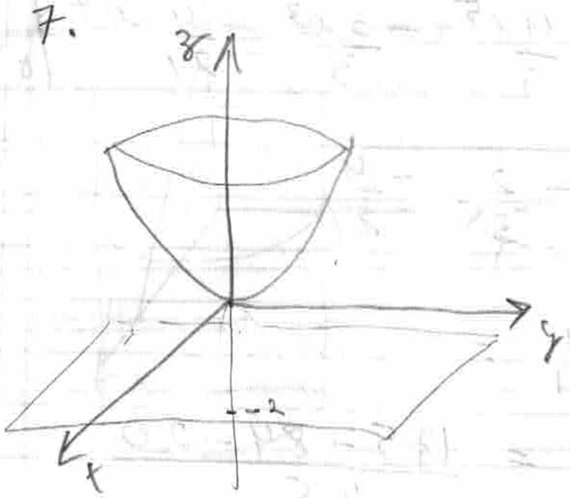
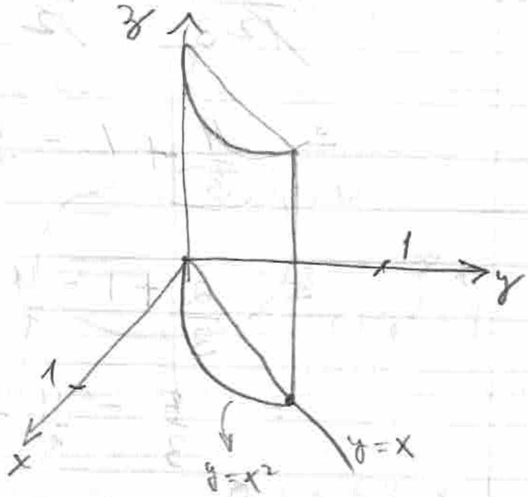
7.





$$\Omega = \{ (x,y,z) \in \mathbb{R}^3 \mid 0 \le x \le 1 \ , \ x^2 \le y \le x \ , \ -2 \le z \le 4($$

$$V = \int_\Omega dx\,dy\,dz = \int_{x=0}^{1} \int_{y=x^2}^{x} \int_{z=-2}^{4(x^2+y^2)} dz\, dy\, dx$$

$$= \int_{x=0}^{1} \int_{y=x^2}^{x} \left(4(x^2+y^2) + 2 \right) dy\, dx$$

$$= \int_{x=0}^{1} (4x^2+2)y + \frac{4y^3}{3} \Big]_{y=x^2}^{x} dx$$

$$= \int_{x=0}^{1} \left((4x^2+2)x + \frac{4}{3}x^3 - (4x^2+2)x^2 - \frac{4x^6}{3} \right) dx$$

$$= \int_{x=0}^{1} \left(\underline{4x^3} + 2x + \underline{\frac{4}{3}x^3} - 4x^4 - 2x^2 - \frac{4x^6}{3} \right) dx$$

$$= \int_{x=0}^{1} \left(\frac{16x^3}{3} + 2x - 4x^4 - 2x^2 - \frac{4}{3}x^6 \right) dx$$

$$= \frac{\overset{4}{\cancel{16}}x^4}{\cancel{12}\,3} + \frac{\cancel{2}x^2}{\cancel{2}} - \frac{4x^5}{5} - \frac{2x^3}{3} - \frac{4}{21}x^7 \Big|_0^1$$

$$= \underline{\frac{4}{3} + 1} - \frac{4}{5} - \underline{\frac{2}{3}} - \frac{4}{21}$$

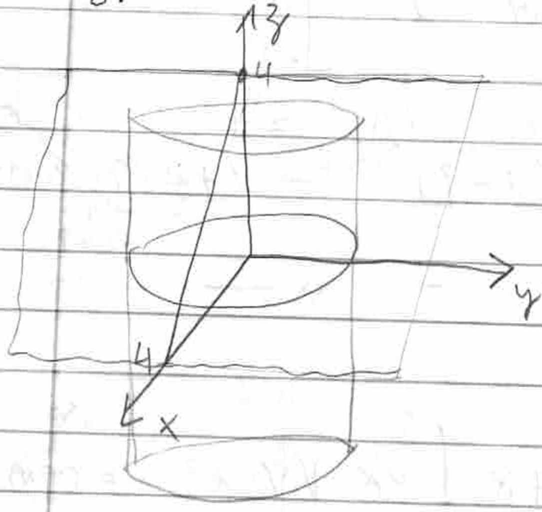
$$= \frac{2}{3} + 1 - \frac{4}{5} - \frac{4}{21} =$$

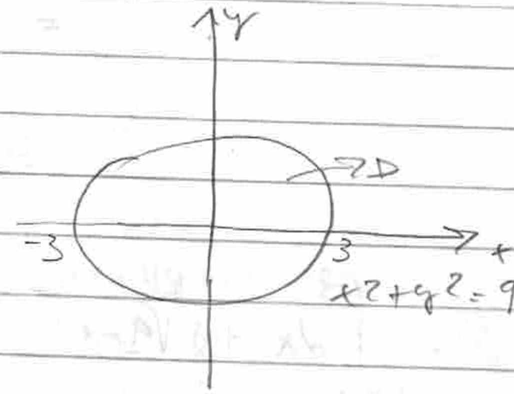
$$= \frac{5}{3} - \frac{4}{5} - \frac{4}{21} = \frac{175 - 84 - 20}{105}$$

$$= \frac{175 - 104}{105} =$$

$$= \frac{71}{105} \,//$$

8.





$$\Omega = \{(x,y,z) \in \mathbb{R}^3 \mid (x,y) \in D, \quad 0 \le z \le 4-x\}$$

$$V = \int_\Omega dV = \int_{(x,y)\in D} \left(\int_0^{4-x} dz \right) dA$$

$$= \int_{(x,y)\in D} z \Big]_0^{4-x} dA = \int_{(x,y)\in D} (4-x)\, dA \quad (*)$$

Seja

$$D = \{(x,y) \in \mathbb{R}^2 \mid -3 \le x \le 3, \quad -\sqrt{9-x^2} \le y \le \sqrt{9-x^2}\}$$

Então,

$$(*) = \int_{-3}^{3} dx \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} dy\, (4-x) = \int_{-3}^{3} dx\, (4-x)\, y \Big]_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}}$$

$$= \int_{-3}^{3} dx\, (4-x)\, 2\sqrt{9-x^2}$$

$$= \int_{-3}^{3} dx \left(\underbrace{-2x\sqrt{9-x^2}}_{(1)} + \underbrace{8\sqrt{9-x^2}}_{(2)} \right)$$

$$① = \int_{-3}^{3} dx - 2x\sqrt{9-x^2} = +\frac{2}{3}(9-x^2)^{3/2}\Big]_{-3}^{3}$$

$$= +\frac{2}{3}(\cancel{9-9})^{3/2} - \left(+\frac{2}{3}(\underbrace{\cancel{9-(-3)^2}}_{9-9})^{3/2}\right)$$

$$= 0$$

$$② = \int_{-3}^{3} dx + 8\sqrt{9-x^2} = +8\int_{-3}^{3} dx\,\sqrt{9-x^2} = (**)$$

Seja

$$x = 3\sin\theta \quad , \quad -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$$

$$\begin{cases} x=-3 \Rightarrow -3 = 3\sin\theta \Rightarrow \theta = -\frac{\pi}{2} \\ x = 3 \Rightarrow 3 = 3\sin\theta \Rightarrow \theta = \frac{\pi}{2} \\ dx = 3\cos\theta\, d\theta \end{cases}$$

$\therefore$

$$(**) = +8\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 3\cos\theta\, d\theta\, \sqrt{9-9\sin^2\theta}$$

$$= +8\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\theta\; 3\cos\theta\; 3\cos\theta$$

$$= +8\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\theta\; 9\cos^2\theta$$

$$= +72\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\theta\; \frac{1+\cos 2\theta}{2}$$

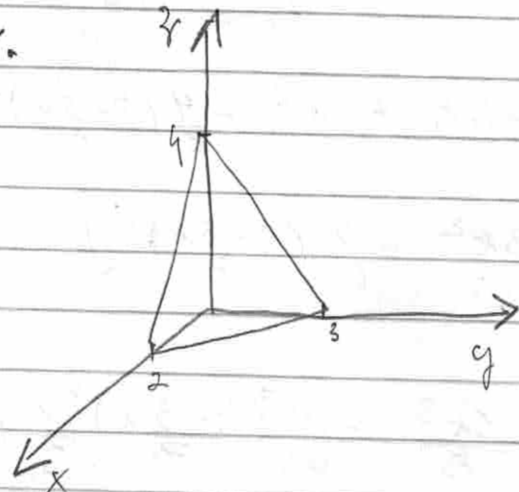
$$= +36\left(\theta + \frac{\sin 2\theta}{2}\right)\Big]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$$

$$= +36\left(\frac{\pi}{2} - \left(-\frac{\pi}{2}\right)\right) = +36\pi$$

8. Cont.

$$\therefore \quad \boxed{V = \int_{\Omega} dV = 36\pi}$$

9.



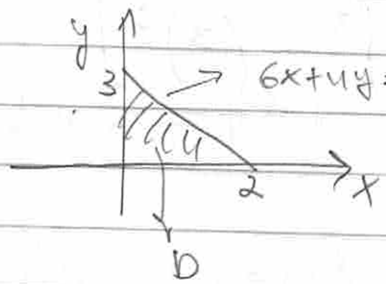
$6x + 4y + 3z = 12$

$\underline{x=0}$: $4y + 3z = 12$

$\underline{y=0}$: $6x + 3z = 12$

$z = 0$: $6x + 4y = 12$

$$\Omega = \left\{ (x,y,z) \in \mathbb{R}^3 \;\middle|\; (x,y) \in D \,,\; 0 \leq z \leq 4 - 2x - \frac{4y}{3} \right\}$$



$\rightarrow 6x + 4y = 12 \quad \therefore \quad y = 3 - \frac{3}{2}x$

$$D = \left\{ (x,y) \in \mathbb{R}^2 \;\middle|\; 0 \leq x \leq 2 ,\; 0 \leq y \leq 3 - \frac{3x}{2} \right\}$$

$$V = \int_{\Omega} dV = \int_0^2 dx \int_0^{3-\frac{3}{2}x} dy \int_0^{4-2x-\frac{4y}{3}} dz$$

$$= \int_0^2 dx \int_0^{3-\frac{3}{2}x} dy \;\; z \Big]_0^{4-2x-\frac{4}{3}y}$$

$$= \int_0^2 dx \int_0^{3-\frac{3}{2}x} dy \left(4 - 2x - \frac{4}{3}y\right)$$

$$= \int_0^2 dx \int_0^{3-\frac{3}{2}x} dy \left(4 - 2x - \frac{4}{3}y\right)$$

$$= \int_0^2 dx \quad 4y - 2xy - \frac{4}{3}\frac{y^2}{2} \Big]_0^{3-\frac{3}{2}x}$$

$$= \int_0^2 dx \left[4(3-\tfrac{3}{2}x) - 2x(3-\tfrac{3}{2}x) - \frac{4}{6}(3-\tfrac{3}{2}x)^2\right]$$

$$= \int_0^2 dx \left[12 - 6x - 6x + 3x^2 - \frac{4}{6}(3-\tfrac{3}{2}x)^2\right]$$

$$= \int_0^2 dx \left[12 - 12x + 3x^2 - \frac{4}{6}(3-\tfrac{3}{2}x)^2\right]$$

$$= 12x - \frac{12x^2}{2} + \frac{\cancel{3}x^3}{\cancel{3}} - \frac{4^2}{6_3}(3-\tfrac{3}{2}x)^3 \frac{1}{3}\left(-\frac{2}{3}\right)\Big]$$

$$= \cancel{24} - 6\cancel{.4} + 8 + \left[\frac{4}{27}(3-\tfrac{3}{2}x)^3\right]_0^2$$

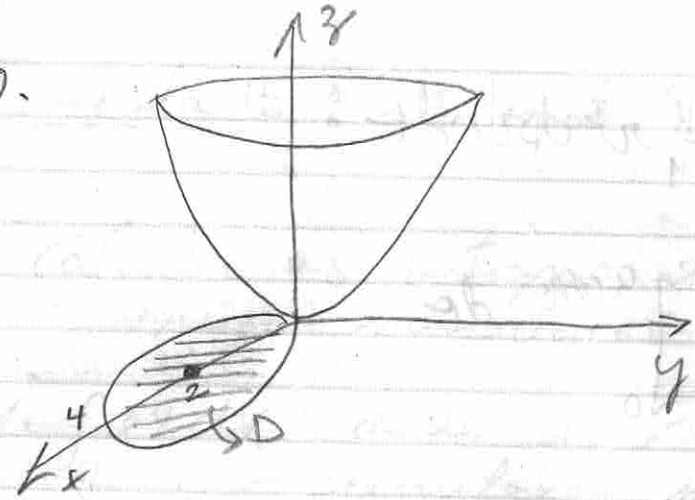
$$= 8 + \frac{4}{27}\left[(3-\cancel{\tfrac{3}{2}\cdot 2}^{\,0})^3 - (3)^3\right]$$

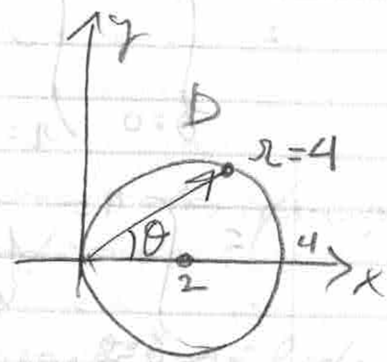
$$= 8 + \frac{4}{\cancel{27}}(-\cancel{27})$$

$$= 8 - 4$$

$$= 4$$

10.





$x^2+y^2=4x$

$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases}$$

$x^2+y^2=4x$

$r^2\cos^2\theta + r^2\sin^2\theta = 4r\cos\theta$

$r^2 = 4r\cos\theta$

$\| r = 4\cos\theta \|$

$D = \{ (x,y) \in \mathbb{R}^2 \; ; \; 0 \le \theta \le \pi \, , \; 0 \le r \le 4\cos\theta \}$

$\Omega = \{ (x,y,z) \in \mathbb{R}^3 \; ; \; 0 \le z \le \frac{1}{4}(x^2+y^2)$

$$\int_{D(r,\theta)} \left(\int_{z=0}^{\frac{1}{4}(x^2+y^2)} dz \right) dA = \int_{D(r,\theta)} z \Big]_0^{\frac{1}{4}(x^2+y^2)} dA$$

$$= \int_{D(r,\theta)} \frac{1}{4}(x^2+y^2) \, \underline{dA} = \int_{\theta=0}^{\pi} \int_{r=0}^{4\cos\theta} \frac{1}{4} r^2 \, \underline{r\,dr\,d\theta}$$

$$= \int_{\theta=0}^{\pi} \int_{r=0}^{4\cos\theta} \frac{1}{4} r^3 \, dr \, d\theta$$

$$= \int_{\theta=0}^{\pi} \frac{1}{4} \frac{r^4}{4} \Big]_0^{4\cos\theta} d\theta$$

$$= \int_{\theta=0}^{\pi} \frac{1}{16} 256 \cos^4\theta \, d\theta$$

$$= \int_{\theta=0}^{\pi} 16 \cos^4\theta \, d\theta = \int_{\theta=0}^{\pi} 16 (\cos^2\theta)^2 \, d\theta$$

$$= \int_0^{\pi} 16 \left(\frac{1+\cos 2\theta}{2}\right)^2 d\theta$$

$$= \int_0^{\pi} \frac{16}{4} (1 + 2\cos 2\theta + \cos^2 2\theta) \, d\theta$$

$$= 4\theta\Big]_0^{\pi} + 8 \underbrace{\frac{\sin 2\theta}{2}\Big]_0^{\pi}}_{0} + 4\int_0^{\pi} \frac{1+\cos 4\theta}{2} d\theta$$

$$= 4\pi + 2\left[\theta + \underbrace{\frac{\sin 4\theta}{4}}_{0}\right]_0^{\pi}$$

$$= 4\pi + 2[\pi] = 6\pi$$

3
16
16
6
9
16
$256 \mid 16$
$96 \mid 16$
00

$$= \int_{\theta=0}^{\pi} \int_{r=0}^{4\cos\theta} \frac{1}{4} r^3 \, dr \, d\theta$$

$$= \int_{\theta=0}^{\pi} \frac{1}{4} \frac{r^4}{4} \Big]_0^{4\cos\theta} d\theta$$

$$= \int_{\theta=0}^{\pi} \frac{1}{16} 256 \cos^4\theta \, d\theta$$

$$\begin{array}{r} 16 \\ 16 \\ \hline 6 \\ 16 \\ \hline 256 \end{array} \quad 256 \mid 16, \; 96 \mid 16$$

$$= \int_{\theta=0}^{\pi} 16 \cos^4\theta \, d\theta = \int_{\theta=0}^{\pi} 16 \, (\cos^2\theta)^2 \, d\theta$$

$$= \int_0^{\pi} 16 \left(\frac{1+\cos 2\theta}{2}\right)^2 d\theta$$

$$= \int_0^{\pi} \frac{16}{4} (1 + 2\cos 2\theta + \cos^2 2\theta) \, d\theta$$

$$= 4\theta\Big]_0^{\pi} + 8 \, \cancel{\frac{\sin 2\theta}{2}}^{\,0} \Big]_0^{\pi} + 4\int_0^{\pi} \frac{1+\cos 4\theta}{2} d\theta$$

$$= 4\pi + 2\left[\theta + \cancel{\frac{\sin 4\theta}{4}}^{\,0}\right]_0^{\pi}$$

$$= 4\pi + 2[\pi] = 6\pi$$