Aula 2

Int. Funções trigonométricas

Exercícios: Leithold pg. 545)

1. $\int tg^2 5x\, dx =$ $\quad (tg^2 x = sec^2 x - 1)$

$= \int (sec^2 5x - 1)\, dx$ $\quad \int sec^2 x\, dx = tg\, x$

$= \int sec^2 5x\, dx - \int dx$

$$\boxed{\int tg^2 5x\, dx = \frac{tg\, 5x}{5} - x + C}$$

2. $\int cotg^2 4t\, dt =$ $\quad \left\{ \begin{array}{l} cotg^2 x = cosec^2 x - 1 \\ \int cosec^2 x\, dx = -cotg\, x \end{array} \right.$

$= \int (cosec^2 4t - 1)\, dt$

$= \int cosec^2 4t\, dt - \int dt$

$$\boxed{\int cotg^2 4t\, dt = \frac{-cotg\, 4t}{4} - t + C}$$

3. $\int x\, cotg^2 2x^2\, dx$

$2x^2 = u \rightarrow du = 4x\, dx$

$\therefore \int x\, cotg^2 2x^2\, dx = \int cotg^2 u \frac{du}{4}$

$= \frac{1}{4} \int cotg^2 u\, du$

$$= \frac{1}{4}\int \cot g^2 u \, du$$

$$= \frac{1}{4}\int (\operatorname{cosec}^2 u - 1)\, du$$

$$= \frac{1}{4}\underbrace{\int \operatorname{cosec}^2 u \, du} - \frac{1}{4}\int du$$

$$= \frac{1}{4}(-\cot g u) - \frac{1}{4}u + C$$

$$= -\frac{1}{4}\cot g\, 2x^2 - \frac{1}{4}2x^2 + C$$

$$\boxed{\int x \cot g^2 2x^2 dx = -\frac{1}{4}\cot g\, 2x^2 - \frac{1}{2}x^2 + C}$$

4. $\int e^x \, tg^2(e^x)\, dx$

Seja $u = e^x \rightarrow du = e^x dx$

Daí,

$$\int \underbrace{e^x}\, tg^2(e^x)\, \underbrace{dx} = \int \underbrace{tg^2 u}\, du$$

$$= \int (\sec^2 u - 1)\, du$$

$$= \underbrace{\int \sec^2 u \, du} - \int du$$

$$= tg\, u - u + C$$

$$\boxed{\int e^x tg^2(e^x)\, dx = tg\, e^x - e^x + C}$$

5. $\int \cot g^3 t \, dt$ $\qquad \int \cot g^n t \operatorname{cosec}^2 t \, dt = -\dfrac{\cot g^{n+1} t}{n+1}$

$= \int \cot g^2 t \cot g \, t \, dt$

$= \int (\operatorname{cosec}^2 t - 1) \cot g \, t \, dt$

$= \underbrace{\int \operatorname{cosec}^2 t \cot g \, t \, dt} - \underbrace{\int \cot g \, t \, dt}$

$= -\frac{1}{2} \cot g^2 t - (-\ln |\operatorname{cosec} t|) + C$

$\therefore$

$$\boxed{\int \cot g^3 t \, dt = -\frac{1}{2} \cot g^2 t + \ln |\operatorname{cosec} t| + C}$$

6. $\int tg^4 x \, dx = \int tg^2 x \, tg^2 x \, dx$

$= \int (\sec^2 x - 1) \, tg^2 x \, dx$

$= \underbrace{\int \sec^2 x \, tg^2 x \, dx} - \underbrace{\int tg^2 x \, dx}$

$= \frac{tg^3 x}{3} - \int (\sec^2 x - 1) \, dx$

$\therefore$ $= \frac{tg^3 x}{3} - \underbrace{\int \sec^2 x \, dx} + \int dx$

$$\boxed{\int tg^4 x \, dx = \frac{tg^3 x}{3} - tg \, x + x + C}$$

obs.: $\int tg^4 mx \, dx = \frac{tg^3 mx}{3m} - \frac{tg \, mx}{m} + x$

7. $\int tg^6 3x\, dx =$

$$\left\{ \int tg^n kx \sec^2 kx\, dx = \frac{tg^{n+1} kx}{k(n+1)} \right.$$

$$= \int \underbrace{tg^2 3x}\, tg^4 3x\, dx$$

$$= \int (\sec^2 3x - 1)\, tg^4 3x\, dx$$

$$= \underbrace{\int \sec^2 3x\, tg^4 3x\, dx} - \underbrace{\int tg^4 3x\, dx}$$

exercício 6

$$= \frac{tg^5 3x}{15} - \int \underbrace{tg^2 3x}\, tg^2 3x\, dx$$

$$= \frac{tg^5 3x}{15} - \int (\sec^2 3x - 1)\, tg^2 3x\, dx$$

$$= \frac{tg^5 3x}{15} - \int \sec^2 3x\, tg^2 3x\, dx + \underbrace{\int tg^2 3x\, dx}$$

$$= \frac{tg^5 3x}{15} - \frac{tg^3 3x}{9} + \int (\sec^2 3x - 1)\, dx$$

$$= \frac{tg^5 3x}{15} - \frac{tg^3 3x}{9} + \underbrace{\int \sec^2 3x\, dx} - \int dx$$

$$\boxed{\int tg^6 3x\, dx = \frac{tg^5 3x}{15} - \frac{tg^3 3x}{9} + \frac{tg 3x}{3} - x + C}$$

$$8.\ \int \cot g^5 2x\, dx$$

$$\int \cot g^n kx\, cossec^2 kx\, dx = -\frac{\cot g^{n+1} kx}{(n+1)k}$$

$$= \int \cot g^2 2x\, \cot g^3 2x\, dx$$

$$= \int (cossec^2 2x - 1)\, \cot g^3 2x\, dx$$

$$= \underbrace{\int cossec^2 2x\, \cot g^3 2x\, dx} - \int \cot g^3 2x\, dx$$

$$= -\frac{\cot g^4 2x}{8} - \int \cot g^2 2x\, \cot g\, 2x\, dx$$

$$= -\frac{\cot g^4 2x}{8} - \int (cossec^2 2x - 1)\, \cot g\, 2x\, dx$$

$$= -\frac{\cot g^4 2x}{8} - \underbrace{\int cossec^2 2x\, \cot g\, 2x\, dx} + \underbrace{\int \cot g\, 2x\, dx}$$

$$= -\frac{\cot g^4 2x}{8} - (-)\frac{\cot g^2 2x}{4} + \frac{\ln|\sin 2x|}{2} + C$$

$$\boxed{\int \cot g^5 2x\, dx = -\frac{\cot g^4 2x}{8} + \frac{\cot g^2 2x}{4} + \frac{\ln|\sin 2x|}{2} + C}$$

$$9.\ \int \sec^{4} x\, dx = \quad \to \text{por (caso 2)}$$

$$\sec^{2k} x = \sec^{2k-2} x\, \sec^2 x = (\sec^2 x)^{(k-1)} \sec^2 x$$

$$= \int \underbrace{\sec^2 x}\, \sec^2 x\, dx$$

$$= \int (1 + tg^2 x)\, \sec^2 x\, dx$$

$$= \underbrace{\int \sec^2 x\, dx} + \underbrace{\int tg^2 x\, \sec^2 x\, dx}$$

$$\boxed{\int \sec^4 x\, dx = tg\, x + \frac{tg^3 x}{3} + C}$$

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10. $\int \operatorname{cosec}^4 x\, dx =$

$= \int \underbrace{\operatorname{cosec}^2 x}\, \operatorname{cosec}^2 x\, dx$

$= \int (1+\operatorname{cotg}^2 x)\, \operatorname{cosec}^2 x\, dx$

$= \underbrace{\int \operatorname{cosec}^2 x\, dx} + \underbrace{\int \operatorname{cotg}^2 x\, \operatorname{cosec}^2 x\, dx}$

$= -\operatorname{cotg} x + (-)\dfrac{\operatorname{cotg}^3 x}{3} + C$

$\therefore$

$$\boxed{\int \operatorname{cosec}^4 x\, dx = -\operatorname{cotg} x - \frac{1}{3}\operatorname{cotg}^3 x + C}$$

(caso 3 : usar int. por partes)

11. $\int \operatorname{cosec}^3 x\, dx$

$u = \operatorname{cosec} x \rightarrow du = -\operatorname{cosec} x \operatorname{cotg} x\, dx$

$dv = \operatorname{cosec}^2 x\, dx \rightarrow v = -\operatorname{cotg} x$

$\therefore$

$\int \operatorname{cosec}^3 x\, dx = -\operatorname{cosec} x \operatorname{cotg} x - \int \operatorname{cotg} x \operatorname{cosec} x \operatorname{cotg} x\, dx$

$= -\operatorname{cosec} x \operatorname{cotg} x - \int \operatorname{cosec} x\, \underbrace{\operatorname{cotg}^2 x}\, dx$

$= -\operatorname{cosec} x \operatorname{cotg} x - \int \operatorname{cosec} x\, (\operatorname{cosec}^2 x - 1)\, dx$

$\underbrace{\int \operatorname{cosec}^3 x\, dx} = -\operatorname{cosec} x \operatorname{cotg} x - \underbrace{\int \operatorname{cosec}^3 x\, dx} + \int \operatorname{cosec} x\, dx$

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11. (continuação)

$$2\int \operatorname{cosec}^3 x\,dx = -\operatorname{cosec} x \operatorname{cotg} x + \int \operatorname{cosec} x\,dx$$

$$= -\operatorname{cosec} x \operatorname{cotg} x + \ln|\operatorname{cosec} x - \operatorname{cotg} x| + C$$

$\therefore$

$$\boxed{\int \operatorname{cosec}^3 x\,dx = -\frac{1}{2}\operatorname{cosec} x \operatorname{cotg} x + \frac{1}{2}\ln|\operatorname{cosec} x - \operatorname{cotg} x| + C}$$

12. $\int \sec^5 x\,dx$ caso 3: int. partes

$$\int \sec^5 x\,dx = \int \sec^3 x \sec^2 x\,dx$$

$$\begin{cases} u = \sec^3 x \rightarrow du = 3\sec^2 x \sec x \operatorname{tg} x\,dx \\ \qquad\qquad\qquad\quad = 3\sec^3 x \operatorname{tg} x\,dx \\ dv = \sec^2 x\,dx \rightarrow v = \operatorname{tg} x \end{cases}$$

$\therefore$

$$\int \sec^5 x\,dx = \sec^3 x \operatorname{tg} x - 3\int \sec^3 x \operatorname{tg}^2 x\,dx$$

$$= \sec^3 x \operatorname{tg} x - 3\int \sec^3 x(\sec^2 x - 1)\,dx$$

$\therefore$

$$\underbrace{\int \sec^5 x\,dx} = \sec^3 x \operatorname{tg} x \underbrace{- 3\int \sec^5 x\,dx} + 3\int \sec^3 x\,dx$$

$$\therefore\ 4\int \sec^5 x\,dx = \sec^3 x \operatorname{tg} x + 3\int \sec^3 x\,dx$$

$\longrightarrow$

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$$4\int \sec^5 x\,dx = \sec^3 x\,tg\,x + 3\underbrace{\int \sec^3 x\,dx}_{\text{já obtido}}$$

$$4\int \sec^5 x\,dx = \sec^3 x\,tg\,x + 3\left(\frac{1}{2}\sec x\,tg\,x + \frac{1}{2}\ln|\sec x + tg\,x|\right)$$

$\therefore$

$$4\int \sec^5 x\,dx = \sec^3 x\,tg\,x + \frac{3}{2}\sec x\,tg\,x + \frac{3}{2}\ln|\sec x + tg\,x|$$

$\therefore$

$$\boxed{\int \sec^5 x\,dx = \frac{1}{4}\sec^3 x\,tg\,x + \frac{3}{8}\sec x\,tg\,x + \frac{3}{8}\ln|\sec x + tg\,x| + C}$$

13) $\int e^x\,tg^4(e^x)\,dx$

$u = e^x \rightarrow du = e^x\,dx$

$$\int e^x\,tg^4(e^x)\,dx = \int tg^4(u)\,du$$

$$= \left(\frac{tg^3 u}{3} - tg\,u + u\right) + C$$

ex. 6

$\therefore$

$$\boxed{\int e^x\,tg^4(e^x)\,dx = \frac{tg^3 e^x}{3} - tg\,e^x + e^x + C}$$

14. $\displaystyle\int \frac{\sec^4(\ln x)}{x}\,dx$

$u = \ln x \longrightarrow du = \frac{1}{x}\,dx$

$\displaystyle\int \frac{\sec^4(\ln x)}{x}\,dx = \underbrace{\int \sec^4 u\,du}$ ex. 9

$= \left(\operatorname{tg} u + \frac{\operatorname{tg}^3 u}{3} + C\right)$

$\therefore$

$$\boxed{\int \frac{\sec^4(\ln x)}{x}\,dx = \operatorname{tg} \ln x + \frac{\operatorname{tg}^3 \ln x}{3} + C}$$

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15. $\displaystyle\int \operatorname{tg}^6 x \sec^4 x\,dx$ Caso 4 $\displaystyle\int \operatorname{tg}^m x \sec^{2n} x\,dx$

$= \displaystyle\int \operatorname{tg}^6 x \sec^2 x \underbrace{\sec^2 x}\,dx$

$= \displaystyle\int \operatorname{tg}^6 x \sec^2 x \,(1+\operatorname{tg}^2 x)\,dx$

$= \underbrace{\displaystyle\int \operatorname{tg}^6 x \sec^2 x\,dx} + \displaystyle\int \operatorname{tg}^8 x \sec^2 x\,dx$

$= \frac{\operatorname{tg}^7 x}{7} + \frac{\operatorname{tg}^9 x}{9} + C$

$\therefore$

$$\boxed{\int \operatorname{tg}^6 x \sec^4 x\,dx = \frac{\operatorname{tg}^7 x}{7} + \frac{\operatorname{tg}^9 x}{9} + C}$$

$\frac{1}{7}$

16. $\int tg^5 x \sec^3 x \, dx =$

$$= \int \underbrace{tg^4 x} \sec^2 x \; tg\, x \sec x \, dx \qquad \frac{d \sec x}{dx} = \sec x \, tg\, x$$

$$= \int (\sec^2 x - 1)^2 \sec^2 x \; tg\, x \sec x \, dx$$

$$= \int (\sec^4 x - 2\sec^2 x + 1) \sec^2 x \; tg\, x \sec x \, dx$$

$$= \underbrace{\int \sec^6 x \; tg\, x \sec x \, dx} - 2 \underbrace{\int \sec^4 x \; tg\, x \sec x \, dx} +$$

$$+ \int \sec^2 x \; tg\, x \sec x \, dx$$

$$= \underbrace{\frac{\sec^7 x}{7}} - 2 \left(\underbrace{\frac{\sec^5 x}{5}} \right) + \frac{\sec^3 x}{3} + C$$

$\therefore$

$$\boxed{\int tg^5 x \sec^3 x \, dx = \frac{1}{7} \sec^7 x - \frac{2}{5} \sec^5 x + \frac{1}{3} \sec^3 x + C}$$

17. $\int cotg^2 3x \, cosec^4 3x \, dx =$

$$= \int cotg^2 3x \, cosec^2 3x \; \underbrace{cosec^2 3x} \, dx$$

$$= \int cotg^2 3x \, cosec^2 3x \; (1 + cotg^2 3x) \, dx$$

$$= \underbrace{\int cotg^2 3x \, cosec^2 3x \, dx} + \int cotg^4 3x \, cosec^2 3x \, dx$$

$$= - \frac{cotg^3 3x}{9} - \frac{cotg^5 3x}{15} + C$$

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$\therefore$

$$\boxed{\int cotg^2 3x \, cosec^4 3x \, dx = - \frac{1}{15} cotg^5 3x - \frac{1}{9} cotg^3 3x + C}$$

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18. $\int (\sec 5x + \operatorname{cosec} 5x)^2\, dx$

$= \int (\sec^2 5x + 2\sec 5x \operatorname{cosec} 5x + \operatorname{cosec}^2 5x)\, dx$

$= \underbrace{\int \sec^2 5x\, dx}_{(*)} + 2\underbrace{\int \sec 5x \operatorname{cosec} 5x\, dx}_{(**)} + \underbrace{\int \operatorname{cosec}^2 5x\, dx}_{(***)}$

$(*) = \int \sec^2 5x\, dx = \frac{\operatorname{tg} 5x}{5}$

$(**) = 2\int \underbrace{\sec 5x \operatorname{cosec} 5x}\, dx$ $\quad (2\operatorname{cosec} 2x = \sec x \operatorname{cosec} x)$

$= 2\int 2\operatorname{cosec} 10x\, dx$ $\quad \sin 2x = 2\sin x \cos x$

$= 4\int \operatorname{cosec} 10x\, dx$ $\quad \frac{1}{\sin 2x} = \frac{1}{2\sin x \cos x}$

$= 4\,\frac{\ln|\operatorname{cosec} 10x - \operatorname{cotg} 10x|}{10}$ $\quad \operatorname{cosec} 2x = \frac{1}{2}\operatorname{cosec} x \sec x$

$2\operatorname{cosec} 2x = \operatorname{cosec} x \sec x$

$= \frac{2}{5}\ln|\operatorname{cosec} 10x - \operatorname{cotg} 10x|$

$(***) = \int \operatorname{cosec}^2 5x\, dx = -\frac{\operatorname{cotg} 5x}{5} + C$

$\therefore$ $\boxed{\int (\sec 5x + \operatorname{cosec} 5x)^2\, dx = \frac{\operatorname{tg} 5x}{5} - \frac{\operatorname{cotg} 5x}{5} + \frac{2}{5}\ln|\operatorname{cosec} 10x - \operatorname{cotg} 10x| + C}$

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19. $\int (tg\,2x + cotg\,2x)^2\,dx =$

$= \int (tg^2 2x + 2\,tg\,2x\,cotg\,2x + cotg^2 2x)\,dx$

$= \underbrace{\int tg^2 2x\,dx}_{(*)} + \underbrace{2\int tg\,2x\,cotg\,2x\,dx}_{(**)} + \underbrace{\int cotg^2 2x\,dx}_{(***)}$

$(*) = \int tg^2 2x\,dx = \int (sec^2 2x - 1)\,dx$

$= \int sec^2 2x\,dx - \int dx$

$= \frac{tg\,2x}{2} - x$ //

$(**) = 2\int tg\,2x\,cotg\,2x\,dx$

$= 2\int dx = 2x$ //

$(***) = \int cotg^2 2x\,dx = \int (cossec^2 2x - 1)\,dx$

$= -\frac{cotg\,2x}{2} - x$ //

$\int (tg\,2x + cotg\,2x)^2\,dx = \frac{tg\,2x}{2} \cancel{-x} + \cancel{2x} - \frac{cotg\,2x}{2} \cancel{-x}$

$$\boxed{\int (tg\,2x + cotg\,2x)^2\,dx = \frac{tg\,2x}{2} - \frac{cotg\,2x}{2} + C}$$

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20. $\int \frac{dx}{1+\cos x}$

Vemos que $\frac{1+\cos 2x}{2} = \cos^2 x$

Daí, $// 1+\cos x = 2\cos^2 \frac{x}{2} //$

$\therefore \int \frac{dx}{1+\cos x} = \int \frac{dx}{2\cos^2 \frac{x}{2}} = \frac{1}{2} \int \underbrace{\sec^2 \frac{x}{2}\, dx}$

$= \frac{1}{2} \left(\operatorname{tg} \frac{x}{2}\right) 2$

$\therefore$ $\boxed{\int \frac{dx}{1+\cos x} = \operatorname{tg} \frac{x}{2} + C}$

21. $\int \frac{2 \sin w - 1}{\cos^2 w}\, dw$

$= 2 \int \frac{\sin w}{\cos^2 w}\, dw - \int \frac{1}{\cos^2 w}\, dw$

$= 2 \int \underbrace{\operatorname{tg} w \sec w \, dw} - \int \underbrace{\sec^2 w \, dw}$

$= 2 \sec w - \operatorname{tg} w$

$\therefore$ $\boxed{\int \frac{2 \sin w - 1}{\cos^2 w}\, dw = 2 \sec w - \operatorname{tg} w + C}$

22. $\int \frac{tg^3\sqrt{x}}{\sqrt{x}}\,dx$

Seja $u = \sqrt{x} \rightarrow du = \frac{1}{2\sqrt{x}}\,dx$

$\therefore$

$$\int \frac{tg^3\sqrt{x}}{\sqrt{x}}\,dx = \int tg^3 u \, 2du$$

$$= 2\int tg^3 u\, du$$

(*)

$$(*) = \int tg^3 u\, du = \int \underbrace{tg^2 u}\, tg\,u\, du$$

$$= \int (sec^2 u - 1)\, tg\,u\, du$$

$$= \int sec^2 u\, tg\,u\, du - \int tg\,u\, du$$

$$= \frac{tg^2 u}{2} - \ln|sec\,u| + C$$

$$= \frac{tg^2\sqrt{x}}{2} - \ln|sec\sqrt{x}| + C$$

$\therefore$

$$\int \frac{tg^3\sqrt{x}}{\sqrt{x}}\,dx = 2\left[\frac{tg^2\sqrt{x}}{2} - \ln|sec\sqrt{x}|\right] + C$$

$\therefore$

$$\boxed{\int \frac{tg^3\sqrt{x}}{\sqrt{x}}\,dx = tg^2\sqrt{x} - 2\ln|sec\sqrt{x}| + C}$$

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23. $\int tg^5 3x \, dx$

$$= \int tg^2 3x \; tg^3 3x \, dx$$

$$= \int (sec^2 3x - 1) \, tg^3 3x \, dx$$

$$= \int sec^2 3x \; tg^3 3x - \int tg^3 3x \, dx$$

$$= \frac{tg^4 3x}{12} - \int tg^3 3x \, dx$$

$$= \frac{tg^4 3x}{12} - \int tg^2 3x \; tg 3x \, dx$$

$$= \frac{tg^4 3x}{12} - \int (sec^2 3x - 1) \, tg 3x \, dx$$

$$= \frac{tg^4 3x}{12} - \int sec^2 3x \; tg 3x \, dx + \int tg 3x \, dx$$

$$\boxed{\int tg^5 3x \, dx = \frac{tg^4 3x}{12} - \frac{tg^2 3x}{6} + \frac{\ln |sec 3x|}{3} + C}$$

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24. $\int \frac{tg^4 y}{sec^5 y} \, dy$

$$= \int tg^4 y \cdot \frac{1}{sec^5 y} \, dy$$

$$= \int \frac{sin^4 y}{cos^4 y} \; \frac{1}{\frac{1}{cos^5 y}} \, dy$$

$$= \int \frac{sin^4 y}{\cancel{cos^4 y}} \; cos^{\cancel{5}} y \, dy = \int sin^4 y \; cos \, y \, dy$$

$$= \frac{sin^5 y}{5} + C$$

25. $\displaystyle\int \frac{du}{1+\sec\frac{u}{2}}$

$$= \int \frac{\left(1-\sec\frac{u}{2}\right)}{\left(1+\sec\frac{u}{2}\right)\left(1-\sec\frac{u}{2}\right)}\,du$$

$$= \int \frac{\left(1-\sec\frac{u}{2}\right)du}{1-\sec^2\frac{u}{2}} \qquad \left(1+tg^2\frac{u}{2} = \sec^2\frac{u}{2}\right)$$

$$= \int \frac{\left(1-\sec\frac{u}{2}\right)}{-tg^2\frac{u}{2}}\,du$$

$$= -\int \frac{1-\sec\frac{u}{2}}{tg^2\frac{u}{2}}\,du$$

$$= -\int \frac{1}{tg^2\frac{u}{2}}\,du + \int \frac{\sec\frac{u}{2}}{tg^2\frac{u}{2}}\,du$$

$$= -\int cotg^2\frac{u}{2}\,du + \int \underbrace{\sec\frac{u}{2}\; cotg^2\frac{u}{2}}\,du$$

$$= -\int cotg^2\frac{u}{2}\,du + \int \frac{1}{\cancel{\cos\frac{u}{2}}}\,\frac{\cos^2\frac{u}{2}}{\sin^2\frac{u}{2}}\,du$$

$$= -\int cotg^2\frac{u}{2}\,du + \int cotg\frac{u}{2}\,cosec\frac{u}{2}\,du$$

$$\qquad (*) \qquad\qquad\qquad (**)$$

Mas

$$// (*) = -\int cotg^2\frac{u}{2}\,du = -\int\left(cosec^2\frac{u}{2} - 1\right)du$$

$$= -\int cosec^2\frac{u}{2}\,du + \int du$$

$$= -\left(-cotg\frac{u}{2}\right)2 + u$$

$$= 2cotg\frac{u}{2} + u \;//$$

$(**) = \int \operatorname{cotg}\frac{u}{2} \operatorname{cosec}\frac{u}{2}\, du =$

$= -2\operatorname{cosec}\frac{u}{2}$

Daí

$$\int \frac{du}{1+\sec\frac{1}{2}u} = 2\operatorname{cotg}\frac{u}{2} + u - 2\operatorname{cosec}\frac{u}{2} + C$$

$$\boxed{\int \frac{du}{1+\sec\frac{u}{2}} = 2\left(\operatorname{cotg}\frac{u}{2} - \operatorname{cosec}\frac{u}{2}\right) + u + C}$$

obs.:

$$\operatorname{cotg}\frac{u}{2} - \operatorname{cosec}\frac{u}{2} = \frac{\cos\frac{u}{2}}{\sin\frac{u}{2}} - \frac{1}{\sin\frac{u}{2}} = \frac{\cos\frac{u}{2} - 1}{\sin\frac{u}{2}}$$

$\frac{1-\cos 2x}{2} = \sin^2 x$

$$= -\frac{1-\cos\frac{u}{2}}{\sin\frac{u}{2}}$$

$$= -\frac{2\sin^2\frac{u}{4}}{\sin\frac{u}{2}}$$

$\sin\frac{u}{2} = 2\sin\frac{u}{4}\cos\frac{u}{4}$

$$= -\frac{\cancel{2}\sin^{\cancel{2}}\frac{u}{4}}{\cancel{2}\cancel{\sin\frac{u}{4}}\cos\frac{u}{4}} = -\operatorname{tg}\frac{u}{4}$$

$\sin^2 2x$

$\therefore$

$$\boxed{\int \frac{du}{1+\sec\frac{u}{2}} = -2\operatorname{tg}\frac{u}{4} + u + C}$$

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26. $\int \frac{\operatorname{cosec}^4 x}{\operatorname{cotg}^2 x}\, dx$

$$= \int \operatorname{cosec}^4 x \; tg^2 x \, dx$$

$$= \int \frac{1}{\sin^{\cancel{4}\,2} x} \, \frac{\cancel{\sin^2 x}}{\cos^2 x} \, dx$$

$$= \int \frac{1}{\sin^2 x \cos^2 x} \, dx =$$

$\sin 2x = 2 \sin x \cos x$

$$= \int \frac{1}{(\sin x \cos x)^2} \, dx$$

$$= \int \frac{1}{\left(\frac{\sin 2x}{2}\right)^2} \, dx$$

$$= 4 \int \frac{1}{\sin^2 2x} \, dx$$

$$= 4 \int \operatorname{cosec}^2 2x \, dx$$

$$= 4 \, (-) \, \frac{\operatorname{cotg} 2x}{2}$$

$$\boxed{\int \frac{\operatorname{cosec}^4 x}{\operatorname{cotg}^2 x}\, dx = -2 \operatorname{cotg} 2x + C}$$

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27. $\int \frac{\sec^3 x}{tg^4 x}\, dx =$

$= \int \sec^3 x \ cotg^4 x \ dx$

$= \int \frac{1}{\cancel{\cos^3 x}} \frac{\cos^4 x}{\sin^4 x}\, dx$

$= \int \frac{\cos x}{\sin x} \frac{1}{\sin^3 x}\, dx$

$= \int cotg x \ cosec^3 x \ dx$

$= \int cotg x \ cosec x \ cosec^2 x \ dx$

$$\boxed{\int \frac{\sec^3 x}{tg^4 x}\, dx = -\frac{cosec^3 x}{3} + C}$$

28. $\int \frac{sen^2 \pi x}{\cos^6 \pi x}\, dx$

$= \int \frac{\sin^2 \pi x}{\cos^2 \pi x} \frac{1}{\cos^4 \pi x}\, dx$

$= \int tg^2 \pi x \ \sec^4 \pi x \ dx$

$= \int tg^2 \pi x \ \sec^2 \pi x \ \underline{\sec^2 \pi x} \ dx$

$= \int tg^2 \pi x \ \sec^2 \pi x \ (1 + tg^2 \pi x)\, dx$

$= \int tg^2 \pi x \ \sec^2 \pi x \ dx + \int tg^4 \pi x \ \sec^2 \pi x \ dx$

$= \frac{tg^3 \pi x}{3\pi} + \frac{tg^5 \pi x}{5\pi} + C \ //$

16.

7. 29. $\int \frac{tg^3(\ln x)\, sec^6(\ln x)}{x}\, dx$

8. Seja $u = \ln x \rightarrow du = \frac{1}{x}dx$

19

$\int \frac{tg^3(\ln x)\, sec^6(\ln x)}{x}\, dx =$

20

$= \int tg^3 u \, sec^6 u \, du$

$= \int tg^3 u \, \underbrace{sec^4 u}_{} \, sec^2 u \, du$

$= \int tg^3 u \, (1+tg^2 u)^2 \, sec^2 u \, du$

$= \int tg^3 u \, (1+2tg^2 u + tg^4 u) \, sec^2 u \, du$

$= \int (tg^3 u \, sec^2 u + 2\, tg^5 u \, sec^2 u + tg^7 u \, sec^2 u)\, du$

$= \frac{tg^4 u}{4} + 2\,\frac{tg^6 u}{6} + \frac{tg^8 u}{8} + C$

$= \frac{tg^4 u}{4} + \frac{tg^6 u}{3} + \frac{tg^8 u}{8} + C$

$= \frac{tg^4 \ln x}{4} + \frac{tg^6 \ln x}{3} + \frac{tg^8 \ln x}{8} + C$ //

tilibra

30. $\displaystyle\int \frac{\sec^4 \omega}{\sqrt{tg\,\omega}}\, d\omega$

$$= \int \sec^4 \omega \; tg\,\omega^{-1/2}\, d\omega$$

$$= \int \sec^2 \omega \; \underbrace{\sec^2 \omega}\; tg\,\omega^{-1/2}\, d\omega$$

$$= \int \sec^2 \omega \,(1 + tg^2 \omega)\; tg\,\omega^{-1/2}\, d\omega$$

$$= \int \sec^2 \omega \; tg\,\omega^{-1/2}\, d\omega + \int \sec^2 \omega \; tg^{3/2} \omega \, d\omega$$

$$\boxed{\int \frac{\sec^4 \omega}{\sqrt{tg\,\omega}}\, d\omega = 2\, tg^{1/2} \omega + \frac{2}{5}\, tg^{5/2} \omega + c}$$

40. $\displaystyle\int cotg\, x \; cosec^n x \, dx = -\frac{cosec^n x}{n} + C \;,\; n \neq 0$

De fato:

$$\int cotg\, x \; cosec^n x \, dx = \int cotg\, x \; cosec\, x \; \underbrace{cosec^{n-1} x}\, dx$$

$$= -\frac{cosec^n x}{n} + C .$$

41. $\int tg^n x \, dx = \frac{tg^{n-1} x}{n-1} - \int tg^{n-2} x \, dx \quad , \; n > 1$

De fato

$n > 1 \rightarrow n \geq 2.$

Daí,

$$\int tg^n x \, dx = \int tg^{n-2} x \; tg^2 x \, dx$$

$$= \int tg^{n-2} x \, (sec^2 x - 1) \, dx$$

$$= \int tg^{n-2} x \, sec^2 x \, dx - \int tg^{n-2} x \, dx$$

$$\boxed{\int tg^n x \, dx = \frac{tg^{n-1} x}{n-1} - \int tg^{n-2} x \, dx}$$

42. $\int tg x \, sec^n x \, dx \quad , \; n \neq 0$

Temos:

$$\int tg x \, sec^n x \, dx = \int tg x \, sec x \; sec^{n-1} x \, dx$$

$$= \frac{sec^n x}{n} + C$$

43. $\int cotg^n x \, dx \overset{n \geq 2}{=} \int cotg^{n-2} x \; cotg^2 x \, dx = \int cotg^{n-2} x \, (cosec^2 x - 1) \, dx$

$$= \int cotg^{n-2} x \, cosec^2 x \, dx - \int cotg^{n-2} x \, dx$$

$$= -\frac{cotg^{n-1} x}{n-1} - \int cotg^{n-2} x \, dx$$

$\therefore$

$$\boxed{\int cotg^n x \, dx = -\frac{cotg^{n-1} x}{n-1} - \int cotg^{n-2} x \, dx}$$