Lista 4 - Cálculo B

Integração das funções racionais por frações parciais : Denominador tem somente fatores lineares

Leithold pg. 560

1. $\int \frac{dx}{x^2-4}$

$$\frac{P(x)}{Q(x)} = \frac{1}{x^2-4} = \frac{1}{(x-2)(x+2)} = \frac{A}{(x-2)} + \frac{B}{x+2}$$

$$\frac{1}{x^2-4} = \frac{A}{x-2} + \frac{B}{x+2}$$

$$\frac{1}{(x^2-4)} = \frac{A(x+2) + B(x-2)}{(x-2)(x+2)}$$

$$\Rightarrow \left\{ \begin{array}{l} 1 = A(x+2) + B(x-2) \\ x = 2 : \quad 1 = 4A \Rightarrow A = 1/4 \\ x = -2 : \quad 1 = -4B \Rightarrow B = -1/4 \end{array} \right.$$

$\therefore$

$$\frac{1}{x^2-4} = \frac{1}{4(x-2)} - \frac{1}{4(x+2)}$$

$\therefore$

$$\int \frac{dx}{x^2-4} = \int \frac{dx}{4(x-2)} - \int \frac{dx}{4(x+2)}$$

$$= \frac{1}{4} \ln|x-2| - \frac{1}{4} \ln|x+2| + C$$

$$\therefore \boxed{\int \frac{dx}{x^2-4} = \frac{1}{4} \ln \left|\frac{x-2}{x+2}\right| + C}$$

2. $$\int \frac{x^2}{x^2+x-6}\,dx =$$

$$\frac{x^2}{x^2+x-6} :$$

$$\begin{array}{r|l} x^2 & x^2+x-6 \\ \cline{2-2} -x^2-x+6 & 1 \\ \hline -x+6 & \end{array}$$

$\frac{p}{q} = M + \frac{r}{q}$

$$\therefore \frac{x^2}{x^2+x-6} = 1 + \frac{-x+6}{x^2+x-6} \quad (*)$$

Mas

$x^2+x-6=0$

$x = \frac{-1 \pm \sqrt{1+24}}{2}$

$= \frac{-1 \pm 5}{2}$

$= -3$ ou 2

$$\frac{-x+6}{x^2+x-6} = \frac{-x+6}{(x+3)(x-2)} = \frac{A}{(x+3)} + \frac{B}{(x-2)}$$

$\therefore$

$$\begin{cases} -x+6 = A(x-2) + B(x+3) \\ x=2: \; -2+6 = A\cdot 0 + B.5 \\ \quad +4 = 5B \implies B = \frac{4}{5} \\ x=-3: \; -(-3)+6 = -5A \\ \quad 9 = -5A \implies A = -\frac{9}{5} \end{cases}$$

$\therefore$

$$\frac{-x+6}{(x+3)(x-2)} = -\frac{9}{5(x+3)} + \frac{4}{5(x-2)} \quad (**)$$

(**) $\to$ (*):

$$\frac{x^2}{x^2+x-6} = 1 - \frac{9}{5(x+3)} + \frac{4}{5(x-2)}$$

$\therefore$

$$\int \frac{x^2}{x^2+x-6}\,dx = \int dx - \frac{9}{5}\int \frac{dx}{x+3} + \frac{4}{5}\int \frac{dx}{x-2}$$

$$= x - \frac{9}{5}\ln|x+3| + \frac{4}{5}\ln|x-2| + C$$

$\therefore$

$$\boxed{\int \frac{x^2}{x^2+x-6}\,dx = x - \frac{9}{5}\ln|x+3| + \frac{4}{5}\ln|x-2| + C}$$

3. $\displaystyle\int \frac{5x-2}{x^2-4}\,dx$

$$\frac{5x-2}{x^2-4} = \frac{5x-2}{(x-2)(x+2)} = \frac{A}{x-2} + \frac{B}{x+2}$$

$\Leftrightarrow$

$$5x-2 = A(x+2) + B(x-2)$$

$$\begin{cases} x=-2: & -12 = -4B \implies B = 3 \\ x=2: & 8 = 4A \implies A = 2 \end{cases}$$

$\therefore$ $$\frac{5x-2}{(x-2)(x+2)} = \frac{2}{x-2} + \frac{3}{x+2} \implies$$

$$\int \frac{5x-2}{x^2-4}\,dx = \int \frac{2}{x-2}\,dx + \int \frac{3}{x+2}\,dx$$

$$= 2\ln|x-2| + 3\ln|x+2| + C \quad //$$

ou

$$= \ln|x-2|^2 + \ln|x+2|^3 + \ln C'$$

$$= \ln\left(|x-2|^2 |x+2|^3 C'\right)$$

$$= \ln|(x-2)^2(x+2)^3 C'| \quad //$$

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4. $\displaystyle\int \frac{(4x-2)}{x^3-x^2-2x}\,dx$

$x^2-x-2=0$

$x = \frac{1 \pm \sqrt{1+8}}{2}$

$= \frac{1 \pm 3}{2}$

$\therefore$

$x=2,\ x=-1$

$$\frac{4x-2}{x^3-x^2-2x} = \frac{4x-2}{x(x^2-x-2)}$$

$$= \frac{4x-2}{x(x-2)(x+1)} = \frac{A}{x} + \frac{B}{x-2} + \frac{C}{x+1}$$

$$4x-2 = A(x-2)(x+1) + Bx(x+1) + Cx(x-2)$$

$$\begin{cases} x=2: & 6 = B\,2\cdot 3 \implies B=1 \\ x=-1: & -6 = C(-1)(-3) \implies C=-2 \\ x=0: & -2 = A(-2)\cdot 1 \implies A=1 \end{cases}$$

$\therefore$

$$\frac{4x-2}{x(x-2)(x+1)} = \frac{1}{x} + \frac{1}{x-2} - \frac{2}{x+1}$$

$\therefore$

$$\int \frac{4x-2}{x^3-x^2-2x}\,dx = \int \frac{4x-2}{x(x-2)(x+1)}\,dx$$

$$= \int \frac{1}{x}\,dx + \int \frac{1}{x-2}\,dx - \int \frac{2}{x+1}\,dx$$

$\therefore$

$$\boxed{\int \frac{4x-2}{x^3-x^2-2x}\,dx = \ln|x| + \ln|x-2| - 2\ln|x+1| + C}$$

5. $\displaystyle\int \frac{4w-11}{2w^2+7w-4}\,dw$

$$\frac{4w-11}{2w^2+7w-4} = \frac{4w-11}{2(w+4)(w-\frac{1}{2})} = \frac{A}{w+4} + \frac{B}{w-\frac{1}{2}}$$

$$\Leftrightarrow$$

$$4w-11 = 2A(w-\tfrac{1}{2}) + 2B(w+4)$$

$$\begin{cases} w=\frac{1}{2}: & -9 = \cancel{2}B\,\frac{9}{\cancel{2}} \Rightarrow B=-1 \\ w=-4: & -27 = 2A(-\frac{9}{2}) \Rightarrow A=3 \end{cases}$$

$2w^2+7w-4=0$

$w = \frac{-7 \pm \sqrt{49+32}}{4}$

$= \frac{-7 \pm \sqrt{81}}{4}$

$= \frac{-7 \pm 9}{4}$

$= -4, \frac{1}{2}$

$\therefore$

$$\frac{4w-11}{2(w+4)(w-\frac{1}{2})} = \frac{3}{w+4} - \frac{1}{w-\frac{1}{2}}$$

$\therefore$

$$\int \frac{4w-11}{2w^2+7w-4}\,dw = \int \frac{3}{w+4}\,dw - \int \frac{1}{w-\frac{1}{2}}\,dw$$

$$= 3\ln|w+4| - 1\ln|w-\tfrac{1}{2}| + C$$

$|x||y| =$
$= |xy|$

$$= \ln\left|\frac{(w+4)^3}{w-\frac{1}{2}}C\right|$$

$$= \ln\left|\frac{(w+4)^3}{2w-1}\,2C\right|$$

$$= \ln\left|\frac{C(w+4)^3}{2w-1}\right|$$

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6. $$\int \frac{9t^2-26t-5}{3t^2-5t-2}\,dt$$

$$\begin{array}{r|l} 9t^2-26t-5 & 3t^2-5t-2 \\ \hline -9t^2+15t+6 & 3 \\ \hline -11t+1 & \end{array}$$

$\frac{f}{q} = m + \frac{r}{q}$

$\therefore$ $$\frac{9t^2-26t-5}{3t^2-5t-2} = 3 + \frac{-11t+1}{3t^2-5t-2}$$

Mas

$3t^2-5t-2=0$

$t = 5 \pm \sqrt{25+24}/6$

$= 5 \pm \sqrt{49}/6$

$= \frac{5\pm 7}{6}$

$= 2, -\frac{1}{3}$

$$3t^2-5t-2 = 3(t-2)(t+\tfrac{1}{3})$$

$\therefore$

$$\frac{9t^2-26t-5}{3t^2-5t-2} = 3 + \frac{-11t+1}{3(t-2)(t+\frac{1}{3})} \quad (*)$$

Mo)

$$\frac{-11t+1}{3(t-2)(t+\frac{1}{3})} = \frac{A}{(t-2)} + \frac{B}{(t+\frac{1}{3})}$$

$$\Leftrightarrow$$

$$-11t+1 = 3(t+\tfrac{1}{3})A + 3(t-2)B$$

$$\begin{cases} t=-\frac{1}{3}: & \frac{11}{3}+1 = 3(-\frac{1}{3}-2)B \\ & \frac{14}{3} = \cancel{3}(-\frac{7}{\cancel{3}})B \implies \| B = -\frac{2}{3} \| \\ t=2: & -22+1 = 3(2+\frac{1}{3})A \\ & -21 = \cancel{3}\,\frac{7}{\cancel{3}}A \implies \| A = -3 \| \end{cases}$$

$\therefore$

$$\frac{-11t+1}{3(t-2)(t+\frac{1}{3})} = \frac{-3}{t-2} - \frac{2}{3(t+\frac{1}{3})} \qquad (**)$$

$(**) \to (*)$:

$$\left\{ \frac{9t^2-26t-5}{3t^2-5t-2} = 3 - \frac{3}{t-2} - \frac{2}{3(t+\frac{1}{3})} \right.$$

$$\therefore \int \frac{9t^2-26t-5}{3t^2-5t-2}\,dt = \int 3\,dt - 3\int \frac{dt}{t-2} - \frac{2}{3}\int \frac{dt}{t+\frac{1}{3}}$$

$$\boxed{\int \frac{9t^2-26t-5}{3t^2-5t-2}\,dt = 3t - 3\ln|t-2| - \frac{2}{3}\ln|t+\tfrac{1}{3}| + C}$$

7. $\displaystyle\int \frac{6x^2-2x-1}{4x^3-x}\,dx$

$4x^2-1=0$

$4x^2=1$

$x=\pm\frac{1}{2}$

$$\frac{6x^2-2x-1}{4x^3-x} = \frac{6x^2-2x-1}{x(4x^2-1)}$$

$$= \frac{6x^2-2x-1}{x4(x+\frac{1}{2})(x-\frac{1}{2})} = \frac{A}{x}+\frac{B}{x+\frac{1}{2}}+\frac{C}{x-\frac{1}{2}}$$

$\therefore$

$$6x^2-2x-1 = 4A(x+\tfrac{1}{2})(x-\tfrac{1}{2}) + 4Bx(x-\tfrac{1}{2}) + 4Cx(x+\tfrac{1}{2})$$

$x=-\frac{1}{2}$: $6\frac{1}{4}-2(-\frac{1}{2})-1 = 4B(-\frac{1}{2})(-1)$

$\frac{3}{2}+\cancel{1}-\cancel{1} = 4\frac{B}{2} \Rightarrow B=\frac{3}{4}$

$x=0$: $-1 = 4A\frac{1}{2}(-\frac{1}{2}) \Rightarrow A=1$

$x=\frac{1}{2}$: $6\frac{1}{4}-2\frac{1}{2}-1 = 4C\frac{1}{2}\times 1$

$\frac{3}{2}-2 = 4\frac{C}{2}$

$-\frac{1}{2} = 4\frac{C}{2} \Rightarrow C=-\frac{1}{4}$

$\therefore$

$$\int\frac{6x^2-2x-1}{x(x+\frac{1}{2})(x-\frac{1}{2})} = \frac{1}{x}+\frac{3}{4(x+\frac{1}{2})}-\frac{1}{4(x-\frac{1}{2})}$$

$\therefore$

$$\int \frac{6x^2-2x-1}{x(x+\frac{1}{2})(x-\frac{1}{2})}\,dx = \int \frac{dx}{x} + \frac{3}{4}\int \frac{dx}{x+\frac{1}{2}} - \frac{1}{4}\int \frac{dx}{x-\frac{1}{2}}$$

$$= \ln|x| + \frac{3}{4}\ln|x+\tfrac{1}{2}| - \frac{1}{4}\ln|x-\tfrac{1}{2}| + C.$$

ou

$$= \frac{1}{4}\left(4\ln|x| + 3\ln|x+\tfrac{1}{2}| - \ln|x-\tfrac{1}{2}| + C\right)$$

$$= \frac{1}{4}\ln\left|\frac{x^4(x+\frac{1}{2})^3}{(x-\frac{1}{2})}C\right|$$

$$= \frac{1}{4}\ln\left|\frac{x^4(2x+1)^3}{(2x-1)}C'\right|$$

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8. $$\int \frac{x^2+x+2}{x^2-1}\,dx$$

$$\begin{array}{r|l} \cancel{x^2}+x+2 & x^2-1 \\ -\cancel{x^2}\quad +1 & 1 \\ \hline x+3 & \end{array}$$

$$\therefore \frac{x^2+x+2}{x^2-1} = 1 + \frac{x+3}{x^2-1}$$ (*)

Mas

$$\frac{x+3}{x^2-1} = \frac{x+3}{(x+1)(x-1)} = \frac{A}{x+1} + \frac{B}{x-1}$$

$$\therefore \quad x+3 = A(x-1) + B(x+1)$$

$$x+3 = A(x-1) + B(x+1)$$

$x=1 \ : \ 4 = 2B \implies B=2$

$x=-1 \ : \ 2 = A(-2) \implies A=-1$

$\therefore$

$$\frac{x+3}{x^2-1} = \frac{-1}{x+1} + \frac{2}{x-1} \quad (**)$$

$(**) \to (*)$:

$$\frac{x^2+x+2}{x^2-1} = 1 - \frac{1}{x+1} + \frac{2}{x-1}$$

$\therefore$

$$\int \frac{x^2+x+2}{x^2-1}\,dx = \int dx - \int \frac{dx}{x+1} + 2\int \frac{dx}{x-1}$$

$$\boxed{\int \frac{x^2+x+2}{x^2-1}\,dx = x - \ln|x+1| + 2\ln|x-1| + C}$$

$$= x + \ln\left|\frac{(x-1)^2}{x+1}C\right|$$

9. $\displaystyle\int \frac{dx}{x^3+3x^2}$

$$\frac{1}{x^3+3x^2} = \frac{1}{x^2(x+3)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+3}$$

$$\Leftrightarrow$$

$$1 = Ax(x+3) + B(x+3) + Cx^2$$

$x=0$: B $\quad 1 = B\,3 \implies // B = 1/3 //$

$x=-3$: C $\quad 1 = C\,9 \implies // C = 1/9 //$

$x=1$: $\quad 1 = A\,4 + B\,4 + C$

$1 = 4A + \frac{4}{3} + \frac{1}{9}$

$-\frac{4}{9} = 4A \implies // A = -\frac{1}{9} //$

$1 - \frac{4}{3} - \frac{1}{9} =$

$= \frac{9-12-1}{9}$

$= \frac{-4}{9}$

$\therefore$

$$\frac{1}{x^3+3x^2} = -\frac{1}{9x} + \frac{1}{3x^2} + \frac{1}{9(x+3)}$$

$\therefore$

$$\int \frac{dx}{x^3+3x^2} = -\frac{1}{9}\int\frac{dx}{x} + \frac{1}{3}\int\frac{dx}{x^2} + \frac{1}{9}\int\frac{dx}{x+3}$$

$$\therefore \quad = -\frac{1}{9}\ln|x| + \frac{1}{3}\frac{x^{-1}}{-1} + \frac{1}{9}\ln|x+3| + C$$

$$\boxed{\int \frac{dx}{x^3+3x^2} = -\frac{1}{9}\ln|x| - \frac{1}{3x} + \frac{1}{9}\ln|x+3| + C} \; //$$

or

$$= -\frac{1}{3x} + \frac{1}{9}\ln\left|\frac{(x+3)}{x}\right| + C \; //$$

10. $\int \frac{x^2+4x-1}{x^3-x}\,dx$

$$\frac{x^2+4x-1}{x^3-x} = \frac{x^2+4x-1}{x(x^2-1)}$$

$$= \frac{x^2+4x-1}{x(x-1)(x+1)} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x+1}$$

$$\Leftrightarrow$$

$$x^2+4x-1 = A(x-1)(x+1) + Bx(x+1) + Cx(x-1)$$

$$\begin{cases} x=1: & 4 = B\cdot 2 \Rightarrow B=2 \\ x=-1: & -4 = C(-1)(-2) \Rightarrow C=-2 \\ x=0: & -1 = A(-1)\,1 \Rightarrow A=1 \end{cases}$$

$\therefore$ $$\frac{x^2+4x-1}{x(x-1)(x+1)} = \frac{1}{x} + \frac{2}{x-1} - \frac{2}{x+1}$$

$\therefore$ $$\int \frac{x^2+4x-1}{x(x-1)(x+1)}\,dx = \int \frac{dx}{x} + 2\int \frac{dx}{x-1} - 2\int \frac{dx}{x+1}$$

$$\int \frac{x^2+4x-1}{x^3-x}\,dx = \ln|x| + 2\ln|x-1| - 2\ln|x+1| + C$$ //

ou

$$= \ln\left|\frac{x(x-1)^2}{(x+1)^2}C\right|$$

11. $\int \frac{dx}{x^2(x+1)^2}$

$$\frac{1}{x^2(x+1)^2} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+1} + \frac{D}{(x+1)^2}$$

$\Leftrightarrow$

mmc: $x^2(x+1)^2$

$$1 = Ax(x+1)^2 + B(x+1)^2 + Cx^2(x+1) + Dx^2$$

$x=0$: $1 = B$

$x=-1$: $1 = D$

$x=1$: $1 = 4A + 4B + 2C + D$

$\Leftrightarrow$ $B=1$

$$\cancel{1} = 4A + 4 + 2C + \cancel{1}$$

$$-4 = 4A + 2C$$

$$\| -2 = 2A + C \|$$

$x=-2$: $1 = A(-2) + B - 4C + 4D$

$$1 = -2A + 1 - 4C + 4$$

$$-4 = -2A - 4C$$

$$\| -2 = -A - 2C \|$$

i.e.,

$-2 = 2A + C \Rightarrow C = -2 - 2A$

$-2 = -A - 2C \Rightarrow \left\{ \begin{array}{l} -2 = -A - 2(-2-2A) \\ -2 = -A + 4 + 4A \\ -6 = 3A \Rightarrow A = -2 \end{array} \right.$

$C = -2 - 2(-2)$

$= -2 + 4$

$C = +2$

$\therefore$

$$\frac{1}{x^2(x+1)^2} = \frac{-2}{x} + \frac{1}{x^2} + \frac{2}{x+1} + \frac{1}{(x+1)^2}$$

$\therefore$

$$\int \frac{dx}{x^2(x+1)^2} = -2\int \frac{dx}{x} + \int \frac{dx}{x^2} + 2\int \frac{dx}{x+1} + \int \frac{dx}{(x+1)^2}$$

$$= -2\ln|x| + \frac{x^{-1}}{-1} + 2\ln|x+1| + \frac{(x+1)^{-1}}{-1} + C$$

$$\boxed{\int \frac{dx}{x^2(x+1)^2} = -\frac{1}{x} - \frac{1}{(x+1)} + 2\ln\left|\frac{x+1}{x}\right| + C}$$

12. $\displaystyle\int \frac{3x^2-x+1}{x^3-x^2}\,dx$

$$\frac{3x^2-x+1}{x^3-x^2} = \frac{3x^2-x+1}{x^2(x-1)} = \frac{A}{x}+\frac{B}{x^2}+\frac{C}{x-1}$$

same = $x^2(x-1)$

$$\Leftrightarrow$$

$$3x^2-x+1 = Ax(x-1)+B(x-1)+Cx^2$$

$$\begin{cases} x=1: & 3 = C \\ x=0: & 1 = -B \\ x=2: & 11 = 2A+B+4C \\ & 11 = 2A-1+12 \Rightarrow A=0 \end{cases}$$

$\therefore$

$$\frac{3x^2-x+1}{x^3-x^2} = -\frac{1}{x^2}+\frac{3}{x-1}$$

$\therefore$

$$\int \frac{3x^2-x+1}{x^3-x^2}\,dx = \int -\frac{1}{x^2}\,dx + 3\int\frac{dx}{x-1}$$

$$= -\frac{x^{-1}}{-1} + 3\ln|x-1| + C$$

$\therefore$

$$\boxed{\int \frac{3x^2-x+1}{x^3-x^2} = \frac{1}{x}+3\ln|x-1|+C}$$

13. $\int \frac{x^2-3x-7}{(2x+3)(x+1)^2}\,dx$

$$\frac{x^2-3x-7}{(2x+3)(x+1)^2} = \frac{A}{(2x+3)} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$$

mmc $(2x+3)(x+1)^2$

$$\Leftrightarrow$$

$$x^2-3x-7 = A(x+1)^2 + B(2x+3)(x+1) + C(2x+3)$$

$x=-1$: $-3 = C$

$x=-\frac{3}{2}$: $\frac{9}{4}+\frac{9}{2}-7 = A\left(-\frac{1}{2}\right)^2$

$$\frac{9+18-28}{4} = A\frac{1}{4}$$

$$-\frac{1}{4} = A\frac{1}{4} \Rightarrow \boxed{A=-1}$$

$x=0$: $-7 = A + 3B + 3C$

$$-7 = -1 + 3B - 9$$

$$3 = 3B \Rightarrow \boxed{B=1}$$

$\therefore$

$$\frac{x^2-3x-7}{(2x+3)(x+1)^2} = \frac{-1}{2x+3} + \frac{1}{x+1} - \frac{3}{(x+1)^2}$$

$\therefore$

$$\int \frac{x^2-3x-7}{(2x+3)(x+1)^2}\,dx = \int \frac{-dx}{2x+3} + \int \frac{dx}{x+1} - 3\int \frac{dx}{(x+1)^2}$$

$$= -\frac{1}{2}\ln|2x+3| + \ln|x+1| - 3\frac{(x+1)^{-1}}{-1}$$

$$\boxed{\int \frac{x^2-3x-7}{(2x+3)(x+1)^2}\,dx = \frac{3}{x+1} + \ln|x+1| - \frac{1}{2}\ln|2x+3| + C}$$

14. $\int \frac{dt}{(t+2)^2(t+1)}$

$$\frac{1}{(t+2)^2(t+1)} = \frac{A}{t+1} + \frac{B}{t+2} + \frac{C}{(t+2)^2}$$

mmc: $(t+2)^2(t+1)$

$$\Leftrightarrow$$

$$1 = A(t+2)^2 + B(t+1)(t+2) + C(t+1)$$

$t=-1$: $1 = A$

$t=-2$: $1 = -C$

$t=0$: $\begin{cases} 1 = A \cdot 4 + 2B + C \\ 1 = 4 + 2B - 1 \end{cases}$

$-2 = 2B \Rightarrow B = -1$

$\therefore$

$$\frac{1}{(t+2)^2(t+1)} = \frac{1}{t+1} - \frac{1}{t+2} - \frac{1}{(t+2)^2}$$

$\therefore$

$$\int \frac{dt}{(t+2)^2(t+1)} = \int \frac{dt}{t+1} - \int \frac{dt}{t+2} - \int \frac{dt}{(t+2)^2}$$

$$= \ln|t+1| - \ln|t+2| - \frac{(t+2)^{-1}}{-1} + C$$

$\therefore$

$$\boxed{\int \frac{dt}{(t+2)^2(t+1)} = \ln\left|\frac{t+1}{t+2}\right| + \frac{1}{t+2} + C}$$

15. $\int \frac{3z+1}{(z^2-4)^2}\,dz$

$$\frac{3z+1}{(z^2-4)^2} = \frac{3z+1}{(z-2)^2(z+2)^2} = \frac{A}{z-2} + \frac{B}{(z-2)^2} + \frac{C}{(z+2)} + \frac{D}{(z+2)^2}$$

mmc: $(z-2)^2(z+2)^2$

$\Leftarrow$

$$3z+1 = A(z-2)(z+2)^2 + B(z+2)^2 + C(z-2)^2(z+2) + D(z-2)^2$$

$z=2$: $7 = B \cdot 16 \Rightarrow B = 7/16$

$z=-2$: $-5 = D \cdot 16 \Rightarrow D = -5/16$

$z=-1$:
$$\begin{cases} -2 = -3A + B + 9C + 9D \\ -2 = -3A + \frac{7}{16} + 9C - \frac{45}{16} \\ \left\| \frac{3}{8} = -3A + 9C \right\| \end{cases}$$

$-2 - \frac{7}{16} + \frac{45}{16}$

$-2 + \frac{38}{16}$

$= \frac{-32+38}{16}$

$= \frac{6}{16} = \frac{3}{8}$

$z=0$:
$$\begin{cases} 1 = A(-2)4 + B \cdot 4 + C \cdot 4 \cdot 2 + D \cdot 4 \\ 1 = -8A + \frac{7}{4} + 8C - \frac{5}{4} \\ \left\| \frac{1}{2} = -8A + 8C \right\| \end{cases}$$

$\therefore$

$$\begin{cases} \frac{\cancel{3}}{8} = -\cancel{3}A + \cancel{9}C \\ \frac{1}{2} = -8A + 8C \end{cases} \Leftarrow \begin{cases} \frac{1}{8} = -A + 3C \\ \frac{1}{16} = -A + C \end{cases}$$

$\frac{1}{8} - \frac{1}{16} = 2C$

$\frac{1}{16} = 2C \Rightarrow C = \frac{1}{32}$

$$\frac{1}{8} = -A + 3C$$

$$= -A + \frac{3}{32} \implies A = \frac{3}{32} - \frac{1}{8}$$

$$A = -\frac{1}{32}$$

$\therefore$

$$\frac{3z+1}{(z-2)^2(z+2)^2} = -\frac{1}{32(z-2)} + \frac{7}{16(z-2)^2} + \frac{1}{32(z+2)} - \frac{5}{16(z+2)^2}$$

$\therefore$

$$\int \frac{3z+1}{(z^2-4)^2}\,dz = -\frac{1}{32}\int \frac{dz}{z-2} + \frac{7}{16}\int \frac{dz}{(z-2)^2} + \frac{1}{32}\int \frac{dz}{z+2} - \frac{5}{16}\int \frac{dz}{(z+2)^2}$$

$$= -\frac{1}{32}\ln|z-2| + \frac{7}{16}\,\frac{-1}{(z-2)} + \frac{1}{32}\ln|z+2| - \frac{5}{16}\,\frac{-1}{z+2}$$

$$= +\frac{1}{32}\ln\left|\frac{z+2}{z-2}\right| + \frac{5}{16}\,\frac{1}{z+2} - \frac{7}{16}\,\frac{1}{z-2} + C$$

$\therefore$

$$\boxed{\int \frac{3z+1}{(z^2-4)^2}\,dz = \frac{1}{32}\ln\left|\frac{z+2}{z-2}\right| + \frac{5}{16}\,\frac{1}{z+2} - \frac{7}{16}\,\frac{1}{z-2} + C}$$

16. $\displaystyle\int \frac{(5x^2-11x+5)\,dx}{x^3-4x^2+5x-2}$

$$\frac{5x^2-11x+5}{x^3-4x^2+5x-2} =$$

$x^3-4x^2+5x-2 \xrightarrow{x=1} 1-4+5-2 = 0 \therefore x=1$ é raiz

$$\therefore \begin{array}{r|l} x^3-4x^2+5x-2 & x-1 \\ -x^3+x^2 & x^2-3x+2 \\ \hline -3x^2+5x-2 & \\ +3x^2-3x & \\ \hline 2x-2 & \\ -2x+2 & \\ \hline 0 & \end{array}$$

$x^2-3x+2=0$

$x = \frac{3 \pm \sqrt{9-8}}{2}$

$= \frac{3 \pm 1}{2} = 2, 1$

$$\therefore \begin{cases} x^3-4x^2+5x-2 = (x-1)(x^2-3x+2) \\ \qquad = (x-1)(x-1)(x-2) \end{cases}$$

$$\therefore \frac{5x^2-11x+5}{x^3-4x^2+5x-2} = \frac{5x^2-11x+5}{(x-1)^2(x-2)}$$

$$\frac{5x^2-11x+5}{x^3-4x^2+5x-2} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x-2}$$

mmc $(x-1)^2(x-2)$

$$\Leftrightarrow$$

$$5x^2-11x+5 = A(x-1)(x-2) + B(x-2) + C(x-1)^2$$

$$\begin{cases} x=1: & -1 = B(-1) \Rightarrow B=1 \\ x=2: & 3 = C \Rightarrow C=3 \\ x=0: & 5 = 2A-2B+C \Rightarrow 5 = 2A-2+3 \Rightarrow A=2 \end{cases}$$

$$\frac{5x^2-11x+5}{x^3-4x^2+5x-2} = \frac{2}{x-1} + \frac{1}{(x-1)^2} + \frac{3}{x-2}$$

$$\therefore \int \frac{5x^2-11x+5}{x^3-4x^2+5x-2}\,dx = \int \frac{2\,dx}{x-1} + \int \frac{dx}{(x-1)^2} + \int \frac{3\,dx}{x-2}$$

$$= 2\ln|x-1| + \frac{-1}{x-1} + 3\ln|x-2| + C$$

$$\boxed{\int \frac{5x^2-11x+5}{x^3-4x^2+5x-2}\,dx = \frac{1}{1-x} + 2\ln|x-1| + 3\ln|x-2| + C}$$

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17. $$\int \frac{x^4+3x^3-5x^2-4x+17}{x^3+x^2-5x+3}\,dx$$

$$\begin{array}{l|l} x^4+3x^3-5x^2-4x+17 & x^3+x^2-5x+3 \\ \hline -x^4-x^3+5x^2-3x & x+2 \\ \hline 0+2x^3+0x^2-7x+17 & \\ \quad -2x^3-2x^2+10x-6 & \\ \hline \quad -2x^2+3x+11 & \end{array}$$

$$\therefore \frac{x^4+3x^3-5x^2-4x+17}{x^3+x^2-5x+3} = x+2 + \frac{-2x^2+3x+11}{x^3+x^2-5x+3} \quad (*)$$

$x^3+x^2-5x+3 \xrightarrow{x=1} 1+1-5+3 = 0$

$\therefore\ x^3+x^2-5x+3 = (x-1)\,M(x)$

$$
\begin{array}{r|l}
x^3 + x^2 - 5x + 3 & x - 1 \\
\underline{-x^3 + x^2 \qquad\qquad} & \overline{x^2 + 2x - 3} \\
2x^2 - 5x + 3 & \\
\underline{-2x^2 + 2x \qquad} & \\
-3x + 3 & \\
\underline{+3x - 3} & \\
0 &
\end{array}
$$

$\therefore$

$$x^3+x^2-5x+3 = (x-1)(x^2+2x-3)$$

$x^2+2x-3 = 0$

$x = -2 \pm \sqrt{4+12}\,/2$

$= \frac{-2 \pm 4}{2}$

$= -3, 1$

$$= (x-1)(x+3)(x-1)$$

$$= (x-1)^2(x+3) \qquad (**)$$

$(**) \to (*)$:

$$\frac{x^4+3x^3-5x^2-4x+17}{x^3+x^2-5x+3} = x+2 + \frac{-2x^2+3x+11}{(x-1)^2(x+3)} \qquad (*)$$

Mas :

$$\frac{-2x^2+3x+11}{(x-1)^2(x+3)} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{(x+3)}$$

mmc $(x-1)^2(x+3)$

$\Leftrightarrow$

$$-2x^2+3x+11 = A(x-1)(x+3) + B(x+3) + C(x-1)^2$$

$x=1$: $\quad 12 = B \cdot 4 \Rightarrow B = 3$

$x=-3$: $\quad -16 = C\,16 \Rightarrow C = -1$

$x=0$: $\quad 11 = -3A + 3B + C$

$11 = -3A + 9 - 1$

$3 = -3A \Rightarrow A = -1$

$$\frac{-2x^2+3x+11}{(x-1)^2(x+3)} = -\frac{1}{x-1} + \frac{3}{(x-1)^2} - \frac{1}{x+3} \quad (☆☆)$$

(☆☆) → (☆) :

$$\frac{x^4+3x^3-5x^2-4x+17}{x^3+x^2-5x+3} = x+2 - \frac{1}{x-1} - \frac{1}{x+3} + \frac{3}{(x-1)^2}$$

∴

$$\int \frac{x^4+3x^3-5x^2-4x+17}{x^3+x^2-5x+3}\,dx =$$

$$= \int (x+2)\,dx - \int \frac{dx}{x-1} - \int \frac{dx}{x+3} + 3\int \frac{dx}{(x-1)^2}$$

$$= \frac{x^2}{2} + 2x - \ln|x-1| - \ln|x+3| + 3\,\frac{-1}{x-1}$$

$$= -\frac{3}{x-1} + 2x + \frac{x^2}{2} - \ln|(x-1)(x+3)| + C$$

$$\boxed{= \frac{1}{2}x^2 + 2x - \frac{3}{x-1} - \ln|x^2+2x-3| + C}$$

$A=3$

$B=C=-1$

$$\frac{x^2}{2} + 2x - \frac{3}{x-1}$$

18. $\displaystyle\int \frac{2x^4-2x+1}{2x^5-x^4}\,dx$

$$\frac{2x^4-2x+1}{2x^5-x^4} = \frac{2x^4-2x+1}{x^4(2x-1)} = \frac{A}{x}+\frac{B}{x^2}+\frac{C}{x^3}+\frac{D}{x^4}+\frac{E}{(2x-1)}$$

mmc $x^4(2x-1)$

$$\Leftrightarrow$$

$$2x^4-2x+1 = Ax^3(2x-1)+Bx^2(2x-1)+Cx(2x-1)+D(2x-1)+Ex^4$$

$x=\frac{1}{2}$: $\frac{2}{16}\cancel{-1}\cancel{+1} = E\frac{1}{16} \Rightarrow E=2$

$x=0$: $1 = -D \Rightarrow D=-1$

$x=1$:
$$\begin{cases} 1 = A+B+C+D+E \\ \quad = A+B+C-1+2 \\ \cancel{1} = A+B+C+\cancel{1} \\ 0 = A+B+C \end{cases}$$

outra maneira :

$$2x^4-2x+1 = Ax^3(2x-1)+Bx^2(2x-1)+Cx(2x-1)+D(2x-1)+Ex^4$$

$$= 2Ax^4-Ax^3+2Bx^3-Bx^2+2Cx^2-Cx+2Dx-D+Ex^4$$

$$2x^4-2x+1 = (2A+E)x^4+(-A+2B)x^3+(-B+2C)x^2+(-C+2D)x-D$$

$$\Rightarrow \begin{cases} 2A+E=2 \\ -A+2B=0 \\ -B+2C=0 \\ -C+2D=-2 \\ D=-1 \end{cases}$$

$$\begin{cases} 2A + E = 2 & \Rightarrow E = 2 \\ -A + 2B = 0 & \Rightarrow -A + 2\overset{0}{B} = 0 \Rightarrow A = 0 \\ -B + 2C = 0 & \Rightarrow -B + 2\overset{0}{C} = 0 \Rightarrow \boxed{B = 0} \\ -C + 2D = -2 & \Rightarrow -C - 2 = -2 \Rightarrow \boxed{C = 0} \\ \boxed{D = -1} & \end{cases}$$

$\therefore$

$$\frac{2x^4 - 2x + 1}{2x^5 - x^4} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{D}{x^4} + \frac{E}{(2x-1)}$$

$$= -\frac{1}{x^4} + \frac{2}{2x-1}$$

$\therefore$

$$\int \frac{2x^4 - 2x + 1}{2x^5 - x^4}\,dx = \int -\frac{dx}{x^4} + 2\int \frac{dx}{2x-1}$$

$$= -\frac{-1}{3x^3} + \cancel{2}\,\frac{\ln|2x-1|}{\cancel{2}} + C$$

$$\boxed{\int \frac{2x^4 - 2x + 1}{2x^5 - x^4}\,dx = \frac{1}{3x^3} + \ln|2x-1| + C}$$

19. $\int \frac{-24x^3+30x^2+52x+17}{9x^4-6x^3-11x^2+4x+4}\,dx$

$9x^4-6x^3-11x^2+4x+4 \xrightarrow{x=1} 9-6-11+4+4=0$

$(x-1)$ é raiz

$$\begin{array}{l|l} 9x^4-6x^3-11x^2+4x+4 & x-1 \\ \underline{-9x^4+9x^3} & 9x^3+3x^2-8x-4 \\ \quad 3x^3-11x^2+4x+4 & \\ \quad \underline{-3x^3+3x^2} & \\ \qquad -8x^2+4x+4 & \\ \qquad \underline{+8x^2-8x} & \\ \qquad\quad -4x+4 & \\ \qquad\quad \underline{+4x-4} & \\ \qquad\qquad 0 & \end{array}$$

$\therefore$

$9x^4-6x^3-11x^2+4x+4 = (x-1)(9x^3+3x^2-8x-4)$ (*)

Mas, $9x^3+3x^2-8x-4 \xrightarrow{x=1} 9+3-8-4=0$

$\Rightarrow$

$(x-1)$ é raiz

$$\begin{array}{l|l} 9x^3+3x^2-8x-4 & x-1 \\ \underline{-9x^3+9x^2} & 9x^2+12x+4 \\ \quad 12x^2-8x-4 & \\ \quad \underline{-12x^2+12x} & \\ \qquad 4x-4 & \\ \qquad \underline{-4x+4} & \\ \qquad\quad 0 & \end{array}$$

$\Rightarrow \therefore 9x^3+3x^2-8x-4 = (x-1)(9x^2+12x+4)$ (**)

Mas

$$9x^2+12x+4=0$$

$36 \cdot 4 = 144$

$$x = -12 \pm \sqrt{144-144} \;/ 18$$

$$= \frac{-12}{18} = -\frac{2}{3} \quad \therefore \; x = -\frac{2}{3} \text{ raiz multiplicidade } 2.$$

$\therefore$

$$9x^2+12x+4 = 9\left(x+\frac{2}{3}\right)^2 = (3x+2)^2 \quad (***)$$

$(***) \to (**)$: $\left\{ 9x^3+3x^2-8x-4 = (x-1)(3x+2)^2 \right.$ $(****)$

$(****) \to (*)$: $\left\{ 9x^4-6x^3-11x^2+4x+4 = (x-1)^2(3x+2)^2 \right.$ $(*)$

Daí :

$$\frac{-24x^3+30x^2+52x+17}{9x^4-6x^3-11x^2+4x+4} = \frac{-24x^3+30x^2+52x+17}{(x-1)^2(3x+2)^2} =$$

mmc : $(x-1)^2(3x+2)^2$

$$\equiv \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{(3x+2)} + \frac{D}{(3x+2)^2}$$

$$\Leftrightarrow$$

$$(-24x^3+30x^2+52x+17) = A(x-1)(3x+2)^2 + B(3x+2)^2 + C(x-1)^2(3x+2) + D(x-1)^2$$

$x=1$: $-24+30+52+17 = 25B$

$$75 = 25B \Rightarrow \boxed{B=3}$$

$x = -\frac{2}{3}$: $-24\left(-\frac{2}{3}\right)^3 + 30\left(-\frac{2}{3}\right)^2 + 52\left(-\frac{2}{3}\right) + 17 = D\left(-\frac{2}{3} - 1\right)^2$

$\overset{8}{\cancel{+24}} \cdot \frac{8}{\cancel{27}\,9} + 30 \cdot \frac{4}{9} - \frac{104}{3} + 17 = D\left(-\frac{5}{3}\right)^2$

$\frac{64}{9} + \frac{120}{9} - \frac{104}{3} + 17 = D\,\frac{25}{9}$

$\frac{64 + 120 - 312}{9} + 17$

$\frac{-128}{9} + 17 = D\,\frac{25}{9}$

$\frac{25}{9} = D\,\frac{25}{9} \Rightarrow D = 1$

$\begin{array}{r} 81 \\ -192 \\ 64 \\ \hline 128 \end{array}$

$x = 0$: $17 = -4A + 4\underset{3}{\underbrace{B}} + 2C + D$

$17 = -4A + 12 + 2C + 1$

$4 = -4A + 2C \Rightarrow \| 2 = -2A + C \|$

$x = -1$: $\underbrace{24 + 30 - 52 + 17} = -2A + B - 4C + 4D$

$19 = -2A + \underbrace{3} - 4C + 4$

$12 = -2A - 4C$

$\| 6 = -A - 2C \|$

$$\therefore \left\{ \begin{array}{l} 2 = -2A + C \implies C = 2 + 2A \\ 6 = -A - 2C \implies 6 = -A - 2(2+2A) \end{array} \right.$$

$$= -A - 4 - 4A$$

$$6 = -5A - 4$$

$$10 = -5A \implies A = -2$$

$$C = 2 + 2A = 2 + 2(-2)$$

$$C = -2$$

$\therefore$

$$\frac{-24x^3 + 30x^2 + 52x + 17}{9x^4 - 6x^3 - 11x^2 + 4x + 4} = \frac{-2}{x-1} + \frac{3}{(x-1)^2} - \frac{2}{(3x+2)} + \frac{1}{(3x+2)^2}$$

$\therefore$

$$\int \frac{-24x^3+30x^2+52x+17}{9x^4-6x^3-11x^2+4x+4}\,dx = -2\int \frac{dx}{x-1} + 3\int \frac{dx}{(x-1)^2} - 2\int \frac{dx}{3x+2} + \int \frac{dx}{(3x+2)^2}$$

$$= \underbrace{-2\ln|x-1|} + 3\,\frac{-1}{(x-1)} - 2\,\frac{\ln|3x+2|}{3} + \frac{-1}{3(3x+2)} + C$$

$$= -\frac{3}{x-1} - \frac{1}{3(3x+2)} - \underbrace{\ln(x-1)^2} - \ln|3x+2|^{2/3} + C$$

$$\boxed{\int \frac{-24x^3+30x^2+52x+17}{9x^4-6x^3-11x^2+4x+4}\,dx = -\frac{3}{x-1} - \frac{1}{3(3x+2)} - \ln\left[(3x+2)^{2/3}(x-1)^2\right] + C}$$

20. $\int \frac{dx}{16x^4-8x^2+1}$

$16x^4-8x^2+1 = 16y^2-8y+1 \qquad (y=x^2)$

$\therefore \quad 16y^2-8y+1=0$

$y = 8 \pm \sqrt{64-64}\,/32$

$= \frac{8}{32} = \frac{1}{4}$

$y = x^2 = \frac{1}{4} \Rightarrow x = \pm\frac{1}{2}.$

$\therefore \quad 16x^4-8x^2+1 = 16\left(x+\frac{1}{2}\right)^2\left(x-\frac{1}{2}\right)^2 = (2x+1)^2(2x-1)^2$

$\therefore$

$\frac{1}{16x^4-8x^2+1} = \frac{1}{(2x+1)^2(2x-1)^2}$ mmc : $(2x+1)^2(2x-1)^2$

$= \frac{A}{(2x+1)} + \frac{B}{(2x+1)^2} + \frac{C}{(2x-1)} + \frac{D}{(2x-1)^2}$

$\Leftrightarrow$

$1 = A(2x+1)(2x-1)^2 + B(2x-1)^2 + C(2x+1)^2(2x-1) + D(2x+1)^2$

$\Leftrightarrow$

$x=0: \quad 1 = A + B - C + D$

$x=\frac{1}{2}: \quad 1 = 4D \Rightarrow D = 1/4$

$x=-\frac{1}{2}: \quad 1 = 4B \Rightarrow B = 1/4$

$x=1: \quad 1 = 3A + B + 9C + 9D$

$$1 = A + B - C + D$$

$$= A + \frac{1}{4} - C + \frac{1}{4}$$

$$= A - C + \frac{1}{2}$$

$$\| \frac{1}{2} = A - C \|$$

$$1 = 3A + B + 9C + 9D$$

$$= 3A + \frac{1}{4} + 9C + \frac{9}{4}$$

$$1 = 3A + 9C + \frac{5}{2}$$

$$-\frac{3}{2} = 3A + 9C \longrightarrow \| -\frac{1}{2} = A + 3C \|$$

$$\therefore \begin{cases} \frac{1}{2} = A - C \\ -\frac{1}{2} = A + 3C \end{cases} \Rightarrow 1 = -4C \Rightarrow C = -\frac{1}{4}$$

$$\frac{1}{2} = A - C$$

$$= A + \frac{1}{4} \Rightarrow A = \frac{1}{2} - \frac{1}{4}$$

$$\therefore \quad A = \frac{1}{4}$$

$$\frac{1}{16x^4 - 8x^2 + 1} = \frac{1}{4(2x+1)} + \frac{1}{4(2x+1)^2} - \frac{1}{4(2x-1)} + \frac{1}{4(2x-1)^2}$$

$\therefore$

$$\int \frac{dx}{16x^4-8x^2+1} = \frac{1}{4}\int \frac{dx}{2x+1} + \frac{1}{4}\int \frac{dx}{(2x+1)^2} - \frac{1}{4}\int \frac{dx}{2x-1}$$

$$+ \frac{1}{4}\int \frac{dx}{(2x-1)^2}$$

$$= \frac{1}{4}\,\frac{\ln|2x+1|}{2} + \frac{1}{4}\,\frac{-1}{2(2x+1)} - \frac{1}{4}\,\frac{\ln|2x-1|}{2}$$

$$+ \frac{1}{4}\,\frac{-1}{2(2x-1)} + C$$

$$= \frac{1}{8}\ln|2x+1| - \frac{1}{8(2x+1)} - \frac{1}{8}\ln|2x-1| - \frac{1}{8(2x-1)} + C$$

$$\boxed{\int \frac{dx}{16x^4-8x^2+1} = -\frac{1}{8(2x+1)} - \frac{1}{8(2x-1)} + \frac{1}{8}\ln\left|\frac{2x+1}{2x-1}\right| + C}$$

$$\frac{1}{8}\ln\left|\frac{x+\frac{1}{2}}{x-\frac{1}{2}}\right| - \frac{1}{16}\,\frac{1}{(x-\frac{1}{2})} - \frac{1}{16}\,\frac{1}{(x+\frac{1}{2})} + C$$