

Lista 7

Integrais Impropias I

Leitold (pg. 672)

$$1. \int_0^{+\infty} e^{-x} dx$$

$$= \lim_{b \rightarrow +\infty} \int_0^b e^{-x} dx$$

$$= \lim_{b \rightarrow +\infty} -e^{-x} \Big|_0^b$$

$$= \lim_{b \rightarrow +\infty} (-e^{-b} - (-e^0))$$

$$= \lim_{b \rightarrow +\infty} (-e^{-b} + 1) \quad \downarrow \quad \lim_{b \rightarrow +\infty} e^{-b} = 0$$

$$= 1 //$$

$$2. \int_{-\infty}^1 e^x dx =$$

$$= \lim_{a \rightarrow -\infty} \int_a^1 e^x dx$$

$$= \lim_{a \rightarrow -\infty} e^x \Big|_a^1$$

$$= \lim_{a \rightarrow -\infty} (e - e^a)$$

$$\downarrow \quad \lim_{a \rightarrow -\infty} e^a = 0$$

$$= e //$$

$$3. \int_{-\infty}^0 x 5^{-x^2} dx =$$

$$= \lim_{a \rightarrow -\infty} \int_a^0 x 5^{-x^2} dx$$

$$(e^x)' = e^x \quad \left| \begin{array}{l} = \lim_{a \rightarrow -\infty} \left. \frac{-1}{2 \ln 5} 5^{-x^2} \right|_a^0 \end{array} \right.$$

$$(a^x)' = a^x \ln a \quad \left| \begin{array}{l} = \lim_{a \rightarrow -\infty} \left(\frac{-1}{2 \ln 5} 5^0 + \frac{1}{2 \ln 5} 5^{-a^2} \right) \end{array} \right.$$

$$= \lim_{a \rightarrow -\infty} \left(\frac{-1}{2 \ln 5} + \frac{5^{-a^2}}{2 \ln 5} \right)$$

Ways

$$\lim_{a \rightarrow -\infty} \frac{5^{-a^2}}{2 \ln 5} = \frac{5^{-\infty}}{2 \ln 5} = 0$$

$$\rightarrow = -\frac{1}{2 \ln 5}$$

$$\int_{-\infty}^0 x 5^{-x^2} dx = -\frac{1}{2 \ln 5}$$

$$4. \int_1^{\infty} 2^{-x} dx$$

$$= \lim_{b \rightarrow \infty} \int_1^b 2^{-x} dx$$

$$(2^{-x})' = -2^{-x} \ln 2$$

$$= \lim_{b \rightarrow \infty} \left. \frac{2^{-x}}{\ln 2} \right|_1^b$$

$$= \lim_{b \rightarrow \infty} \left(\frac{2^{-b}}{\ln 2} + \frac{1}{2 \ln 2} \right)$$

$$= \frac{\lim_{b \rightarrow \infty} 2^{-b}}{\ln 2} + \frac{1}{2 \ln 2}$$

$$= \frac{1}{2 \ln 2} //$$

$$5. \int_0^{\infty} x 2^{-x} dx =$$

$$= \lim_{b \rightarrow \infty} \int_0^b x 2^{-x} dx$$

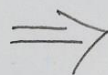
Mas

$$\int x 2^{-x} dx = \frac{-x 2^{-x}}{\ln 2} + \int \frac{2^{-x}}{\ln 2} dx$$

$$u = x \rightarrow du = dx$$

$$dv = 2^{-x} dx \rightarrow v = \frac{2^{-x}}{-\ln 2}$$

$$= \frac{-x 2^{-x}}{\ln 2} + \frac{1}{\ln 2} \frac{2^{-x}}{(-\ln 2)}$$



$$\int_0^b x 2^{-x} dx = \left[\frac{-x 2^{-x}}{\ln 2} - \frac{2^{-x}}{(\ln 2)^2} \right]_0^b$$

$$= \frac{-b 2^{-b}}{\ln 2} - \frac{2^{-b}}{(\ln 2)^2} + \frac{1}{(\ln 2)^2}$$

$$\lim_{b \rightarrow +\infty} \int_0^b x 2^{-x} dx = \lim_{b \rightarrow +\infty} \left(\frac{-b 2^{-b}}{\ln 2} - \frac{2^{-b}}{(\ln 2)^2} + \frac{1}{(\ln 2)^2} \right)$$

$$= \frac{1}{(\ln 2)^2}$$

$$\int_0^{+\infty} x 2^{-x} dx = \frac{1}{(\ln 2)^2}$$

$$6. \int_5^{+\infty} \frac{dx}{\sqrt{x-1}} =$$

$$= \lim_{b \rightarrow +\infty} \int_5^b \frac{dx}{\sqrt{x-1}}$$

$$= \lim_{b \rightarrow +\infty} \left. 2\sqrt{x-1} \right|_5^b$$

$$= \lim_{b \rightarrow +\infty} (2\sqrt{b-1} - 4)$$

$$= +\infty \quad (\text{Divergente})$$

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7. $\int_{-\infty}^{\infty} x \cosh x dx$

$$\int x \cosh x dx = \int x \frac{e^x + e^{-x}}{2} dx$$

$$= \frac{1}{2} \int (xe^x + xe^{-x}) dx$$

$$\int xe^x dx = xe^x - \int e^x dx$$

$$u = x \rightarrow du = dx \quad \Bigg| \quad = xe^x - e^x$$

$$dv = e^x dx \rightarrow v = e^x$$

$$\int xe^{-x} dx = -xe^{-x} + \int e^{-x} dx$$

$$u = x \rightarrow du = dx \quad \Bigg| \quad = -xe^{-x} - e^{-x}$$

$$dv = e^{-x} dx \rightarrow v = -e^{-x}$$

∴ ∴

$$= \frac{1}{2} (xe^x - e^x - xe^{-x} - e^{-x})$$

∴ ∴

$$\int x \cosh x dx = \frac{1}{2} e^x (x-1) - \frac{1}{2} e^{-x} (x+1)$$

$$\int_{-\infty}^{+\infty} x \cosh x dx = \lim_{a \rightarrow -\infty} \int_a^0 x \cosh x dx + \lim_{b \rightarrow +\infty} \int_0^b x \cosh x dx$$

(★) (★★)

Mod

$$(★) = \lim_{a \rightarrow -\infty} \int_a^0 x \cosh x dx =$$

$$= \lim_{a \rightarrow -\infty} \left(\frac{1}{2} e^x(x-1) - \frac{1}{2} e^{-x}(x+1) \right) \Big|_a^0$$

$$= \lim_{a \rightarrow -\infty} \left(\frac{1(-1) - \frac{1}{2}(1)}{2} - \frac{1}{2} e^a(a-1) + \frac{1}{2} e^{-a}(a+1) \right)$$

$$= \lim_{a \rightarrow -\infty} \left(-1 - \frac{1}{2} e^a(a-1) + \frac{1}{2} e^{-a}(a+1) \right)$$

$$= -\infty$$

∴ ∴

$$\int_{-\infty}^{+\infty} x \cosh x dx = \text{diverge}$$

1 / 1

$$8. \int_{-\infty}^0 x^2 e^x dx =$$

$$= \lim_{a \rightarrow -\infty} \int_a^0 x^2 e^x dx$$

May

$$\int x^2 e^x dx = x^2 e^x - \int 2x e^x dx \quad (*)$$

$$u = x^2 \rightarrow du = 2x dx$$

$$dv = e^x dx \rightarrow v = e^x$$

$$\int x e^x dx = x e^x - \int e^x dx = x e^x - e^x \quad (**)$$

$$u = x \rightarrow du = dx$$

$$dv = e^x dx \rightarrow v = e^x$$

Substituyendo ~~(*)~~ \rightarrow ~~(**)~~ :

$$\int x^2 e^x dx = x^2 e^x - 2(x e^x - e^x)$$

$$= x^2 e^x - 2x e^x + 2e^x$$

$$= (x^2 - 2x + 2) e^x //$$

Daí,

$$\int_a^0 x^2 e^x dx = (x^2 - 2x + 2) e^x \Big|_a^0$$

$$= 2 - (a^2 - 2a + 2) e^a$$

$$\lim_{a \rightarrow -\infty} \int_a^0 x^2 e^x dx = \lim_{a \rightarrow -\infty} (2 - (a^2 - 2a + 2) e^a)$$

$$= 2$$

$$\int_{-\infty}^0 x^2 e^x dx = 2$$

9. $\int_5^{+\infty} \frac{x dx}{\sqrt[3]{9-x^2}}$

$$= \lim_{b \rightarrow +\infty} \int_5^b \frac{x dx}{\sqrt[3]{9-x^2}}$$

$$= \lim_{b \rightarrow +\infty} \left. \frac{-3}{4} (9-x^2)^{2/3} \right|_5^b$$

$$= \frac{-3}{4} \lim_{b \rightarrow +\infty} \left((9-b^2)^{2/3} - 4\sqrt[3]{2} \right)$$

$$= -\infty \quad \text{diverge}$$

$$\left[\frac{-3}{4} (9-x^2)^{2/3} \right] =$$

$$= \frac{-3}{4} \times \frac{2}{3} (9-x^2)^{\frac{2}{3}-1} (-2x)$$

$$= -\frac{1}{2} (-2x) (9-x^2)^{-1/3}$$

$$= \frac{x}{\sqrt[3]{9-x^2}}$$

$$(9-25)^{2/3} = -16^{2/3}$$

$$= \sqrt[3]{16^2}$$

$$= \sqrt[3]{2^8} = 2^{8/3}$$

$$= 4\sqrt[3]{2}$$

$$10. \int_{-\infty}^{+\infty} \frac{3x \, dx}{(3x^2+2)^3}$$

formamos
c=0

$$= \lim_{a \rightarrow -\infty} \int_a^0 \frac{3x}{(3x^2+2)^3} \, dx + \lim_{b \rightarrow +\infty} \int_0^b \frac{3x}{(3x^2+2)^3} \, dx$$

Mas

$$\int \frac{3x \, dx}{(3x^2+2)^3} = \frac{1}{-4 \cdot (3x^2+2)^2}$$

$$\lim_{a \rightarrow -\infty} \int_a^0 \frac{3x}{(3x^2+2)^3} \, dx = \lim_{a \rightarrow -\infty} \left. -\frac{1}{4(3x^2+2)^2} \right|_a^0$$

$$= \lim_{a \rightarrow -\infty} \left(-\frac{1}{4 \times 4} + \frac{1}{4(3a^2+2)^2} \right)$$

$$= -\frac{1}{16}$$

$$\lim_{b \rightarrow +\infty} \int_0^b \frac{3x}{(3x^2+2)^3} \, dx = \lim_{b \rightarrow +\infty} \left. -\frac{1}{4(3x^2+2)^2} \right|_0^b$$

$$= \lim_{b \rightarrow +\infty} \left(-\frac{1}{4(3b^2+2)^2} + \frac{1}{4 \times 4} \right)$$

$$= \frac{1}{16}$$

~~$$\int_{-\infty}^{+\infty} \frac{3x}{(3x^2+2)^3} \, dx = -\frac{1}{16} + \frac{1}{16} = 0$$~~

$$11. \int_{\sqrt{3}}^{+\infty} \frac{3 dx}{x^2+9} =$$

$$= \lim_{b \rightarrow +\infty} \int_{\sqrt{3}}^b \frac{3 dx}{x^2+9}$$

$$\int \frac{dx}{x^2+a^2} = \frac{1}{a} \arctan \frac{x}{a}$$

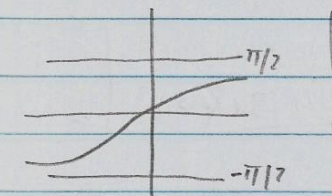
Mas,

$$\int_{\sqrt{3}}^b \frac{3 dx}{x^2+9} = 3 \int_{\sqrt{3}}^b \frac{dx}{x^2+9}$$

$$= \frac{3}{3} \arctan \frac{x}{3} \Big|_{\sqrt{3}}^b$$

$$= \arctan \frac{b}{3} - \arctan \frac{\sqrt{3}}{3}$$

$$= \arctan \frac{b}{3} - \frac{\pi}{6}$$



$$\lim_{x \rightarrow -\infty} \arctan \frac{x}{3} = -\frac{\pi}{2}$$

$$\lim_{x \rightarrow +\infty} \arctan \frac{x}{3} = \frac{\pi}{2}$$

$$y = \arctan \frac{x}{3} \Leftrightarrow x = 3 \tan y$$

$$y = \arctan \frac{\sqrt{3}}{3} \Leftrightarrow \frac{\sqrt{3}}{3} = 3 \tan y$$

$$\downarrow$$

$$y = \frac{\pi}{6}$$

$$\lim_{b \rightarrow +\infty} \int_{\sqrt{3}}^b \frac{3 dx}{x^2+9} =$$

$$= \lim_{b \rightarrow +\infty} \left(\arctan \frac{b}{3} - \frac{\pi}{6} \right)$$

$$\lim_{b \rightarrow +\infty} \arctan \frac{b}{3} = \arctan \infty = \frac{\pi}{2}$$

$$= \frac{\pi}{2} - \frac{\pi}{6}$$

$$= \frac{3\pi - \pi}{6} = \frac{2\pi}{6} = \frac{\pi}{3}$$

1
2

$$\int_{\sqrt{3}}^{+\infty} \frac{3 dx}{x^2+9} = \frac{\pi}{3}$$

/ /

$$12. \int_0^{+\infty} \frac{dx}{x \ln x} =$$

$$= \lim_{b \rightarrow +\infty} \int_e^b \frac{dx}{x \ln x}$$

Mo3

$$\int \frac{dx}{x \ln x} = \int \frac{du}{u} = \ln |u| = \ln |\ln x|$$

$$u = \ln x \rightarrow du = \frac{1}{x} dx$$

$$\Rightarrow \lim_{b \rightarrow +\infty} \int_e^b \frac{dx}{x \ln x} = \lim_{b \rightarrow +\infty} \ln |\ln x| \Big|_e^b$$

$$= \lim_{b \rightarrow +\infty} (\ln |\ln b| - \ln |\ln e|)$$

$$= \lim_{b \rightarrow +\infty} (\ln |\ln b| - \ln 1)$$

$$= \lim_{b \rightarrow +\infty} (\ln |\ln b| - 0)$$

$$= \lim_{b \rightarrow +\infty} \ln |\ln b|$$

$$= +\infty$$

\Rightarrow

$$\int_e^{+\infty} \frac{dx}{x \ln x} = +\infty \text{ diverge}$$

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B. $\int_{-\infty}^{+\infty} e^{-|x|} dx =$

$$= \lim_{a \rightarrow -\infty} \int_a^0 e^{-|x|} dx + \lim_{b \rightarrow +\infty} \int_0^b e^{-|x|} dx$$

$$= \lim_{a \rightarrow -\infty} \int_a^0 e^x dx + \lim_{b \rightarrow +\infty} \int_0^b e^{-x} dx$$

$$= \lim_{a \rightarrow -\infty} e^x \Big|_a^0 + \lim_{b \rightarrow +\infty} -e^{-x} \Big|_0^b$$

$$= \lim_{a \rightarrow -\infty} (1 - e^a) + \lim_{b \rightarrow +\infty} (-e^{-b} + 1)$$

$$= 1 + 1$$

$$= 2 //$$

14. $\int_{-\infty}^{+\infty} x e^{-x^2} dx =$

$$= \lim_{a \rightarrow -\infty} \int_a^0 x e^{-x^2} dx + \lim_{b \rightarrow +\infty} \int_0^b x e^{-x^2} dx$$

$$= \lim_{a \rightarrow -\infty} \frac{-1}{2} e^{-x^2} \Big|_a^0 + \lim_{b \rightarrow +\infty} \frac{-1}{2} e^{-x^2} \Big|_0^b$$

$$= \lim_{a \rightarrow -\infty} \left(-\frac{1}{2} + \frac{1}{2} e^{-a^2} \right) + \lim_{b \rightarrow +\infty} \left(-\frac{1}{2} e^{-b^2} + \frac{1}{2} \right)$$

$$= -\frac{1}{2} + \frac{1}{2} = 0 //$$

→ só era esperada pois $x e^{-x^2}$ é função ímpar.

$$15. \int_e^{+\infty} \frac{dx}{x(\ln x)^2} =$$

$$\rightarrow = \lim_{b \rightarrow +\infty} \int_e^b \frac{dx}{x(\ln x)^2}$$

Met 1:

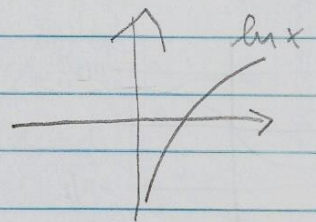
$$u = \ln x \rightarrow du = \frac{1}{x} dx$$

$$\therefore \int \frac{dx}{x(\ln x)^2} = \int \frac{du}{u^2} = -\frac{1}{u} = -\frac{1}{\ln x}$$

$$\rightarrow = \lim_{b \rightarrow +\infty} \left[-\frac{1}{\ln x} \right]_e^b$$

$$= \lim_{b \rightarrow +\infty} \left(-\frac{1}{\ln b} + 1 \right)$$

$$= 1$$



$$\lim_{b \rightarrow +\infty} \frac{1}{\ln b} = \frac{1}{\infty} = 0$$

$$\therefore \boxed{\int_e^{+\infty} \frac{dx}{x(\ln x)^2} = 1}$$

$$16. \int_{-\infty}^{+\infty} \frac{dx}{16+x^2} =$$

$$= \lim_{a \rightarrow -\infty} \int_a^0 \frac{dx}{16+x^2} + \lim_{b \rightarrow +\infty} \int_0^b \frac{dx}{16+x^2}$$

$$= \lim_{a \rightarrow -\infty} \left. \frac{1}{4} \arctan \frac{x}{4} \right|_a^0 + \lim_{b \rightarrow +\infty} \left. \frac{1}{4} \arctan \frac{x}{4} \right|_0^b$$

$$\int \frac{dx}{x^2+a^2} = \frac{1}{a} \arctan \frac{x}{a} \quad = \lim_{a \rightarrow -\infty} \left(\frac{1}{4} \arctan \frac{0}{4} - \frac{1}{4} \arctan \frac{a}{4} \right) + \lim_{b \rightarrow +\infty} \left(\frac{1}{4} \arctan \frac{b}{4} - \frac{1}{4} \arctan \frac{0}{4} \right)$$

$$= \lim_{a \rightarrow -\infty} -\frac{1}{4} \arctan \frac{a}{4} + \lim_{b \rightarrow +\infty} \frac{1}{4} \arctan \frac{b}{4}$$

$$= -\frac{1}{4} \lim_{a \rightarrow -\infty} \arctan \frac{a}{4} + \frac{1}{4} \lim_{b \rightarrow +\infty} \arctan \frac{b}{4}$$

$$\text{Dom } \arctan = \mathbb{R}$$

$$\text{Im } \arctan = \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

$$\arctan x = y \Leftrightarrow x = \tan y$$

$$= -\frac{1}{4} \left(-\frac{\pi}{2}\right) + \frac{1}{4} \left(\frac{\pi}{2}\right)$$

$$= \frac{\pi}{8} + \frac{\pi}{8} = \frac{\pi}{4}$$

/ /

$$17. \int_1^{+\infty} \ln x \, dx =$$

$$= \lim_{b \rightarrow +\infty} \int_1^b \ln x \, dx$$

Now

$$\int \ln x \, dx = x \ln x - \int dx$$

$$u = \ln x \rightarrow du = \frac{1}{x} dx \quad = x \ln x - x$$

$$dv = dx \rightarrow v = x$$

$$\lim_{b \rightarrow +\infty} \int_1^b \ln x \, dx = \lim_{b \rightarrow +\infty} (x \ln x - x) \Big|_1^b$$

$$= \lim_{b \rightarrow +\infty} (b \ln b - b - x \ln 1 + 1)$$

$$= \lim_{b \rightarrow +\infty} (b \ln b - b + 1)$$

$$= \lim_{b \rightarrow +\infty} b (\ln b - 1 + \frac{1}{b}) = \infty \cdot \infty = \infty$$

$$\int_1^{+\infty} \ln x \, dx = +\infty \quad \text{diverge}$$

$$18. \int_0^{+\infty} e^{-x} \cos x \, dx$$

$$= \lim_{b \rightarrow +\infty} \int_0^b e^{-x} \cos x \, dx$$

Mas,

$$\int e^{-x} \cos x \, dx = \sin x e^{-x} + \int \sin x e^{-x} \, dx \quad (*)$$

$$u = e^{-x} \rightarrow du = -e^{-x} dx$$

$$dv = \cos x \, dx \rightarrow v = \sin x$$

Moş

$$\int \sin x e^{-x} \, dx = -\cos x e^{-x} - \int \cos x e^{-x} \, dx \quad (**)$$

$$u = e^{-x} \rightarrow du = -e^{-x} dx$$

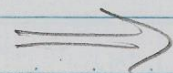
$$dv = \sin x \, dx \rightarrow v = -\cos x$$

Substituindo $(**)$ \rightarrow $(*)$:

$$\int e^{-x} \cos x \, dx = \sin x e^{-x} - \cos x e^{-x} - \int \cos x e^{-x} \, dx$$

$$2 \int e^{-x} \cos x \, dx = (\sin x - \cos x) e^{-x}$$

$$\therefore \int e^{-x} \cos x \, dx = \frac{1}{2} (\sin x - \cos x) e^{-x}$$



1 / 1

$$\lim_{b \rightarrow +\infty} \int_0^b e^{-x} \cos x \, dx =$$

$$= \lim_{b \rightarrow +\infty} \left. \frac{1}{2} (\sin x - \cos x) e^{-x} \right|_0^b$$

$$= \lim_{b \rightarrow +\infty} \left(\frac{1}{2} (\sin b - \cos b) e^{-b} - \frac{1}{2} (\sin 0 - \cos 0) e^{-0} \right)$$

$$= \lim_{b \rightarrow +\infty} \left(\frac{1}{2} (\sin b - \cos b) e^{-b} - \frac{1}{2} (-1) \right)$$

$$= \lim_{b \rightarrow +\infty} \left(\frac{1}{2} (\sin b - \cos b) e^{-b} + \frac{1}{2} \right)$$

$$b \rightarrow +\infty : \left| \frac{1}{2} (\sin b - \cos b) \right| < 2$$

$$e^{-b} \rightarrow 0$$

∴

$$\lim_{b \rightarrow +\infty} \left(\frac{1}{2} (\sin b - \cos b) e^{-b} + \frac{1}{2} \right) = \frac{1}{2}$$

∴

$$\int_0^{+\infty} e^{-x} \cos x \, dx = \frac{1}{2}$$

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$$19) a) \int_{-\infty}^{+\infty} \sin x \, dx =$$

$$= \lim_{a \rightarrow -\infty} \int_a^0 \sin x \, dx + \lim_{b \rightarrow +\infty} \int_0^b \sin x \, dx$$

$$= \lim_{a \rightarrow -\infty} -\cos x \Big|_a^0 + \lim_{b \rightarrow +\infty} -\cos x \Big|_0^b$$

$$= \lim_{a \rightarrow -\infty} (-1 + \cos a) + \lim_{b \rightarrow +\infty} (-\cos b + 1)$$

Uma vez que $\left\{ \begin{array}{l} \lim_{a \rightarrow -\infty} \cos a \text{ não existe} \\ \lim_{b \rightarrow +\infty} \cos b \end{array} \right.$

temos que

$$\boxed{\int_{-\infty}^{+\infty} \sin x \, dx \text{ não existe}}$$

$$b) // \lim_{\pi \rightarrow -\infty} \int_{-\pi}^{\pi} \sin x \, dx =$$

$$= \lim_{\pi \rightarrow -\infty} \left[-\cos x \Big|_{-\pi}^{\pi} \right]$$

$$= \lim_{\pi \rightarrow -\infty} \left(-\cos \pi - (-\cos(-\pi)) \right)$$

$$= \lim_{\pi \rightarrow -\infty} \left(-\cos \pi + \cos \pi \right) = 0 //$$

20.

Seja $\int_{-b}^b f(x) dx$ convergente.

Queremos demonstrar que

$$\int_{-b}^{+\infty} f(-x) dx = \int_{-\infty}^b f(x) dx.$$

De fato,

Por definição,

$$\int_{-b}^{+\infty} f(-x) dx = \lim_{a \rightarrow +\infty} \int_{-b}^a f(-x) dx \quad (*)$$

Fazemos agora a mudança de variável

$x \rightarrow y = -x$ na integral $\int_{-b}^a f(-x) dx$,

$$\int_{-b}^a f(-x) dx = \int_b^{-a} f(y) dy$$

$$\begin{array}{l} x = -b \rightarrow y = b \\ x = a \rightarrow y = -a \end{array} \quad \left| \quad = - \int_b^{-a} f(y) dy \right.$$

$$= \int_{-a}^b f(y) dy$$

$$= \int_{-a}^b f(x) dx$$

↓ q o variável
muda

$$\| \int_{-b}^a f(-x) dx = \int_{-a}^b f(x) dx \| \quad (**)$$

Subst. (x) in $(*)$ erhalten, 100

$$\int_b^{+\infty} f(-x) dx = \lim_{a \rightarrow +\infty} \int_b^a f(-x) dx$$

$$= \lim_{a \rightarrow +\infty} \int_{-a}^b f(x) dx$$

$$= \int_{-\infty}^b f(x) dx$$

$$\int_b^{+\infty} f(-x) dx = \int_{-\infty}^b f(x) dx$$

21.

a) $\int_{-\infty}^{+\infty} x(1+x^2)^{-2} dx$ ist konvergent

b) $\int_{-\infty}^{+\infty} x(1+x^2)^{-1} dx$ ist divergent

Beispiel

a) $\int_{-\infty}^{+\infty} x(1+x^2)^{-2} dx =$

$$= \lim_{a \rightarrow -\infty} \int_a^0 x(1+x^2)^{-2} dx + \lim_{b \rightarrow +\infty} \int_0^b x(1+x^2)^{-2} dx$$

21. (continuacao)

$$\int x(1+x^2)^{-2} dx = -\frac{1}{2} (1+x^2)^{-1}$$

$$= \lim_{a \rightarrow -\infty} \left. \frac{-1}{2(1+x^2)} \right|_a^0 + \lim_{b \rightarrow +\infty} \left. \frac{-1}{2(1+x^2)} \right|_0^b$$

$$= \lim_{a \rightarrow -\infty} \left(\frac{-1}{2} + \frac{1}{2(1+a^2)} \right) + \lim_{b \rightarrow +\infty} \left(\frac{-1}{2} + \frac{1}{2(1+b^2)} \right)$$

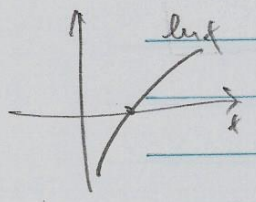
$$= -\frac{1}{2} - \frac{1}{2} = -1$$

$$\int_{-\infty}^{+\infty} x(1+x^2)^{-2} dx = -1$$

b) $\int_{-\infty}^{+\infty} x(1+x^2)^{-1} dx$

$$= \lim_{a \rightarrow -\infty} \int_a^0 x(1+x^2)^{-1} dx + \lim_{b \rightarrow +\infty} \int_0^b x(1+x^2)^{-1} dx$$

$$= \lim_{a \rightarrow -\infty} \left. \frac{1}{2} \ln |1+x^2| \right|_a^0 + \lim_{b \rightarrow +\infty} \left. \frac{1}{2} \ln |1+x^2| \right|_0^b$$



$$= \lim_{a \rightarrow -\infty} \left(\frac{1}{2} \ln 1 - \frac{1}{2} \ln(1+a^2) \right) + \lim_{b \rightarrow +\infty} \left(\frac{1}{2} \ln 1 - \frac{1}{2} \ln(1+b^2) \right)$$

$$= \lim_{a \rightarrow -\infty} \left(-\frac{1}{2} \ln(1+a^2) \right) + \lim_{b \rightarrow +\infty} \left(-\frac{1}{2} \ln(1+b^2) \right)$$

$$= -\infty + -\infty = -\infty \Rightarrow$$

$$\int_{-\infty}^{+\infty} x(1+x^2)^{-1} dx = -\infty \quad \text{diverge}$$

22. $\int_1^{+\infty} \frac{dx}{x^n}$

c) $n < 0$

$$\int_1^{+\infty} \frac{dx}{x^n} = \lim_{b \rightarrow +\infty} \int_1^b \frac{dx}{x^n}$$

$$= \lim_{b \rightarrow +\infty} \left. \frac{x^{-n+1}}{-n+1} \right|_1^b$$

$$= \lim_{b \rightarrow +\infty} \left(\frac{b^{-n+1}}{-n+1} - \frac{1}{-n+1} \right)$$

$$n < 0 \Rightarrow -n > 0 \Rightarrow \lim_{b \rightarrow +\infty} b^{-n+1} = +\infty$$

r.p.

$$\int_1^{+\infty} \frac{dx}{x^n} = +\infty \quad \text{if } n < 0$$

ii) $n=0$

$$\int_1^{+\infty} \frac{dx}{x^0} = \int_1^{+\infty} dx = \infty \quad \text{diverge}$$

iii) $n > 0$

$$\int_1^{+\infty} \frac{dx}{x^n} = \lim_{b \rightarrow +\infty} \left(\frac{b^{-n+1}}{-n+1} - \frac{1}{-n+1} \right)$$

Moş

$$\lim_{b \rightarrow +\infty} b^{-n+1} = 0 \quad (n > 0)$$

lyo

$$\int_1^{+\infty} \frac{dx}{x^n} = \lim_{b \rightarrow +\infty} \left(\frac{b^{-n+1}}{-n+1} - \frac{1}{-n+1} \right) \\ = -\frac{1}{-n+1}$$

$$\int_1^{+\infty} \frac{dx}{x^n} = \frac{1}{n-1} \quad (n > 1)$$

converge