Lista 19 (parte I)

Exercícios

Encontre o comprimento de arco das respectivas curvas sobre os intervalos dados:

1. $y = \frac{x^3}{3} + \frac{1}{4x} \quad ; \quad x \in [1, 2]$

$$s = \int_1^2 \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$\left(\frac{dy}{dx} = x^2 - \frac{1}{4x^2}\right)$$

$$= \int_1^2 \sqrt{1 + \left(x^2 - \frac{1}{4x^2}\right)^2}\, dx$$

$$= \int_1^2 \sqrt{1 + x^4 - \frac{1}{2} + \frac{1}{16x^4}}\, dx$$

$$= \int_1^2 \sqrt{\frac{1}{2} + x^4 + \frac{1}{16x^4}}\, dx$$

$$= \int_1^2 \sqrt{\left(x^2 + \frac{1}{4x^2}\right)^2}\, dx$$

$$= \int_1^2 \left(x^2 + \frac{1}{4x^2}\right) dx = \left.\frac{x^3}{3} - \frac{1}{4x}\right]_1^2$$

$$\frac{8}{3} - \frac{1}{8} - \frac{1}{3} + \frac{1}{4} = \frac{7}{3} + \frac{1}{8} = \frac{56+3}{24} = \frac{59}{24}$$

$$= \frac{8}{3} - \frac{1}{8} - \left(\frac{1}{3} - \frac{1}{4}\right) = \frac{59}{24}$$

$2.\quad y = \frac{e^x + e^{-x}}{2}, \quad x \in [0, b]$

$$s = \int_0^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$\frac{dy}{dx} = \frac{1}{2}e^x - \frac{e^{-x}}{2}$$

$$\Rightarrow \quad s = \int_0^b \sqrt{1 + \frac{1}{4}(e^x - e^{-x})^2}\, dx$$

$$= \int_0^b \sqrt{1 + \frac{1}{4}e^{2x} - \frac{1}{2} + \frac{1}{4}e^{-2x}}\, dx$$

$$= \int_0^b \sqrt{\frac{1}{2} + \frac{1}{4}e^{2x} + \frac{1}{4}e^{-2x}}\, dx$$

$$= \int_0^b \sqrt{\left(\frac{e^x + e^{-x}}{2}\right)^2}\, dx$$

$$= \int_0^b \frac{e^x + e^{-x}}{2}\, dx$$

$$= \left(\frac{e^x}{2} - \frac{e^{-x}}{2}\right)\Bigg]_0^b = \frac{e^b}{2} - \frac{e^{-b}}{2} - \left(\cancel{\frac{1}{2}} - \cancel{\frac{1}{2}}\right)$$

$$= \frac{e^b - e^{-b}}{2} \quad //$$

3. $y = \frac{x^2}{2} - \frac{\ln x}{4}, \quad x \in [2,3]$

$$s = \int_2^3 \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$\frac{dy}{dx} = x - \frac{1}{4x}$$

$$= \int_2^3 \sqrt{1 + \left(x - \frac{1}{4x}\right)^2}\, dx$$

$$= \int_2^3 \sqrt{1 + x^2 - \frac{1}{2} + \frac{1}{16x^2}}\, dx$$

$$= \int_2^3 \sqrt{\frac{1}{2} + x^2 + \frac{1}{16x^2}}\, dx$$

$$= \int_2^3 \sqrt{\left(x + \frac{1}{4x}\right)^2}\, dx$$

$$= \int_2^3 \left(x + \frac{1}{4x}\right) dx$$

$$= \frac{x^2}{2} + \frac{1}{4}\ln x \Big]_2^3$$

$$= \frac{9}{2} + \frac{1}{4}\ln 3 - \left(\frac{4}{2} + \frac{1}{4}\ln 2\right)$$

$$= \frac{5}{2} + \frac{1}{4}\ln\frac{3}{2} \;//$$

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4. $x = \cos^3 t$, $y = \sin^3 t$, $t \in [0, \frac{\pi}{2}]$

$$s = \int_0^{\pi/2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$$\frac{dx}{dt} = -3\cos^2 t \sin t \quad , \quad \frac{dy}{dt} = 3\sin^2 t \cos t$$

$$= \int_0^{\pi/2} \sqrt{9\cos^4 t \sin^2 t + 9\sin^4 t \cos^2 t}\, dt$$

$$= \int_0^{\pi/2} \sqrt{9\cos^2 t \sin^2 t\,(\cos^2 t + \sin^2 t)}\, dt$$

$$= \int_0^{\pi/2} \sqrt{9\cos^2 t \sin^2 t}\, dt$$

$$= \int_0^{\pi/2} 3\cos t \sin t\, dt$$

$$= \int_0^{\pi/2} \frac{3}{2}\, \underbrace{2\cos t \sin t}\, dt = \frac{3}{2}\int_0^{\pi/2} \sin 2t\, dt$$

$$= \frac{3}{2}\,(-)\frac{\cos 2t}{2}\Big]_0^{\pi/2}$$

$$= \frac{-3}{4}\cos\pi + \frac{3}{4}\cos 0$$

$$= \frac{-3}{4}(-1) + \frac{3}{4}$$

$$= \frac{3}{2} \,//$$

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5. $\begin{cases} x = \cos t + t \sin t \\ y = \sin t - t \cos t \end{cases}$, $t \in [\frac{\pi}{6}, \frac{\pi}{4}]$

$$\frac{dx}{dt} = \cancel{-\sin t} + \cancel{\sin t} + t \cos t = t \cos t$$

$$\frac{dy}{dt} = \cancel{\cos t} - \cancel{\cos t} + t \sin t = t \sin t$$

$$I = \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \sqrt{\left(\frac{dy}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2} \, dt$$

$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \sqrt{t^2 \sin^2 t + t^2 \cos^2 t} \, dt$$

$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} t \, dt = \frac{t^2}{2} \Big]_{\pi/6}^{\pi/4}$$

$$= \frac{1}{2}\left(\frac{\pi}{4}\right)^2 - \frac{1}{2}\left(\frac{\pi}{6}\right)^2$$

$$= \frac{\pi^2}{32} - \frac{\pi^2}{72} =$$

$$= \frac{72\pi^2 - 32\pi^2}{2304} = \frac{40\pi^2}{2304} = \frac{20\pi^2}{1152} = \frac{10\pi^2}{576}$$

$$= \frac{5\pi^2}{288}$$

1
32
×72
164
224
2304

2304 | 2 → 1152 | 2 → 576 | 2 → 288

6. $y = \frac{2}{3}(x^2+1)^{3/2}$, $x \in [0,2]$

$$L = \int_0^2 \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx \quad ; \quad \frac{dy}{dx} = 2x\sqrt{x^2+1}$$

$$= \int_0^2 \sqrt{1 + 4x^2(x^2+1)}\, dx$$

$$= \int_0^2 \sqrt{1 + 4x^4 + 4x^2}\, dx$$

$$= \int_0^2 \sqrt{(2x^2+1)^2}\, dx$$

$$= \int_0^2 2x^2+1\, dx = \left. \frac{2x^3}{3} + x \right]_0^2$$

$$= \frac{2.8}{3} + 2 = \frac{16}{3} + 2$$

$$= \frac{22}{3} \,//$$

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7. $x = \frac{2}{3}(y-1)^{3/2} \quad , \quad y \in [1, 5]$

$$s = \int_1^5 \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy \quad ; \quad \frac{dx}{dy} = \sqrt{y-1}$$

$$= \int_1^5 \sqrt{1 + (y-1)}\, dy \qquad \left(\frac{dx}{dy}\right)^2 = y-1$$

$$= \int_1^5 \sqrt{y}\, dy = \frac{2}{3} y^{3/2} \Big|_1^5$$

$$= \frac{2}{3}\left(5^{3/2} - 1\right)$$

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8. $y = \frac{x^3}{6} + \frac{1}{2x} \quad , \quad x \in [1, 3]$

$$s = \int_1^3 \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx \qquad \frac{dy}{dx} = \frac{3x^2}{6} - \frac{1}{2x^2}$$

$$= \frac{x^2}{2} - \frac{1}{2x^2}$$

$$= \int_1^3 \sqrt{1 + \left(\frac{x^2}{2} - \frac{1}{2x^2}\right)^2}\, dx$$

$$= \int_1^3 \sqrt{1 + \frac{x^4}{4} - \frac{1}{2} + \frac{1}{4x^4}}\, dx$$

$$= \int_1^3 \sqrt{\frac{x^4}{4} + \frac{1}{2} + \frac{1}{4x^4}}\, dx = \int_1^3 \sqrt{\left(\frac{x^2}{2} + \frac{1}{2x^2}\right)^2}\, dx$$

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$$= \int_1^3 \left(\frac{x^2}{2} + \frac{1}{2x^2}\right) dx$$

$$= \frac{x^3}{6} - \frac{1}{2x} \Big]_1^3$$

$$= \frac{27}{6} - \frac{1}{2 \cdot 3} - \left(\frac{1}{6} - \frac{1}{2}\right)$$

$$= \frac{27}{6} - \frac{1}{6} - \frac{1}{6} + \frac{1}{2}$$

$$= \frac{25}{6} + \frac{1}{2/3} = \frac{28}{6} = \frac{14}{3} \,//$$

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9. $x = \frac{y^4}{8} + \frac{1}{4y^2}$, $y \in [1,2]$

$$I = \int_1^2 \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy \quad ; \quad \frac{dx}{dy} = \frac{4y^3}{8} - \frac{1}{2y^3} = \frac{y^3}{2} - \frac{1}{2y^3}$$

$$= \int_1^2 \sqrt{1 + \frac{y^6}{4} - \frac{1}{2} + \frac{1}{4y^6}}\, dy$$

$$= \int_1^2 \sqrt{\frac{y^6}{4} + \frac{1}{2} + \frac{1}{4y^6}}\, dy = \int_1^2 \sqrt{\left(\frac{y^3}{2} + \frac{1}{2y^3}\right)^2} dy$$

$$= \int_1^2 \left(\frac{y^3}{2} + \frac{1}{2y^3}\right) dy = \frac{y^4}{8} - \frac{1}{4y^2}\Big]_1^2$$

$$= \frac{16}{8} - \frac{1}{4 \cdot 4} - \left(\frac{1}{8} - \frac{1}{4}\right) = \frac{33}{16} \,//$$

$2 - \frac{1}{16} - \left(-\frac{1}{8}\right)$

$= 2 - \frac{1}{16} + \frac{1}{8}$

$= \frac{32 - 1 + 2}{16} = \frac{33}{16}$

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10. $8x^2y - 2x^6 = 1$, $(1, \frac{3}{8})$ à $(2, \frac{129}{32})$

$y = \dfrac{1+2x^6}{8x^2}$, $\dfrac{dy}{dx} = -\dfrac{1}{4x^3} + x^3$

$= \dfrac{1}{8x^2} + \dfrac{x^4}{4}$; $\left(\dfrac{dy}{dx}\right)^2 = \dfrac{1}{16x^6} - \dfrac{1}{2} + x^6$

$$S = \int_1^2 \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$= \int_1^2 \sqrt{1 + \frac{1}{16x^6} - \frac{1}{2} + x^6}\, dx$$

$$= \int_1^2 \sqrt{\frac{1}{16x^6} + \frac{1}{2} + x^6}\, dx$$

$$= \int_1^2 \sqrt{\left(\frac{1}{4x^3} + x^3\right)^2}\, dx = \int_1^2 \left(\frac{1}{4x^3} + x^3\right) dx$$

$$= -\frac{1}{8x^2} + \frac{x^4}{4} \Big]_1^2$$

$$= -\frac{1}{8 \cdot 4} + \frac{16}{4} - \left(-\frac{1}{8} + \frac{1}{4}\right)$$

$$= -\frac{1}{32} + 4 - \frac{1}{8} = \frac{-1 + 128 - 4}{32} = \frac{123}{32}$$

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11. $12xy - 4y^4 = 3$; $\left(\frac{7}{12}, 1\right)$ à $\left(\frac{67}{24}, 2\right)$

$$x = \frac{3 + 4y^4}{12y} = \frac{1}{4y} + \frac{y^3}{3}$$

$$\begin{cases} \dfrac{dx}{dy} = -\dfrac{1}{4y^2} + y^2 \\ \left(\dfrac{dx}{dx}\right)^2 = \dfrac{1}{16y^4} - \dfrac{1}{2} + y^4 \end{cases}$$

$$L = \int_1^2 \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

$$= \int_1^2 \sqrt{1 + \frac{1}{16y^4} - \frac{1}{2} + y^4}\, dy$$

$$= \int_1^2 \sqrt{\frac{1}{16y^4} + \frac{1}{2} + y^4}\, dy$$

$$= \int_1^2 \sqrt{\left(\frac{1}{4y^2} + y^2\right)^2}\, dy = \int_1^2 \left(\frac{1}{4y^2} + y^2\right) dy$$

$$= -\frac{1}{4y} + \frac{y^3}{3}\Big]_1^2 = -\frac{1}{8} + \frac{8}{3} - \left(-\frac{1}{4} + \frac{1}{3}\right)$$

$$= -\frac{1}{8} + \frac{1}{4} + \frac{8}{3} - \frac{1}{3}$$

$$= \frac{1}{8} + \frac{7}{3} = \frac{59}{24}$$

12. $y^3 = 8x^2$, $(1,2)$ a $(8,8)$

$$\begin{cases} y = 2x^{2/3} , \quad \dfrac{dy}{dx} = \dfrac{4}{3}x^{-\frac{1}{3}} \end{cases}$$

$$s = \int_1^8 \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$= \int_1^8 \sqrt{1 + \frac{16}{9}x^{-\frac{2}{3}}}\, dx \qquad ?$$

Maneira mais fácil:

$x = t^3 \longrightarrow y = 2t^2$, $t \in [1, 2]$

$\therefore$

$$s = \int_1^2 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$$= \int_1^2 \sqrt{(3t^2)^2 + (4t)^2}\, dt$$

$$= \int_1^2 \sqrt{9t^4 + 16t^2}\, dt = \int_1^2 t\sqrt{9t^2 + 16}\, dt$$

$$= \int_1^2 3t\sqrt{t^2 + \frac{16}{9}}\, dt$$

$$= 3\,\frac{\cancel{2}}{3}\frac{1}{\cancel{2}}\left(t^2 + \frac{16}{9}\right)^{3/2}\Bigg]_1^2$$

$$= \left(4 + \frac{16}{9}\right)^{3/2} - \left(1 + \frac{16}{9}\right)^{3/2} = \boxed{\left(\frac{52}{9}\right)^{3/2} - \left(\frac{25}{9}\right)^{3/2}}$$

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13. $(y-3)^2 = 4(x+2)^3$; $(-1,5)$ à $(2,19)$

$$y-3 = 2(x+2)^{3/2}$$

$$y = 3+2(x+2)^{3/2}$$

$$\frac{dy}{dx} = 3(x+2)^{1/2} \quad ; \quad \left(\frac{dy}{dx}\right)^2 = 9(x+2)$$

$\therefore$

$$l = \int_{-1}^{2} \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$$

$$= \int_{-1}^{2} \sqrt{1+9(x+2)}\, dx$$

$$= \int_{-1}^{2} \sqrt{1+9x+18}\, dx$$

$$= \int_{-1}^{2} \sqrt{19+9x}\, dx$$

$$= \frac{2}{3}\frac{1}{9}(19+9x)^{3/2}\Big]_{-1}^{2}$$

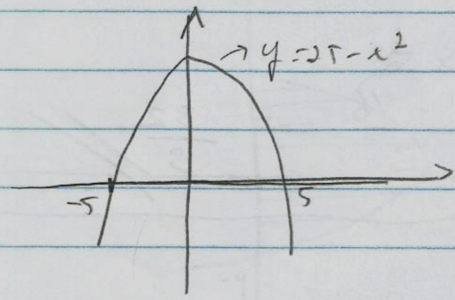
$$= \frac{2}{27}\left[(19+18)^{3/2} - (19-9)^{3/2}\right]$$

$$= \frac{2}{27}\left[37^{3/2} - 10^{3/2}\right] \quad //$$

Lista 9 (Parte II)

Área de uma região plana

14. $y=0$, $y=25-x^2$



$$A=\int_{-5}^{5}(25-x^2)\,dx$$

$$=25x-\frac{x^3}{3}\Big]_{-5}^{5}$$

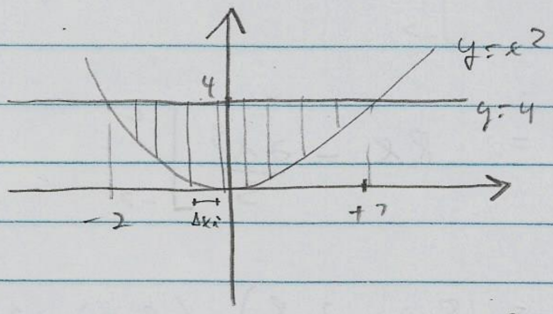
$$=\left(25\cdot 5-\frac{5^3}{3}\right)-\left(25(-5)-\frac{(-5)^3}{3}\right)$$

$$=125-\frac{125}{3}+125-\frac{125}{3}$$

$$=250-\frac{250}{3}$$

$$=\frac{750-250}{3}=\frac{500}{3}$$

15. $y=x^2$, $y=4$



$x^2=4 \Rightarrow x=\pm 2$

$$A=\lim_{\Delta x_i\to 0}\sum_{i=1}^{\infty}(4-x_i^2)\Delta x_i=\int_{-2}^{2}(4-x^2)\,dx$$

$$=4x-\frac{x^3}{3}\Big]_{-2}^{2}$$

$$=\left(4\cdot 2-\frac{8}{3}\right)-\left(4\cdot(-2)-\frac{(-2)^3}{3}\right)$$

$=$

$$= \left(8 - \frac{8}{3}\right) - \left(-8 + \frac{8}{3}\right)$$

$$= 8 - \frac{8}{3} + 8 - \frac{8}{3}$$

$$= 16 - \frac{16}{3} = \frac{48 - 16}{3} = \frac{32}{3}$$

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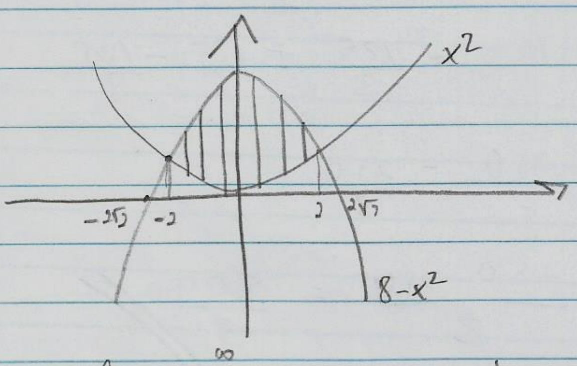
16. $y = x^2$, $y = 8 - x^2$



$$\left\{ \begin{array}{l} x^2 = 8 - x^2 \Rightarrow 2x^2 = 8 \\ x^2 = 4 \\ x = \pm 2 \end{array} \right.$$

$$A = \lim_{\Delta x \to 0} \sum_{i=1}^{\infty} \left[(8 - x_i^2) - x_i^2 \right] \Delta x = \int_{-2}^{2} (8 - x^2 - x^2)\, dx$$

$$= \int_{-2}^{2} (8 - 2x^2)\, dx$$

$$= 8x - \frac{2x^3}{3} \Big]_{-2}^{2}$$

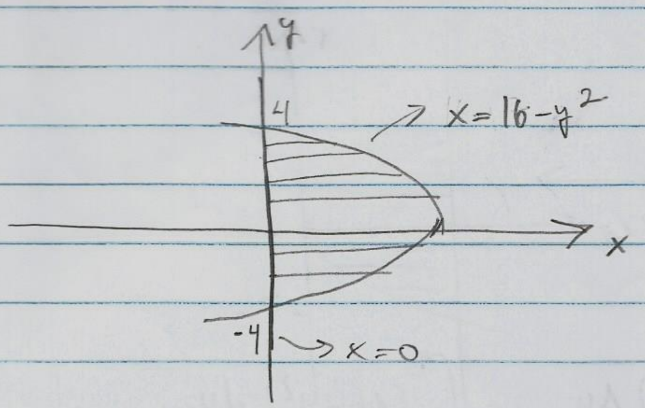
$$= \left(8 \times 2 - \frac{2 \times 8}{3}\right) - \left(8(-2) - \frac{2(-2)^3}{3}\right)$$

$$= 16 - \frac{16}{3} + 16 - \frac{16}{3}$$

$$= 32 - \frac{32}{3} = \frac{96 - 32}{3}$$

$$= \frac{64}{3}$$

17. $x = 0$, $x = 16 - y^2$



$$A = \lim_{\Delta y \to 0} \sum_{i=1}^{+\infty} (16 - y^2 - 0)\,\Delta y = \int_{-4}^{4} (16 - y^2)\,dy$$

$$= 16y - \frac{y^3}{3} \Big]_{-4}^{4}$$

$$= 16 \times 4 - \frac{4^3}{3} - \left(16(-4) - \frac{(-4)^3}{3}\right)$$

$$= 64 - \frac{64}{3} - \left(-64 + \frac{64}{3}\right)$$

$$= 64 - \frac{64}{3} + 64 - \frac{64}{3}$$

$$= 128 - \frac{128}{3}$$

$$= \frac{384 - 128}{3} = \frac{256}{3}$$ //

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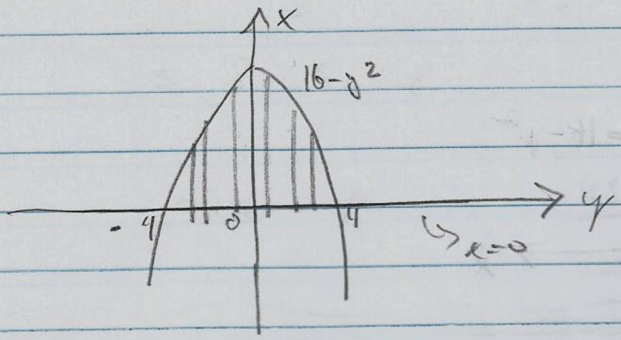
Ob 1 $x = 0$ $x = 16 - y^2$

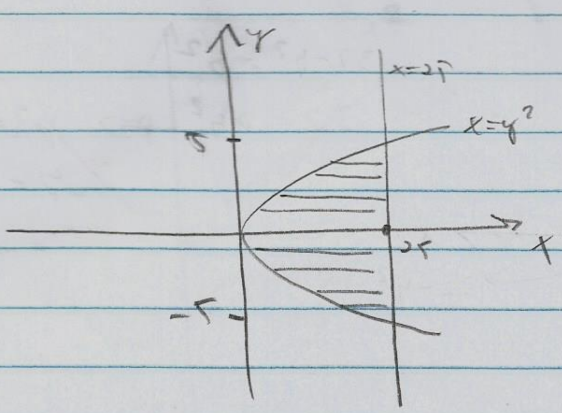


$$A = \lim_{\Delta y \to 0} \sum_{i=1}^{\infty} (16 - y^2 - 0)\,\Delta y = \int_{-4}^{4} (16 - y^2)\,dy$$

$$= 16y - \frac{y^3}{3} \Bigg]_{-4}^{4}$$

$$= \ldots \quad \frac{256}{3}$$ (Mesma resposta!!)

18. $x = y^2, \quad x = 25$



$y^2 = 25$

$y = \pm 5$

$$A = \int_{-5}^{5} (25 - y^2)\, dy = 25y - \frac{y^3}{3} \Big]_{-5}^{5}$$

$$= \left(25(5) - \frac{125}{3}\right) - \left(25(-5) - \frac{(-5)^3}{3}\right)$$

$$= 125 - \frac{125}{3} + 125 - \frac{125}{3}$$

$$= 250 - \frac{250}{3}$$

$$= \frac{750 - 250}{3} = \frac{500}{3}$$

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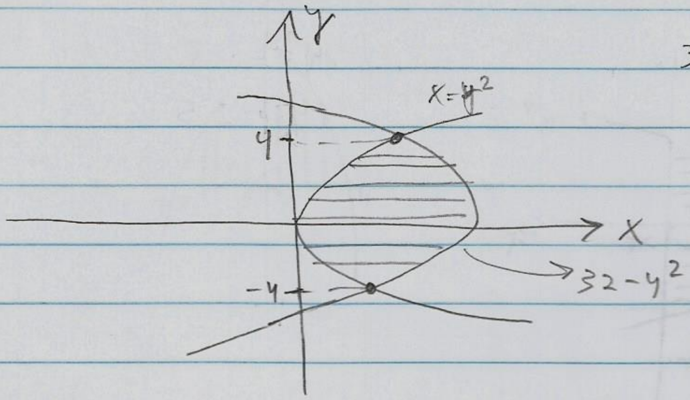
19. $x = y^2$, $x = 32 - y^2$



$32 - y^2 = y^2$

$32 = 2y^2 \Rightarrow y^2 = 16$

$y = \pm 4$

$$A = \lim_{\Delta y \to 0} \sum_{i=1}^{\infty} (32 - y_i^2 - y_i^2)\,\Delta y$$

$$= \int_{-4}^{4} (32 - y^2 - y^2)\,dy$$

$$= \int_{-4}^{4} (32 - 2y^2)\,dy = 32y - \frac{2}{3}y^3 \Big]_{-4}^{4}$$

$$= \left(32 \times 4 - \frac{2}{3}64\right) - \left(32(-4) - \frac{2}{3}(-64)\right)$$

$$= 128 - \frac{128}{3} + 128 - \frac{128}{3}$$

$$= 256 - \frac{256}{3}$$

$$= \frac{768 - 256}{3} = \frac{512}{3}$$

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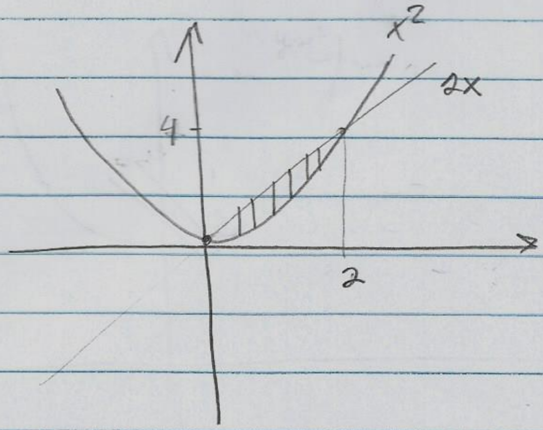
20. $y = x^2$, $y = 2x$

$x^2 = 2x$

$x(x-2) = 0$

$x = 0$, $x = 2$



$$A = \lim_{\Delta x \to 0} \sum_{i=1}^{\infty} (2x_i - x_i^2)\,\Delta x$$

$$= \int_0^2 (2x - x^2)\,dx$$

$$= \frac{\cancel{2}x^2}{\cancel{2}} - \frac{x^3}{3}\Big]_0^2 = 4 - \frac{8}{3}$$

$$= \frac{12-8}{3} = \frac{4}{3}$$

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21. $y = x^2$, $x = y^2$

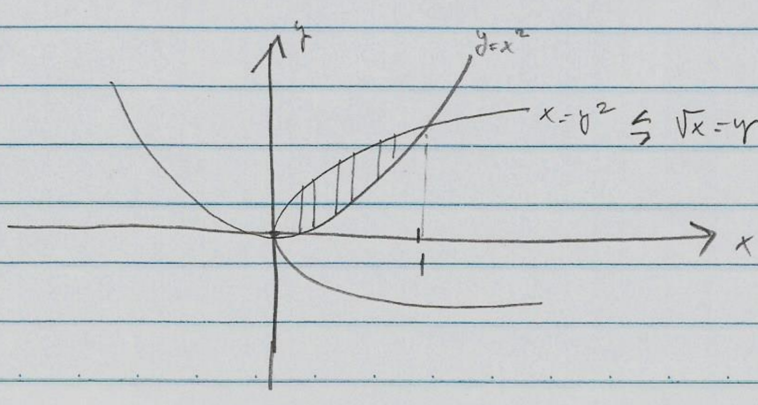
$x^2 = \sqrt{x}$

$\Leftrightarrow$

$x^4 = x$

$x(x^3 - 1) = 0$

$x = 0$ $x = 1$.



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$$A = \lim_{\Delta x \to 0} \sum_{i=1}^{\infty} (\sqrt{x_i} - x_i^2)\, \Delta x$$

$$= \int_0^1 (\sqrt{x} - x^2)\, dx$$

$$= \frac{2}{3}x^{3/2} - \frac{x^3}{3}\Big]_0^1 = \frac{2}{3} - \frac{1}{3} = \frac{1}{3}$$ //

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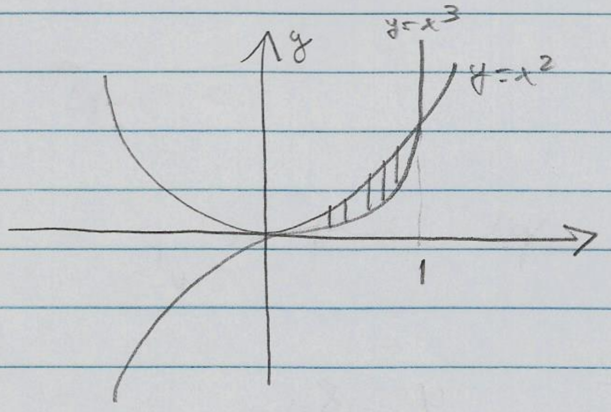
22. $y = x^2$, $y = x^3$



$x^2 = x^3$

$\Leftrightarrow$

$x^2(1-x) = 0$

$x = 0$, $x = 1$

$$A = \int_0^1 (x^2 - x^3)\, dx = \frac{x^3}{3} - \frac{x^4}{4}\Big]_0^1$$

$$= \frac{1}{3} - \frac{1}{4} = \frac{4-3}{12} = \frac{1}{12}$$ //

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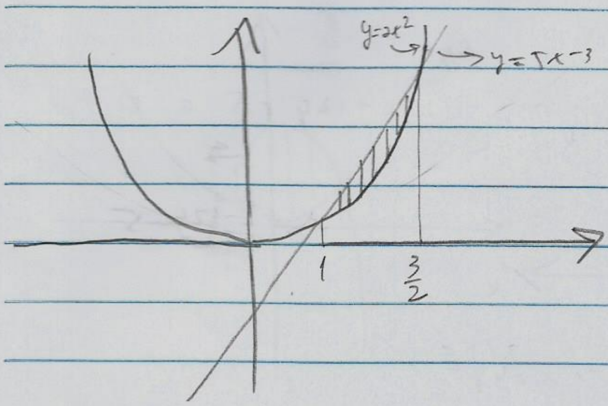
23. $y = 2x^2, \quad y = 5x - 3$



$2x^2 = 5x - 3$

$\Leftrightarrow$

$2x^2 - 5x + 3 = 0$

$x = 5 \pm \sqrt{25 - 24} \,/4$

$= \dfrac{5 \pm 1}{4} \nearrow \frac{6}{4} = \frac{3}{2}$, $\searrow 1$

$$A = \int_1^{3/2} (5x - 3 - 2x^2)\,dx$$

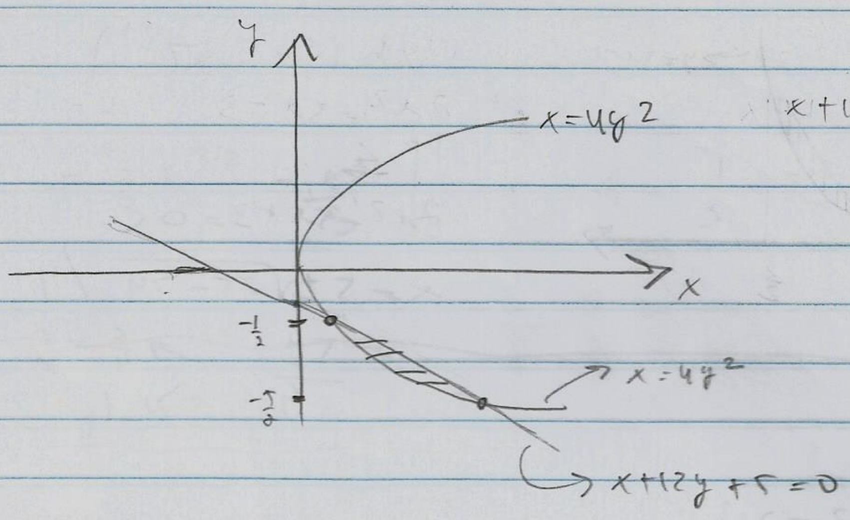
$$= \left(\frac{5x^2}{2} - 3x - \frac{2x^3}{3}\right)\Bigg]_1^{3/2}$$

$$= \frac{5}{2}\cdot\frac{9}{4} - 3\cdot\frac{3}{2} - \frac{\cancel{2}}{\cancel{3}}\,\frac{\overset{9}{\cancel{27}}}{\underset{4}{\cancel{8}}} - \frac{5}{2} + 3 + \frac{2}{3}$$

$$= \frac{45}{8} - \frac{9}{2} - \frac{9}{4} - \frac{5}{2} + 3 + \frac{2}{3} = \frac{1}{24}$$

/ /

24. $x = 4y^2$, $x + 12y + 5 = 0$



$x + 12y + 5 = 0$

$\Leftrightarrow$

$x = -12y - 5$

$4y^2 = -12y - 5$
$\Leftrightarrow$
$4y^2 + 12y + 5 = 0$

$y = -12 \pm \sqrt{144 - 80} / 8$

$= -12 \pm \sqrt{64} / 8$

$\therefore$

$\underline{\underline{y}} = \dfrac{-12 \pm 8}{8}$ $\nearrow \frac{-4}{8} = -\frac{1}{2}$; $\searrow \frac{-20}{8} = -\frac{5}{2}$

$A = \lim_{\Delta y \to 0} \sum_{i=1}^{\infty} (-12y - 5 - 4y^2)\Delta y$

$= \int_{-\frac{5}{2}}^{-\frac{1}{2}} (-4y^2 - 12y - 5)\, dy$

$= -\dfrac{4y^3}{3} - \dfrac{\cancel{12}^{6} y^2}{\cancel{2}} - 5y \Big]_{-5/2}^{-1/2} =$

$= -\frac{4}{3}(-\frac{1}{2})^3 - 6(-\frac{1}{2})^2 - 5(-\frac{1}{2}) - \left(-\frac{4}{3}(-\frac{5}{2})^3 - 6(-\frac{5}{2})^2 - 5(-\frac{5}{2})\right)$

$= -\frac{4}{3}\frac{(-1)}{8} - 6\frac{1}{4} + \frac{5}{2} - \left(+\frac{500}{24} - \frac{150}{4} + \frac{25}{2}\right) = 16/3$ //

$\underbrace{\frac{1}{6} - \frac{3}{2} + \frac{5}{2}}_{1} - \frac{500}{24} + \frac{150}{4/6} - \frac{25}{2} =$

$= \dfrac{4 + 24 - 500 + 900 - 300}{24}$

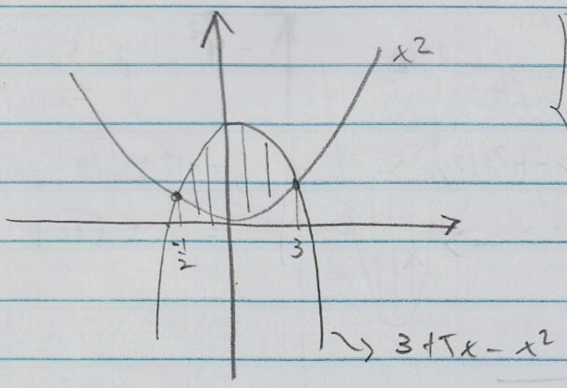
$= \dfrac{4 + 24 + 100}{24}$

$= \frac{128}{24} = \frac{64}{12} = \frac{32}{6} = \frac{16}{3}$

25
×12
150
25
300

tilibra

25. $y = x^2$, $y = 3+5x-x^2$



$$-x^2+5x+3=0$$
$$x = -5 \pm \sqrt{25+12}\ /-2$$
$$= \frac{-5 \pm \sqrt{37}}{-2}$$

$$x^2 = 3+5x-x^2$$
$$2x^2-5x-3=0$$
$$x = 5 \pm \sqrt{25+24}\ /4$$
$$= \frac{5 \pm 7}{4} \nearrow 3, \searrow -\frac{1}{2}$$

$$A = \int_{-\frac{1}{2}}^{3} (3+5x-x^2-x^2)\,dx$$

$$= \int_{-\frac{1}{2}}^{3} (3+5x-2x^2)\,dx$$

$$= \left(3x + \frac{5x^2}{2} - \frac{2x^3}{3}\right)\Bigg|_{-\frac{1}{2}}^{3}$$

$$= 3.3 + \frac{5.9}{2} - 2.\frac{\cancel{27}^{9}}{\cancel{3}} - \left(3.\left(-\frac{1}{2}\right) + \frac{5}{2}\left(-\frac{1}{2}\right)^2 - \frac{2}{3}\left(-\frac{1}{2}\right)^3\right)$$

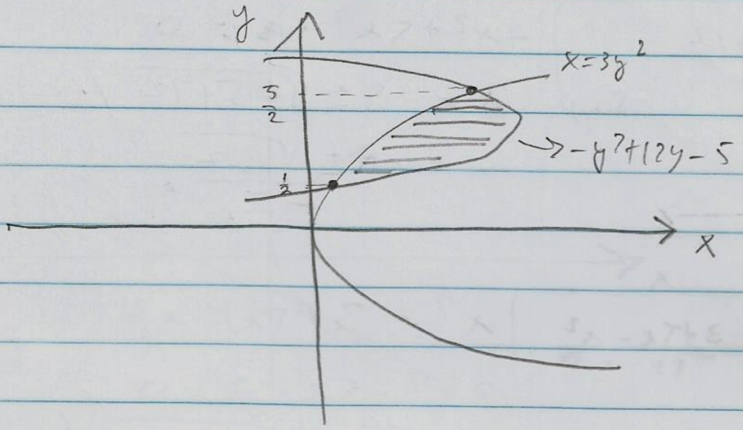
$$= 9 + \frac{45}{2} - 18 + \frac{3}{2} - \frac{5}{8} - \frac{1}{12}$$

$$= -9 + 24 - \frac{15+2}{24}$$

$$= 15 - \frac{17}{24} = \frac{360-17}{24} = \frac{343}{24}$$ //

/ /

26. $x = 3y^2$, $x = -y^2 + 12y - 5$



$-y^2 + 12y - 5 = 0$
$\Leftrightarrow$
$y = -12 \pm \sqrt{144 - 20} \, / -2$
$= -12 \pm \sqrt{124} \, / -2$

$-y^2 + 12y - 5 = 3y^2$
$\Leftrightarrow$
$4y^2 - 12y + 5 = 0$
$y = 12 \pm \sqrt{144 - 80} \, / 8$
$= 12 \pm \sqrt{64} \, / 8$
$= \frac{12 \pm 8}{8} \nearrow \frac{5}{2}$, $\searrow \frac{1}{2}$

$$A = \int_{\frac{1}{2}}^{\frac{5}{2}} (-y^2 + 12y - 5 - 3y^2) \, dy$$

$$= \int_{\frac{1}{2}}^{\frac{5}{2}} (-4y^2 + 12y - 5) \, dy$$

$$= \frac{-4y^3}{3} + \frac{\overset{6}{\cancel{12}}y^2}{\cancel{2}} - 5y \Big]_{\frac{1}{2}}^{\frac{5}{2}}$$

$$= \frac{-4}{3}\left(\frac{5}{2}\right)^3 + 6\left(\frac{5}{2}\right)^2 - 5.\frac{5}{2} - \left(\frac{-4}{3}\left(\frac{1}{2}\right)^3 + 6\left(\frac{1}{2}\right)^2 - 5\left(\frac{1}{2}\right)\right)$$

$$= \frac{-\cancel{4}}{3} \cdot \frac{125}{\cancel{8}2} + \overset{3}{\cancel{6}} \cdot \frac{25}{\cancel{4}2} - \frac{25}{2} + \frac{1}{6} - \frac{3}{2} + \frac{5}{2}$$

$$= -\frac{125}{6} + \frac{75}{2} - \frac{25}{2} + \frac{1}{6} + 1$$

$$= -\frac{124}{6} + \frac{50}{2} + 1$$

$$= -\frac{62}{3} + 25 + 1$$

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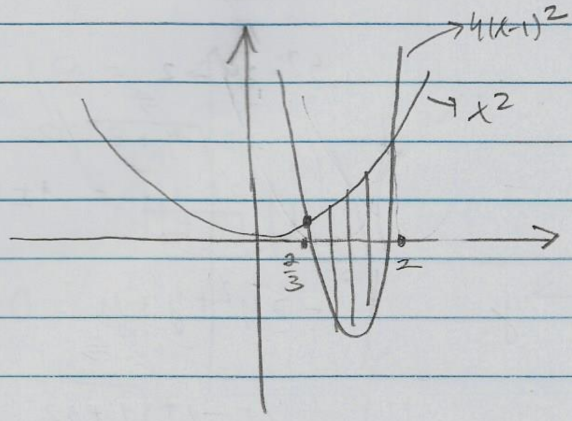
$$= -\frac{62}{3} + 26 = \frac{-62 + 78}{3} = \frac{16}{3}$$

27. $y = x^2 \quad , \quad y = 4(x-1)^2$



$x^2 = 4x^2 - 8x + 4$

$\Leftrightarrow$

$3x^2 - 8x + 4 = 0$

$x = 8 \pm \sqrt{64-48}\ /6$

$= 8 \pm \sqrt{16}\ /6$

$= \frac{8 \pm 4}{6} \begin{cases} 2 \\ \frac{2}{3} \end{cases}$

$$A = \int_{\frac{2}{3}}^{2} \left(x^2 - 4(x-1)^2\right) dx$$

$$= \int_{\frac{2}{3}}^{2} (-3x^2 + 8x - 4)\ dx$$

$$= \frac{-\cancel{3}x^3}{\cancel{3}} + \frac{8x^2}{2} - 4x \Big]_{\frac{2}{3}}^{2}$$

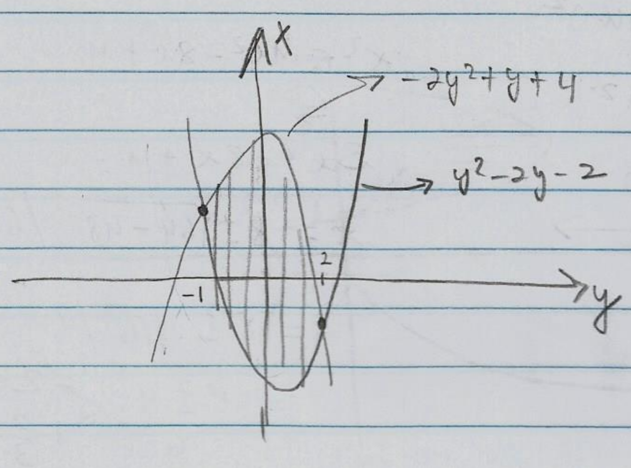
$$= -x^3 + 4x^2 - 4x \Big]_{\frac{2}{3}}^{2}$$

$$= -8 + 4.4 - 4.2 - \left(-\left(\frac{2}{3}\right)^3 + 4\left(\frac{2}{3}\right)^2 - 4.\frac{2}{3}\right)$$

$$= \cancel{-8} + \cancel{16} \cancel{-8} - \left(-\frac{8}{27} + \frac{16}{9} - \frac{8}{3}\right)$$

$$= \frac{8}{27} - \frac{16}{9_{/3}} + \frac{8}{3_{/9}} = \frac{8-48+72}{27} = \frac{32}{27} \ //$$

$28.\quad x = y^2 - 2y - 2 \quad , \quad x = -2y^2 + y + 4$



$y^2 - 2y - 2 = 0$
$\Leftrightarrow$
$y = 2 \pm \sqrt{4+8} \,/2$
$= \dfrac{2 \pm \sqrt{12}}{2} = 1 \pm \sqrt{3}$

$-2y^2 + y + 4 = 0$
$\Leftrightarrow$
$y = -1 \pm \sqrt{1+32} \,/-4$
$= \dfrac{-1 \pm \sqrt{33}}{-4} = \dfrac{-1 \pm 6}{-4}$ $\nearrow \frac{7}{4}$ $\searrow -\frac{5}{4}$

$3 \,|\, \frac{4}{1}$

$\frac{7}{30} \,|\, \frac{4}{1.75}$

$y^2 - 2y - 2 = -2y^2 + y + 4$
$\Leftrightarrow$
$3y^2 - 3y - 6 = 0$
$y^2 - y - 2 = 0$
$\therefore$
$y = 1 \pm \sqrt{1+8} \,/2$
$= \dfrac{1 \pm 3}{2}$ $\nearrow 2$ $\searrow -1$

$$A = \int_{-1}^{2} \left(-2y^2 + y + 4 - (y^2 - 2y - 2) \right) dy$$

$$= \int_{-1}^{2} \left(-3y^2 + 3y + 6 \right) dy$$

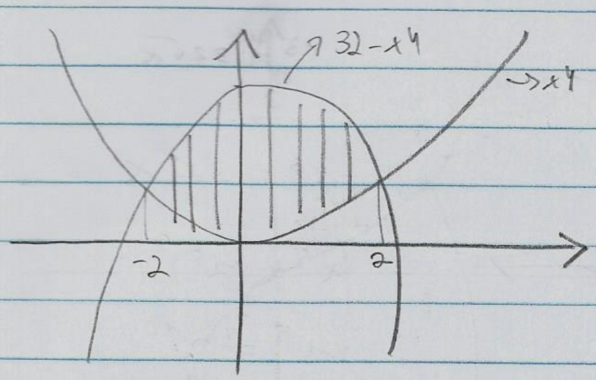
$$= -\frac{\cancel{3}y^3}{\cancel{3}} + \frac{3y^2}{2} + 6y \Big]_{-1}^{2}$$

$$= -8 + \frac{3 \cdot \cancel{4}}{\cancel{2}} + 6 \cdot 2 - \left(-(-1)^3 + \frac{3}{2}(-1)^2 + 6(-1) \right)$$

$$= -8 + 6 + 12 - \left(+1 + \frac{3}{2} - 6 \right)$$

$$= 10 - \left(-5 + \frac{3}{2} \right) = 15 - \frac{3}{2} = \frac{30-3}{2} = \frac{27}{2} \,//$$

29. $y = x^4$, $y = 32 - x^4$



$$x^4 = 32 - x^4$$
$$\Leftrightarrow$$
$$2x^4 = 32$$
$$x^4 = 16$$
$$\Leftrightarrow$$
$$x = \pm\sqrt[4]{16} = \pm 2$$

$$A = \int_{-2}^{2} (32 - x^4 - x^4)\, dx =$$

$$= \int_{-2}^{2} (32 - 2x^4)\, dx$$

$$= 32x - \frac{2x^5}{5} \Big]_{-2}^{2}$$

$$= 32 \cdot 2 - \frac{2}{5} \cdot 2^5 - \left(32(-2) - \frac{2}{5}(-2)^5\right)$$

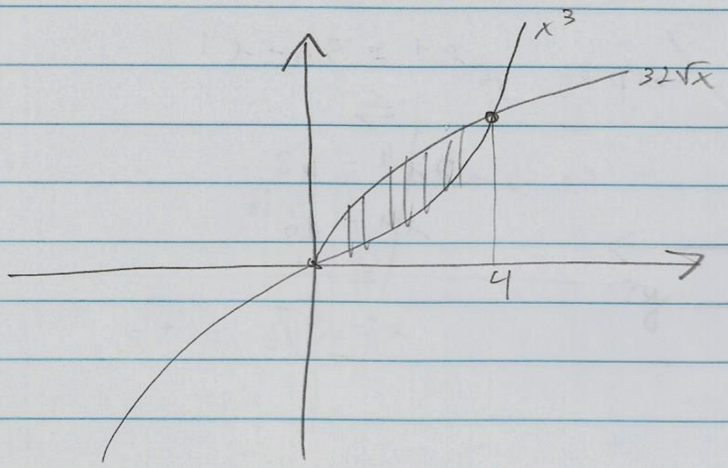
$$= 64 - \frac{64}{5} + 64 - \frac{64}{5}$$

$$= 128 - \frac{128}{5}$$

$$= 128\left(1 - \frac{1}{5}\right) = 128\left(\frac{4}{5}\right)$$

$$= \frac{512}{5}$$ //

30. $y = x^3$, $y = 32\sqrt{x}$



$x^3 = 32\sqrt{x}$

$\Leftrightarrow$

$x^6 = 32^2 x$

$x^5 = (2^5)^2$

$x^5 = 2^{10}$

$\Leftrightarrow$

$\begin{cases} x = 4 \\ x = 0 \end{cases}$

$$A = \int_0^4 (32\sqrt{x} - x^3)\, dx$$

$$= 32 \cdot \frac{2}{3} x^{3/2} - \frac{x^4}{4} \Big]_0^4$$

$$= 32 \cdot \frac{2}{3} 4^{3/2} - \frac{4^4}{4}$$

$$= 32 \cdot \frac{2}{3} 2^3 - 4^3$$

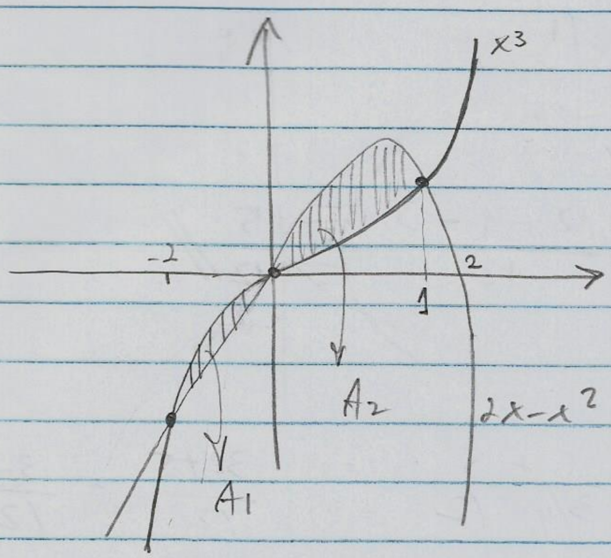
$$= \frac{64 \cdot 8}{3} - 64 = 64\left(\frac{8}{3} - 1\right)$$

$$= 64\left(\frac{5}{3}\right)$$

$$= \frac{320}{3}$$

1 . 1

31. $y = x^3$, $y = 2x - x^2$



$y = 2x - x^2$
$\Leftarrow$
$2x - x^2 = 0$
$x(2-x) = 0$
$\Rightarrow x=0 , x=2$

$x^3 = 2x - x^2$

$x^3 + x^2 - 2x = 0$

$x(x^2 + x - 2) = 0$

$\Rightarrow$ $x=0$

$x^2 + x - 2 = 0 \Leftarrow x = \frac{-1 \pm \sqrt{1+8}}{2}$

$x = \frac{-1 \pm 3}{2}$ $\nearrow -2$ $\searrow 1$

$A = A_1 + A_2$

$A_1 = \int_{-2}^{0}\left(x^3 - (2x - x^2)\right) dx = \int_{-2}^{0}(x^3 - 2x + x^2)\, dx$

$= \frac{x^4}{4} - \cancel{\frac{2x^2}{2}} + \frac{x^3}{3} \Big]_{-2}^{0}$

$= -\left(\frac{1}{4}(-2)^4 - (-2)^2 + \frac{(-2)^3}{3}\right)$

$= -\left(\cancel{\frac{16}{4}} - \cancel{4} - \frac{8}{3}\right)$

$= +\frac{8}{3}$

$$A_2 = \int_0^1 (2x - x^2 - x^3)\,dx$$

$$= \frac{\cancel{2}x^2}{\cancel{2}} - \frac{x^3}{3} - \frac{x^4}{4}\Big]_0^1$$

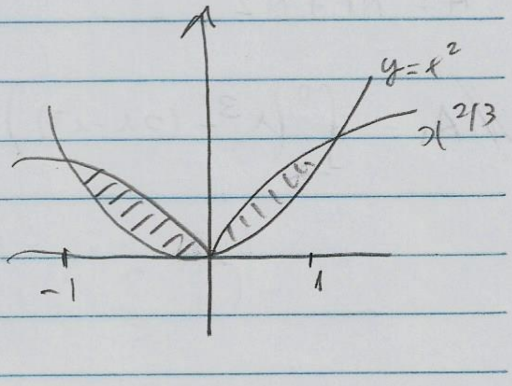
$$= 1 - \frac{1}{3} - \frac{1}{4} = \frac{12-4-3}{12} = \frac{5}{12}$$

$\therefore$

$$A = A_1 + A_2 = \frac{8}{3/4} + \frac{5}{12} = \frac{32+5}{12} = \frac{37}{12}$$

$\therefore$ $\boxed{A = \frac{37}{12}}$

32. $y = x^2$, $y = x^{2/3}$



$\frac{2}{3} + 1 = \frac{5}{3}$

$$A = \int_{-1}^{1} \left(x^{2/3} - x^2\right) dx = \frac{3}{5}x^{\frac{5}{3}} - \frac{x^3}{3}\Big]_{-1}^{1} = \frac{3}{5} - \frac{1}{3} - \left(\frac{3}{5}(-1) - \frac{1}{3}(-1)\right)$$

$$= \frac{3}{5} - \frac{1}{3} + \frac{3}{5} - \frac{1}{3}$$

$$= \frac{6}{5} - \frac{2}{3} = \frac{18-10}{15} = \frac{8}{15}$$

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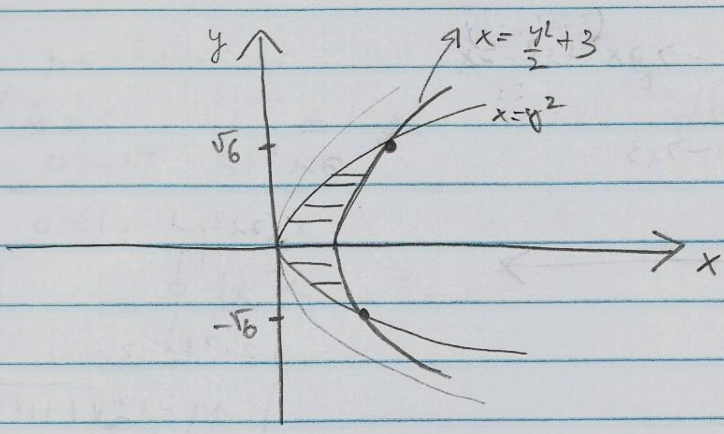
33. $y^2 = x \quad , \quad y^2 = 2(x-3)$



$\left\{ \begin{array}{l} y^2 = 2x - 6 \\ x = \frac{y^2}{2} + 3 \end{array} \right.$

$y^2 = \frac{y^2}{2} + 3$

$\frac{y^2}{2} = 3$

$y^2 = 6$

$y = \pm\sqrt{6}$

$$A = \int_{-\sqrt{6}}^{\sqrt{6}} \left(\frac{y^2}{2} + 3 - y^2 \right) dy$$

$$= \int_{-\sqrt{6}}^{\sqrt{6}} \left(-\frac{y^2}{2} + 3 \right) dy = -\frac{y^3}{6} + 3y \Big]_{-\sqrt{6}}^{\sqrt{6}}$$

$$= -\frac{\cancel{6}\sqrt{6}}{\cancel{6}} + 3\sqrt{6} - \left(-\frac{(-\sqrt{6})^3}{6} + 3(-\sqrt{6}) \right)$$

$$= -\sqrt{6} + 3\sqrt{6} - \left(\frac{\cancel{6}\sqrt{6}}{\cancel{6}} - 3\sqrt{6} \right)$$

$$= -2\sqrt{6} + 6\sqrt{6}$$

$$= 4\sqrt{6}$$

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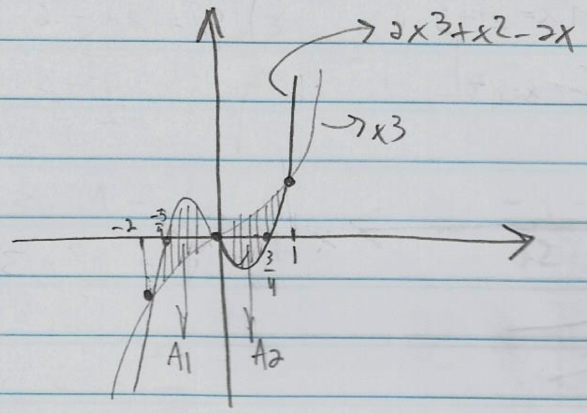
34. $y = x^3, \quad y = 2x^3 + x^2 - 2x$



$y = 2x^3 + x^2 - 2x$

$\Leftrightarrow$

$2x^3 + x^2 - 2x = 0$

$x(2x^2 + x - 2) = 0$

$\Rightarrow x = 0$

$2x^2 + x - 2 = 0$

$x = -1 \pm \sqrt{1+16} / 4$

$= \frac{-1 \pm \sqrt{17}}{4} \approx \frac{-1 \pm 4}{4} \to -\frac{5}{4}, \ \frac{3}{4}$

$x^3 = 2x^3 + x^2 - 2x$

$\Leftrightarrow$

$x^3 + x^2 - 2x = 0$

$x(x^2 + x - 2) = 0 \Rightarrow x = 0$, $\quad x^2 + x - 2 = 0$

$\Leftrightarrow$

$x = -1 \pm \sqrt{1+8} / 2$

$= \frac{-1 \pm 3}{2} \to -2, \ 1$

$A = A_1 + A_2$

$$A_1 = \int_{-2}^{0} (2x^3 + x^2 - 2x - x^3)\, dx = \int_{-2}^{0} (x^3 + x^2 - 2x)\, dx$$

$$= \frac{x^4}{4} + \frac{x^3}{3} - \frac{\cancel{2}x^2}{\cancel{2}} \Big]_{-2}^{0}$$

$$= -\left[\frac{(-2)^4}{4} + \frac{(-2)^3}{3} - (-2)^2\right]$$

$$= -\left[\cancel{4} - \frac{8}{3} - \cancel{4}\right] = +\frac{8}{3}$$

34. (continuação)

$$A_2 = \int_0^1 \left[x^3 - (2x^3 + x^2 - 2x)\right] dx$$

$$= \int_0^1 (x^3 - 2x^3 - x^2 + 2x)\, dx$$

$$= \int_0^1 (-x^3 - x^2 + 2x)\, dx$$

$$= -\frac{x^4}{4} - \frac{x^3}{3} + \frac{\cancel{2}x^2}{\cancel{2}} \Bigg|_0^1$$

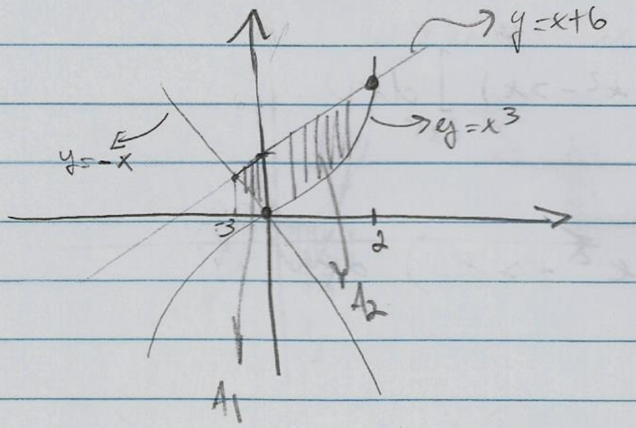
$$= -\frac{1}{4}_{/3} - \frac{1}{3}_{/4} + \frac{1}{1}_{/12} = \frac{-3-4+12}{12} = \frac{5}{12}$$

$\therefore$

$$A = A_1 + A_2 = \frac{8}{3}_{/4} + \frac{5}{12} = \frac{32+5}{12} = \frac{37}{12}$$

$\therefore$ $\boxed{A = \frac{37}{12}}$

35. $y = x^3$, $x+y=0$, $y = x+6$



$\begin{cases} y = x^3 \;;\; y = -x \\ x^3 = -x \Leftrightarrow x(x^2+1)=0 \\ x=0 \end{cases}$

$\begin{cases} y = x^3 \;;\; y = x+6 \\ \Leftrightarrow \\ x^3 = x+6 \\ x^3 - x - 6 = 0 \\ \underline{x=2} \text{ é raiz} \end{cases}$

$$\begin{array}{l|l} x^3 - x - 6 & x-2 \\ \hline -x^3+2x^2 & x^2+2x+3 \\ 2x^2 - x - 6 & \\ -2x^2+4x & \\ 3x-6 & \\ -3x+6 & \\ 0\;0 & \end{array}$$

$\therefore$

$x^3 - x - 6 = 0$

$(x-2)(x^2+2x+3) = 0$

$\begin{cases} x^2+2x+3 = 0 \rightarrow \text{não há raízes} \\ x = -2 \pm \sqrt{4-12} \,/2 \end{cases}$

$y = -x$, $y = x+6$

$-x = x+6 \Rightarrow -2x = 6 \Rightarrow x = -3$

$A = A_1 + A_2$

$A_1 = \int_{-3}^{0} (x+6-(-x))\, dx = \int_{-3}^{0} (2x+6)\, dx = \frac{\cancel{2}x^2}{\cancel{2}} + 6x \Big]_{-3}^{0}$

$= -\left((-3)^2 + 6(-3)\right)$

$= -(9-18) = 9$ //

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$$// A_2 = \int_0^2 (x+6-x^3)\, dx$$

$$= \frac{x^2}{2} + 6x - \frac{x^4}{4} \Big]_0^2$$

$$= \frac{4}{2} + 6 \cdot 2 - \frac{2^4}{4}$$

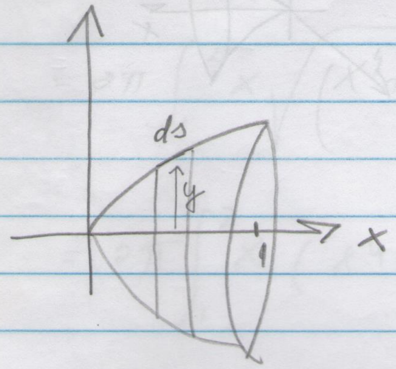
$$= 2 + 12 - 4 = 10 //$$

$\therefore \quad A = A_1 + A_2 = 9 + 10 = 19$

$\therefore \quad \boxed{A = 19}$

Lista 9 (Parte III)

36. $y = \sqrt{x}$; $0 \leq x \leq 1$ eixo x



$$S = \int_0^1 2\pi y(x) \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{x}}$$

$$S = \int_0^1 2\pi \sqrt{x} \sqrt{1 + \frac{1}{4x}}\, dx$$

$$= \int_0^1 2\pi \sqrt{x} \sqrt{\frac{1+4x}{4x}}\, dx$$

$$= \int_0^1 \cancel{2}\pi \cancel{\sqrt{x}} \frac{\sqrt{1+4x}}{\cancel{2\sqrt{x}}}\, dx$$

$$= \int_0^1 \pi \sqrt{1+4x}\, dx = \pi \frac{2}{3} \cdot \frac{1}{4} (1+4x)^{3/2} \Big]_0^1$$

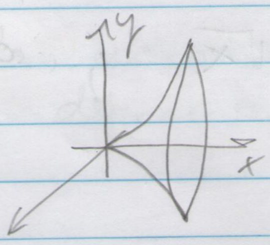
$$= \frac{\pi}{6}(1+4)^{3/2} - \frac{\pi}{6}$$

$$= \frac{\pi}{6}\left(5^{3/2} - 1\right)$$

/ /

37. $y = x^3$; $1 \leq x \leq 2$ eixo x

$\frac{dy}{dx} = 3x^2$



$$S = \int_1^2 2\pi\, y(x) \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$= \int_1^2 2\pi x^3 \sqrt{1 + 9x^4}\, dx$$

$$= 2\pi \frac{\cancel{2}}{3} \frac{1}{\cancel{36}\,9} (1 + 9x^4)^{3/2} \Big]_1^2$$

16
144

$$= \frac{\pi}{27} \left\{ (1 + 9 \cdot 16)^{3/2} - (1 + 9)^{3/2} \right\}$$

$$= \frac{\pi}{27} \left((145)^{3/2} - 10^{3/2} \right)$$

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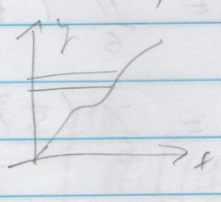
38. $y = \frac{x^5}{5} + \frac{1}{12x^3}$; $1 \leq x \leq 2$, eixo y



$\frac{dy}{dx} = x^4 - \frac{1}{4x^4}$

$$S = \int_1^2 2\pi x \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$= 2\pi \int_1^2 x \sqrt{1 + x^8 - \frac{1}{2} + \frac{1}{16x^8}}\, dx$$

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38. cont.

$$= 2\pi \int_1^2 x \sqrt{x^8 + \frac{1}{2} + \frac{1}{16x^8}}\, dx$$

$$= 2\pi \int_1^2 x \sqrt{\left(x^4 + \frac{1}{4x^4}\right)^2}\, dx$$

$$= 2\pi \int_1^2 x \left(x^4 + \frac{1}{4x^4}\right) dx$$

$$= 2\pi \int_1^2 \left(x^5 + \frac{1}{4x^3}\right) dx$$

$$= 2\pi \left(\frac{x^6}{6} - \frac{1}{8x^2}\right)\Bigg]_1^2$$

$\frac{2\,2}{4}\ \frac{2\,2}{4}\ \frac{2\,2}{4}$

$$= 2\pi \left(\frac{2^6}{6} - \frac{1}{8 \cdot 4} - \left(\frac{1}{6} - \frac{1}{8}\right)\right)$$

6, 32

6, 16x2

$$= 2\pi \left(\frac{64}{6} - \frac{1}{32} - \frac{1}{6} + \frac{1}{8/4}\right)$$

$$= 2\pi \left(\frac{63}{6} + \frac{3}{32}\right)$$

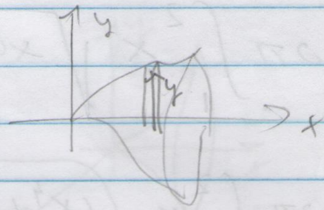
$$= 2\pi \left(\frac{63 \times 32 + 3 \times 6}{32 \times 6}\right) = 2\pi \left(\frac{2016 + 18}{192}\right)$$

$$= 2\pi \frac{2034}{192} = 2\pi \frac{1017}{96} = \cancel{2}\pi \frac{339}{\cancel{32}}$$

$$= \frac{339\pi}{16}$$

39. $x = \dfrac{y^4}{8} + \dfrac{1}{4y^2}$, $1 \leq y \leq 2$, eixo x

$$S = \int_1^2 2\pi y \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$



$$= \int_1^2 2\pi y \sqrt{1 + \left(\frac{y^3}{2} - \frac{1}{2y^3}\right)^2}\, dy \qquad \frac{dy}{dx} = \frac{y^3}{2} - \frac{1}{2y^3}$$

$$= \int_1^2 2\pi y \sqrt{1 + \frac{y^6}{4} - \frac{1}{2} + \frac{1}{4y^6}}\, dy$$

$$= \int_1^2 2\pi y \sqrt{\frac{y^6}{4} + \frac{1}{2} + \frac{1}{4y^6}}\, dy$$

$$= \int_1^2 2\pi y \sqrt{\left(\frac{y^3}{2} + \frac{1}{2y^3}\right)^2}\, dy$$

$$= \int_1^2 2\pi y \left(\frac{y^3}{2} + \frac{1}{2y^3}\right) dy$$

$$= 2\pi \int_1^2 \left(\frac{y^4}{2} + \frac{1}{2y^2}\right) dy$$

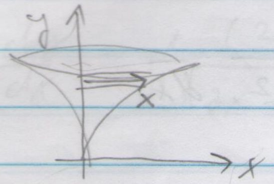
$$= 2\pi \left(\frac{y^5}{10} - \frac{1}{2y}\right)\Bigg]_1^2$$

$$= 2\pi \left(\frac{2^5}{10} - \frac{1}{2.2} - \left(\frac{1}{10} - \frac{1}{2.1}\right)\right)$$

$$\frac{32}{10} - \frac{1}{4} - \frac{1}{10} + \frac{1}{2} = \frac{31}{10} + \frac{1}{4} = \frac{124+10}{40} = \frac{134}{40} = \frac{67}{20}$$

$$= 2\pi \left(\frac{32}{10} - \frac{1}{4} - \frac{1}{10} + \frac{1}{2}\right) = \cancel{2}\pi \frac{67}{\cancel{20}} = \frac{67\pi}{10}$$

40. $y^3 = 3x$; $0 \le x \le 9$, eixo y



$$A = \int_0^9 2\pi x \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$y = \sqrt[3]{3x}$

$\frac{dy}{dx} = \frac{1}{\cancel{3}}(3x)^{-\frac{2}{3}} \cdot \cancel{3}$

$= (3x)^{-2/3}$

$\left(\frac{dy}{dx}\right)^2 = (3x)^{-4/3}$

?

Tentemos ; $y^3 = 3x$, $0 \le x \le 9 \Rightarrow$; $0 \le y \le 3$

$$A = \int_0^3 2\pi x(y) \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

$x = \frac{1}{3}y^3$

$$= \int_0^3 2\pi \frac{1}{3}y^3 \sqrt{1 + y^4}\, dy$$

$\frac{dx}{dy} = y^2$

$$= \frac{2\pi}{3}\int_0^3 y^3 \sqrt{1+y^4}\, dy$$

$\frac{3\,3\,3\,3}{9\;9} = 81$

$$= \frac{\cancel{2}\pi}{3}\,\frac{\cancel{2}_1}{3\cancel{4}}(1+y^4)^{3/2}\Big]_0^3 = \frac{\pi}{9}\left((1+3^4)^{3/2} - 1\right)$$

$9^3 = \frac{9\,9\,9}{81} = 729$

$$= \frac{\pi}{9}\left(82^{3/2} - 1\right) = \frac{\pi}{9}\left(82\sqrt{82} - 1\right)$$ //

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41. $y = \frac{2}{3} x^{3/2}$, $1 \le x \le 2$ (eixo y)

$\frac{dy}{dx} = x^{1/2}$; $S = \int_1^2 2\pi x \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$

$= \int_1^2 2\pi x \sqrt{1 + x}\, dx$

$u = 1 + x$

$x = 1 \rightarrow u = 2$

$x = 2 \rightarrow u = 3$

$= \int_2^3 2\pi (u-1) \sqrt{u}\, du$

$= 2\pi \int_2^3 (u^{3/2} - u^{1/2})\, du$

$= 2\pi \left(\frac{2}{5} u^{5/2} - \frac{2}{3} u^{3/2} \right)_2^3$

$= \frac{4\pi}{5} \sqrt{3^5} - \frac{4\pi}{3} \sqrt[2]{3^3} - \left(\frac{4\pi}{5} \sqrt{2^5} - \frac{4\pi}{3} \sqrt{2^3} \right)$

$= \frac{4\pi}{5} \times 9\sqrt{3} - \frac{4\pi}{3} 3\sqrt{3} - \frac{4\pi}{5} 4\sqrt{2} + \frac{4\pi}{3} 2\sqrt{2}$

$= \frac{36\pi}{5} \sqrt{3} - \frac{12\pi}{3} \sqrt{3} - \frac{16\pi}{5} \sqrt{2} + \frac{8\pi}{3} \sqrt{2}$

$= \frac{108\pi\sqrt{3} - 60\pi\sqrt{3}}{15} - \frac{48\pi\sqrt{2}}{15} + \frac{40\pi\sqrt{2}}{15}$

$= \frac{48\pi\sqrt{3}}{15} - \frac{8\pi\sqrt{2}}{15}$ //

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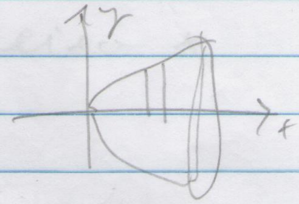
/ /

42. $y = (2x - x^2)^{1/2}$, $0 \leq x \leq 2$, eixo x

$$\frac{dy}{dx} = \frac{1}{2} \frac{(2-2x)}{\sqrt{2x-x^2}} = \frac{1-x}{\sqrt{2x-x^2}}$$



$$S = \int_0^2 2\pi\, y(x) \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$= \int_0^2 2\pi \sqrt{2x-x^2} \sqrt{1 + \frac{(1-x)^2}{2x-x^2}}\, dx$$

$$= \int_0^2 2\pi \sqrt{2x-x^2} \sqrt{\frac{\cancel{2x} - \cancel{x^2} + 1 - \cancel{2x} + \cancel{x^2}}{2x-x^2}}\, dx$$

$$= \int_0^2 2\pi \sqrt{2x-x^2} \frac{1}{\sqrt{2x-x^2}}\, dx$$

$$= 2\pi \cdot 2 = 4\pi$$

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Lista 9 (Parte IV)

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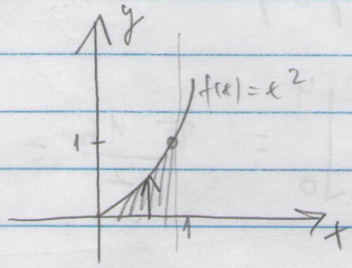
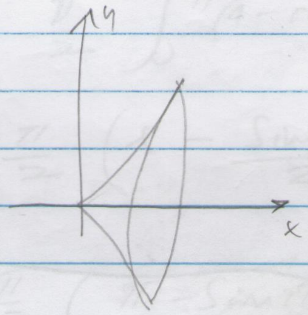
Exercícios

Nos exercícios a seguir, determine o volume do sólido gerado pela rotação em torno do eixo indicado da região plana limitada pelas curvas.

43. $y = x^2$, $y = 0$, $x = 1$; em torno do eixo x

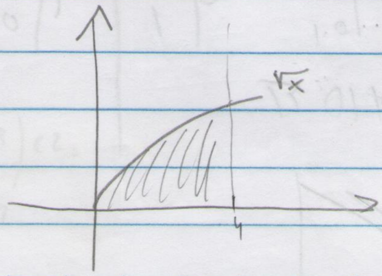




$$V = \int_0^1 \pi f(x)^2 \, dx = \int_0^1 \pi x^4 \, dx = \pi \frac{x^5}{5} \Big]_0^1$$

$$\therefore \boxed{V = \frac{\pi}{5}}$$

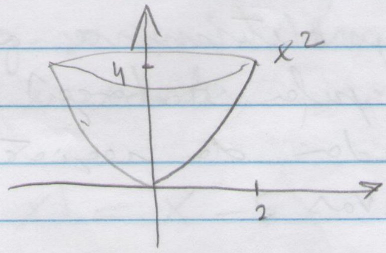
44. $y = \sqrt{x}$, $y = 0$, $x = 4$; em torno do eixo x



$$V = \int_0^4 \pi x \, dx = \frac{\pi x^2}{2} \Big]_0^4$$

$$\therefore \boxed{V = 8\pi}$$

45. $y = x^2$, $0 \leq x \leq 2$; eixo y



$$V = \int_0^4 \pi \underbrace{f^2(y)} \, dy$$

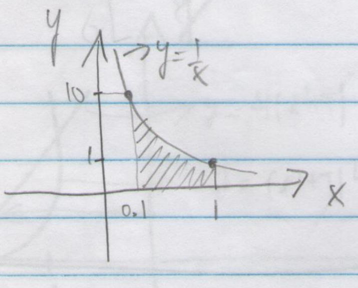
$$= \int_0^4 \pi (\sqrt{y})^2 \, dy$$

$y = x^2 \Leftrightarrow x = \sqrt{y}$

$$= \int_0^4 \pi y \, dy$$

$$= \frac{\pi y^2}{2} \Big|_0^4 = \frac{\pi \, 16}{2} = 8\pi$$

46. $y = \frac{1}{x}$, $y = 0$, $x = \frac{1}{10}$, $x = 1$; eixo x



$$V = \int_{0.1}^{1} \pi \left(\frac{1}{x}\right)^2 dx$$

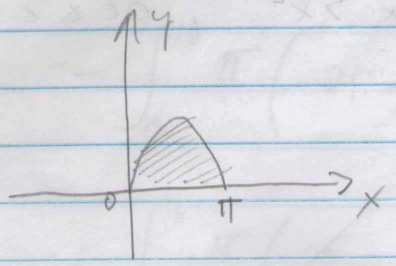
$$= \pi \int_{0.1}^{1} \frac{1}{x^2} \, dx$$

$$= \pi \, \frac{-1}{x} \Big|_{0.1}^{1} = -\frac{\pi}{1} - \left(\frac{-\pi}{0.1}\right)$$

$$= -\pi + 10\pi$$

$$= 9\pi$$

47. $y = \sin x$, $0 \le x \le \pi$, $y = 0$; em torno do eixo x



$$V = \int_0^{\pi} \pi \sin^2 x \, dx$$

$$= \pi \int_0^{\pi} \frac{1 - \cos 2x}{2} \, dx$$

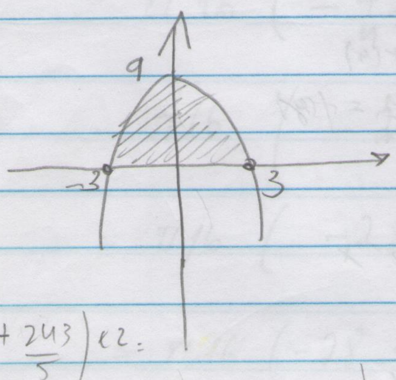
$$= \frac{\pi}{2} \int_0^{\pi} (1 - \cos 2x) \, dx$$

$$= \frac{\pi}{2} \left(x - \frac{\sin 2x}{2} \right)_0^{\pi}$$

$$= \frac{\pi}{2} \left(\pi - \cancel{\frac{\sin 2\pi}{2}}^{\,0} - \left(0 - \cancel{\frac{\sin 0}{2}}^{\,0}\right) \right)$$

$$= \frac{\pi^2}{2}$$

48. $y = 9 - x^2$, $y = 0$; eixo x



$$V = \int_{-3}^{3} \pi (9 - x^2)^2 \, dx$$

$$= \pi \int_{-3}^{3} (81 - 18x^2 + x^4) \, dx$$

$$= \pi \left(81x - 6x^3 + \frac{x^5}{5} \right)_{-3}^{3}$$

$$= \pi \left(81 \cdot 3 - 6 \cdot 27 + \frac{3^5}{5} - 81(-3) - 6(-3)^3 - \frac{(-3)^5}{5} \right)$$

$$= \frac{1296\pi}{5}$$

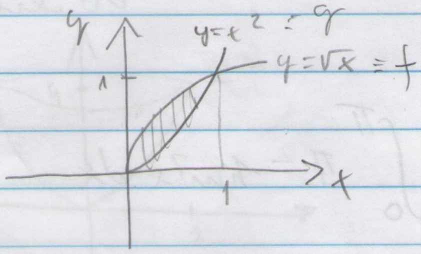
$\left(243 - 162 + \frac{243}{5}\right) \times 2 =$

$= \left(81 + \frac{243}{5}\right) \times 2$

$\frac{(405 + 243)}{5} \times 2 = \frac{648}{5} \times 2 = \frac{1296}{5}$

11

49. $y = x^2$, $x = y^2$; eixo x



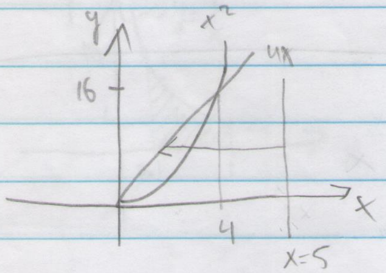
$\sqrt{x} > x^2$, $0 \le x \le 1$

$$V = \int_0^1 \pi\left((\sqrt{x})^2 - (x^2)^2\right) dx$$

$$= \int_0^1 \pi(x - x^4)\, dx = \pi\left(\frac{x^2}{2} - \frac{x^5}{5}\right)\Big|_0^1$$

$$= \pi\left(\frac{1}{2} - \frac{1}{5}\right) = \frac{3\pi}{10}$$

50. $y = x^2$, $y = 4x$, em torno da linha $x = 5$



Dois métodos são possíveis:

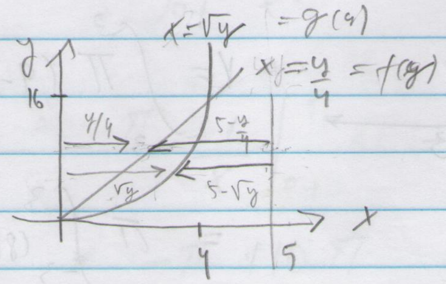
$x^2 = 4x$

$x^2 - 4x = 0$

$x(x-4) = 0$

$x = 0$, $x = 4$

i)



$$V = \int_0^{16} \pi\left(f^2(y) - g^2(y)\right) dy =$$

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$$V = \int_0^{16} \pi \left(\left(5 - \frac{y}{4}\right)^2 - (5-\sqrt{y})^2 \right) dy$$

$$= \int_0^{16} \pi \left(25 - \frac{10y}{4} + \frac{y^2}{16} - (25 - 10\sqrt{y} + y) \right) dy$$

$$= \int_0^{16} \pi \left(\cancel{25} - \frac{10y}{4} + \frac{y^2}{16} - \cancel{25} + 10\sqrt{y} - y \right) dy$$

$$= \int_0^{16} \pi \left(-\frac{14y}{4} + \frac{y^2}{16} + 10\sqrt{y} \right) dy$$

$\frac{7}{2}$

$$= \pi \left(-\frac{7}{2}\frac{y^2}{2} + \frac{y^3}{48} + 10\frac{2}{3}y^{3/2} \right)\Big|_0^{16}$$

$4^3 = 16 \cdot 4$

$$= \pi \left(-\frac{7}{4} \times 16^2 + \frac{16^3}{48} + \frac{20}{3} 16^{3/2} \right)$$

$16^{3/2} = 4^3$
$= 16 \cdot 4$

$$= \pi \left(-\frac{7}{4} \times 16^2 + \frac{16^3}{48} + \frac{20}{3} \times 16 \times 4 \right)$$

$$= \pi 16 \left(-\frac{7}{4} \times 16 + \frac{16^2}{48} + \frac{20}{3} \times 4 \right)$$

$$= \pi 16 \left(-28 + \frac{16 \times \cancel{16}}{\cancel{48}\,3} + \frac{80}{3} \right)$$

$$= \pi 16 \left(-28 + \frac{16}{3} + \frac{80}{3} \right)$$

$$= \pi 16 \left(-28 + \frac{96}{3} \right) = \pi 16 (-28 + 32)$$

$$= \pi 16 \cdot 4$$

$$= \underline{\underline{64\pi}}$$

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2º método

Faz-se uma translação do eixo x de 5 unidades, isto é, define-se

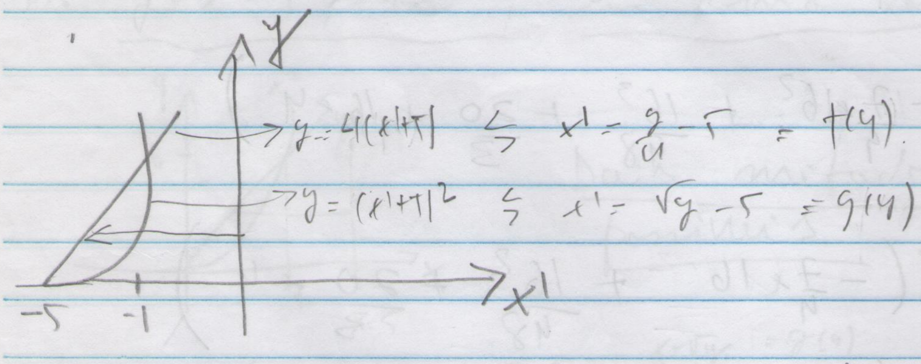
$$x' = x - 5$$

de modo que neste novo sistema de eixos (x', y) tem-se $x = 5 \rightarrow x' = 0$, isto é, far-se-á uma rotação em torno do eixo y.

Ora,

$$y = x^2 \longrightarrow y = (x'+5)^2 \Leftrightarrow x' = \sqrt{y} - 5$$

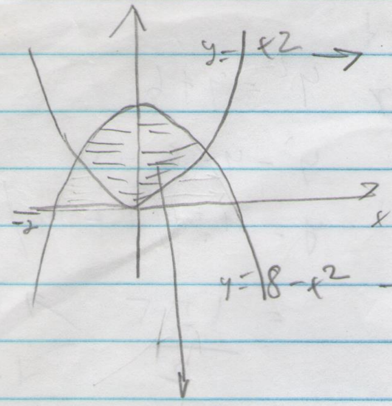
$$y = 4x \longrightarrow y = 4(x'+5) \Leftrightarrow x' = \frac{y}{4} - 5$$



$$V = \int_0^{16} \pi \left(f(y)^2 - g(y)^2 \right) dy = \int_0^{16} \pi \left(\left(\frac{y}{4} - 5\right)^2 - \left(\sqrt{y} - 5\right)^2 \right) dy$$

$$= 64\pi$$

51. $y = x^2$, $y = 8 - x^2$; eixo x



$y = x^2 \rightarrow x = \sqrt{y}$

$y = 8 - x^2 \rightarrow x = \sqrt{8-y}$

$x^2 = 8 - x^2$

$\Leftrightarrow$

$2x^2 = 8$

$x^2 = 4$

$x = \pm 2$

$V = 2V'$

$$V' = \int_0^2 \pi\left((8-x^2)^2 - (x^2)^2\right) dx$$

$$= \int_0^2 \pi\left((8-x^2)^2 - x^4\right) dx$$

$$= \int_0^2 \pi\left(64 - 16x^2 + \cancel{x^4} - \cancel{x^4}\right) dx$$

$$= \int_0^2 \pi\left(64 - 16x^2\right) dx$$

$$= \pi 64x - \pi 16 \frac{x^3}{3} \Big]_0^2$$

$$= \pi 64 \times 2 - \pi \frac{16}{3} \times 8$$

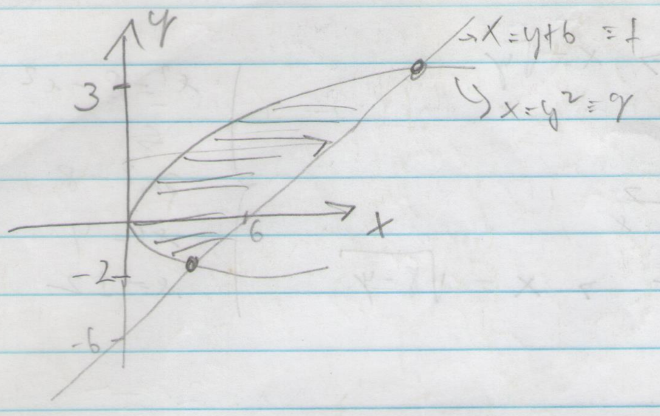
$$= 128\pi - \frac{128\pi}{3}$$

$$V' = 128\pi \frac{2}{3} = \frac{256\pi}{3} \Rightarrow V = 2V'$$

$\therefore \boxed{V = \frac{512}{3}\pi}$

/ /

52. $x = y^2$, $x = y + 6$; eixo y



$y^2 = y + 6$

$y^2 - y - 6 = 0$

$y = 1 \pm \sqrt{1 + 24} / 2$

$= \frac{1 \pm 5}{2} \nearrow 3 \searrow -2$

$$V = \int_{-2}^{3} \pi \left((y+6)^2 - (y^2)^2 \right) dy$$

$$= \pi \frac{(y+6)^3}{3} - \pi \frac{y^5}{5} \Big]_{-2}^{3}$$

$$= \pi \left(\frac{9^3}{3} - \frac{3^5}{5} \right) - \pi \left(\frac{4^3}{3} - \frac{(-2)^5}{5} \right)$$

$3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 =$ 9 9 $81 \times 3 = 243$

$$= \pi \left(\frac{729}{3} - \frac{243}{5} \right) - \pi \left(\frac{64}{3} + \frac{32}{5} \right)$$

$275 \,|\, 5$ $25 \,|\, 55$

$$= \pi \left(\frac{729 - 64}{3} \right) - \pi \left(\frac{243 + 32}{5} \right)$$

$$= \pi \left(\frac{665}{3} \right) - \pi \left(\frac{275}{5} \right)$$

665 165

$$= \pi \frac{665}{3} - \pi 55 = \pi \left(\frac{665 - 165}{3} \right)$$

$$= \frac{\pi 500}{3}$$