

Cálculo B - Lista 11

Funções de várias variáveis

Encontre o domínio das funções $f(x, y)$ representando-o como uma região do plano R^2 .

1. $f(x, y) = \frac{1}{x^2 + y^2 - 1}$
 2. $f(x, y) = \sqrt{1 - x^2 - y^2}$
 3. $f(x, y) = \sqrt{x^2 - y^2 - 1}$
 4. $f(x, y) = \sqrt{x^2 + y^2 - 1}$
 5. $f(x, y) = \frac{1}{\sqrt{1 - x^2 - y^2}}$
 6. $f(x, y) = \frac{x^4 - y^4}{x^2 - y^2}$
 7. $f(x, y) = \ln(xy - 1)$
 8. $f(x, y) = \sqrt{xy}$
 9. $f(x, y) = \frac{e^x - e^y}{e^x + e^y}$
 10. $f(x, y) = \ln(xy)$
 11. $f(x, y) = \frac{1}{\sqrt{y - x^2}}$
 12. $f(x, y) = \sqrt{9 - x^2} - \sqrt{4 - y^2}$
 13. $f(x, y) = \frac{2}{\sqrt{9 - (x^2 + y^2)}}$
- Ache o domínio das funções $f(x, y, z)$
14. $f(x, y, z) = \frac{x + y + z}{x - y - z}$
 15. $f(x, y, z) = \sqrt{16 - x^2 - 4y^2 - z^2}$
 16. $f(x, y, z) = \ln(4 - x^2 - y^2) + |z|$
 17. $f(x, y, z) = \ln(x + 2y + 3z)$
 18. $f(x, y, z) = \cos x + \cos y + \cos z$
 19. $f(x, y, z) = \frac{x + y + z}{|x + y + z|}$
 20. $f(x, y, z) = \frac{z^2}{x^2 - y^2}$
 21. $f(x, y, z) = -\frac{z^2}{\sqrt{x^2 - y^2}}$
 22. $f(x, y, z) = \frac{\sqrt{1 - x^2} + \sqrt{4 - y^2}}{1 + \sqrt{9 - z^2}}$
 23. $f(x, y, z) = \ln(x + 2y + 3z)$

$$24. f(x, y, z) = e^{\sqrt{4 - x^2 - y^2 - z^2}}$$

Identifique e faça um esboço das curvas de nível para cada uma das funções a seguir

25. $f(x, y) = x - y$
26. $f(x, y) = x^2 - y$
27. $f(x, y) = \frac{x}{x + y}$
28. $f(x, y) = x^3 - y$
29. $f(x, y) = x^2 - y^2$
30. $f(x, y) = y^2$
31. $f(x, y) = \ln(x^2 + y^2)$
32. $f(x, y) = \frac{\ln y}{x^2}$
33. $f(x, y) = \frac{x^2}{x^2 + y^2}$

Identifique e faça um esboço das superfícies de nível para cada função $f(x, y, z)$ correspondentes aos valores de c dados.

34. $f(x, y, z) = z(x^2 + y^2)^{-\frac{1}{2}}$, $c = 1$
35. $f(x, y, z) = 4x^2 + 9y^2 - 72z$, $c = 0$
36. Identifique as superfícies de nível de $f(x, y, z) = x^2 + y^2 - z^2$ para (i) $c < 0$ (ii) $c = 0$ (iii) $c > 0$.

Faça um esboço do gráfico de cada uma das funções

37. $f(x, y) = x^{\frac{1}{3}}$
38. $f(x, y) = \sqrt{4 - x^2 - y^2}$

Faça um esboço das superfícies quádricas a seguir

39. $\frac{x^2}{4} + y^2 + \frac{z^2}{9} = 1$
40. $x^2 + 2y^2 + 3z^2 = 6$
41. $x^2 + z^2 = 4$
42. $y^2 + z^2 = 9$
43. $z = x^2 + \frac{y^2}{9}$
44. $x = y^2 + \frac{z^2}{4}$
45. $z^2 = x^2 + 4y^2$

46. $x^2 = 9y^2 + 4z^2$

47. $y = 1 - x^2$

48. $x = z^2 + 3$

49. $z = y^2 - 4x^2$

50. $x = 4z^2 - y^2$

51. $y^2 - x^2 = 4$

52. $z^2 - y^2 = 9$

53. $z^2 + 4y^2 - 2x^2 = 1$

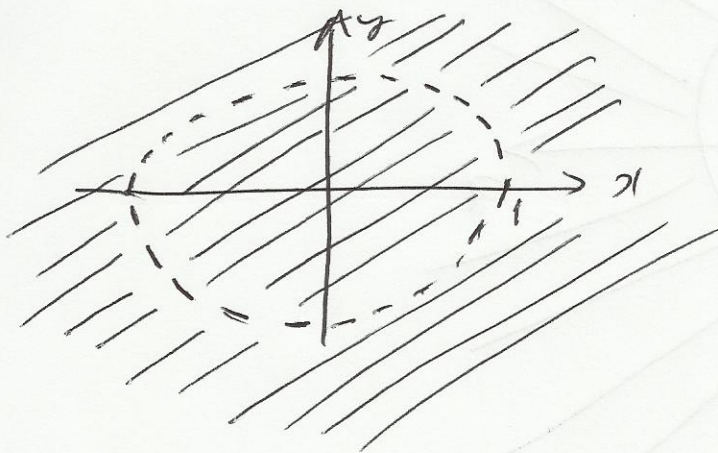
54. $4x^2 + y^2 - z^2 = 16$

55. $z^2 - 4y^2 - x^2 = 1$

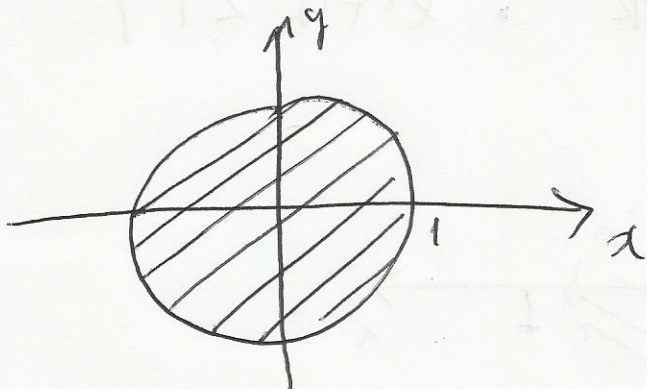
56. $x^2 - 9y^2 - 4z^2 = 36$

Respostas

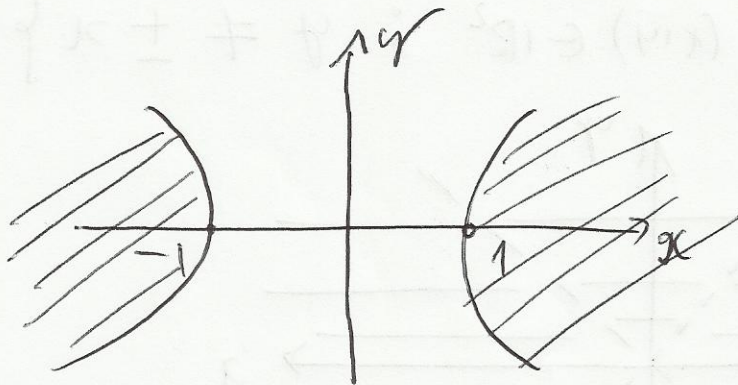
1. Dom $f = \{ (x,y) \in \mathbb{R}^2 : x^2 + y^2 \neq 1 \}$



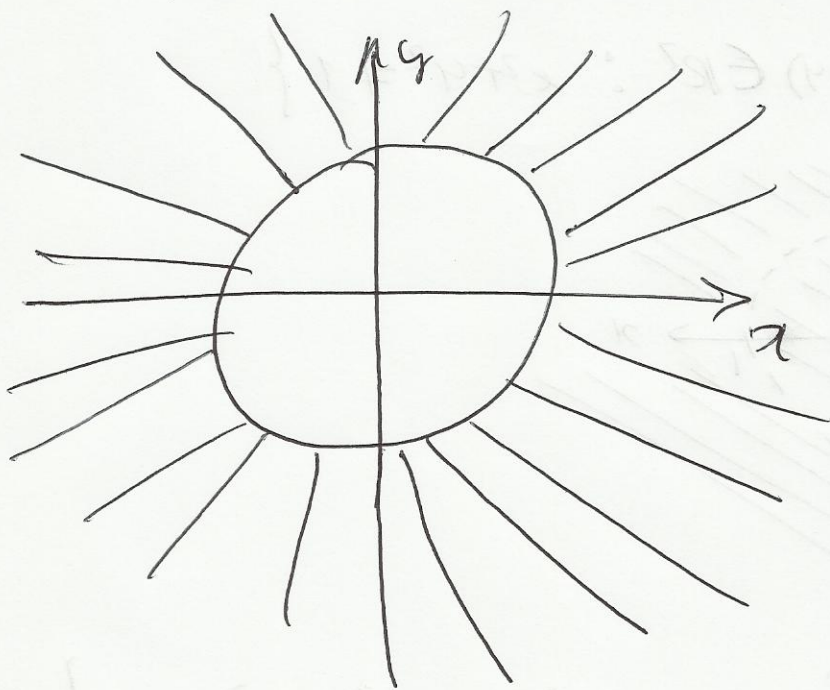
2. Dom $f = \{ (x,y) \in \mathbb{R}^2 : x^2 + y^2 \leq 1 \}$



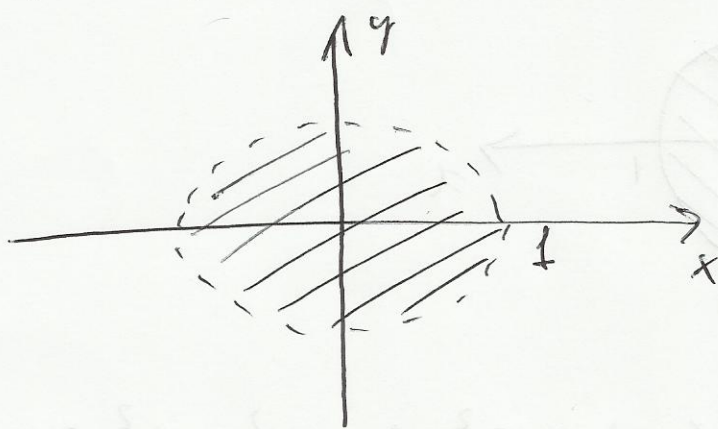
3. Dom $f = \{ (x,y) \in \mathbb{R}^2 : x^2 - y^2 > 1 \}$



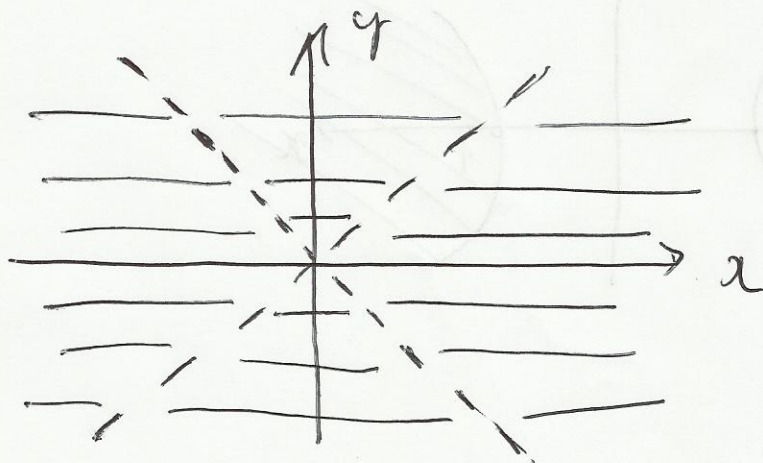
4. Dom $f = \{ (x,y) \in \mathbb{R}^2 : x^2 + y^2 > 1 \}$



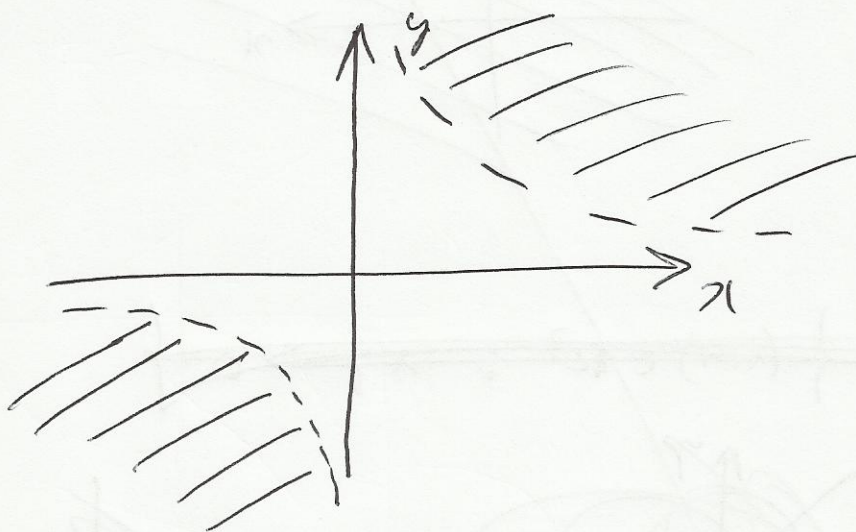
5. Dom $f = \{ (x,y) \in \mathbb{R}^2 : x^2 + y^2 < 1 \}$



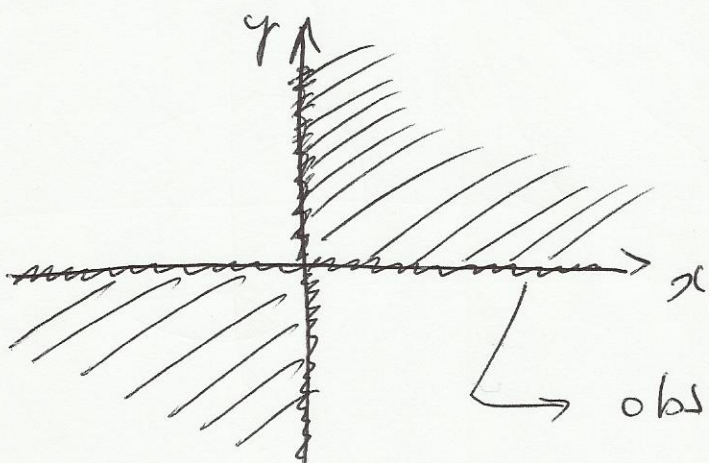
6. Dom $f = \{ (x,y) \in \mathbb{R}^2 : y \neq \pm x \}$



$$7. \text{ Dom } f = \left\{ (x|y) \in \mathbb{R}^2 : xy > 1 \right\}$$

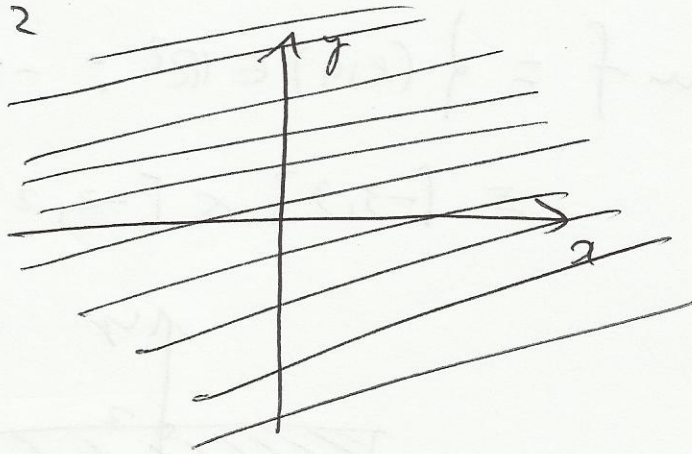


$$8. \text{ Dom } f = \left\{ (x|y) \in \mathbb{R}^2 : xy > 0 \right\}$$

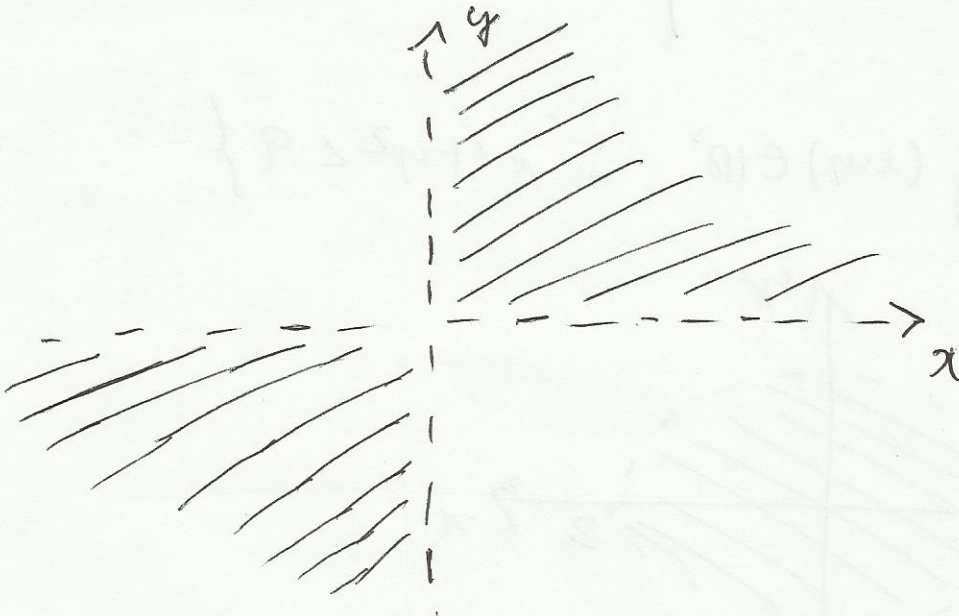


obs. : Esta linha
 Simplesmente desenhada
 nos eixos x e y
 servem para
 indicar que
 estes pontos estão
 no conjunto.

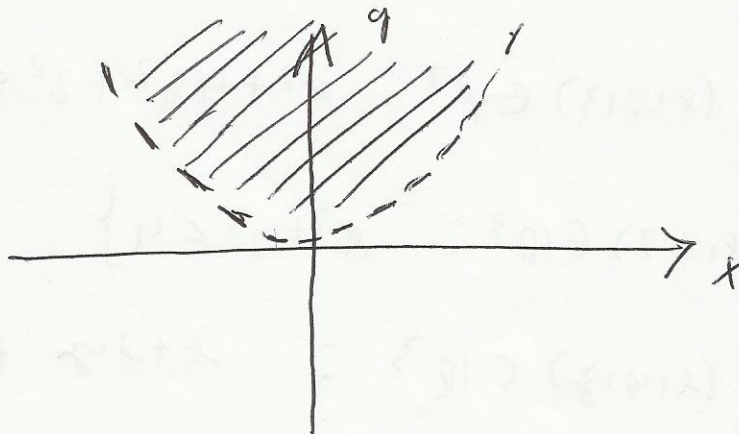
9. $\text{Dom } f = \mathbb{R}^2$



10. $\text{Dom } f = \{ (x, y) \in \mathbb{R}^2 : xy > 0 \}$



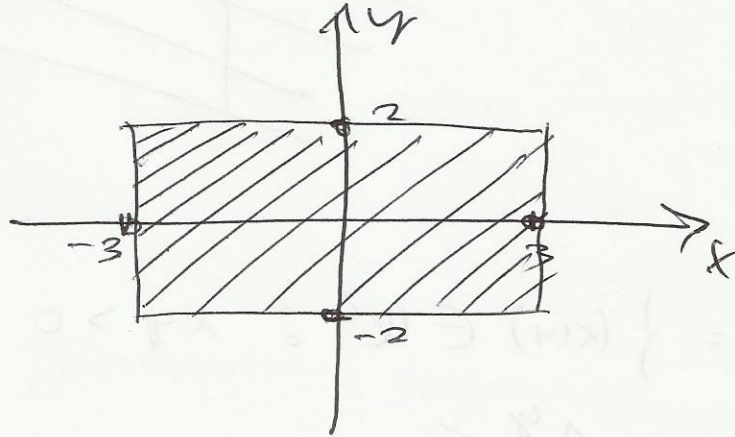
11. $\text{Dom } f = \{ (x, y) \in \mathbb{R}^2 : y > x^2 \}$



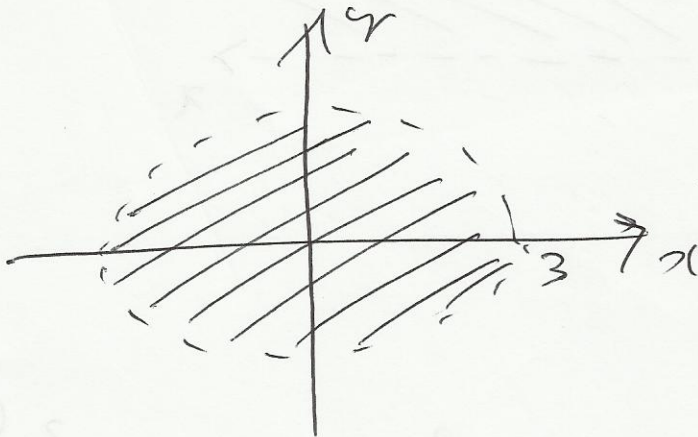
12.

$$\text{Dom } f = \{ (x, y) \in \mathbb{R}^2 : -3 \leq x \leq 3, -2 \leq y \leq 2 \}$$

$$\equiv [-3, 3] \times [-2, 2]$$



13. $\text{Dom } f = \{ (x, y) \in \mathbb{R}^2 : x^2 + y^2 < 9 \}$



14. $\text{Dom } f = \{ (x, y, z) \in \mathbb{R}^3 : x - y - z \neq 0 \}$

15. $\text{Dom } f = \{ (x, y, z) \in \mathbb{R}^3 : x^2 + 4y^2 + z^2 \leq 16 \}$

16. $\text{Dom } f = \{ (x, y, z) \in \mathbb{R}^3 : x^2 + y^2 < 4 \}$

17. $\text{Dom } f = \{ (x, y, z) \in \mathbb{R}^3 : x + 2y + 3z > 0 \}$

18. $\text{Dom } f = \mathbb{R}^3$

19. Dom $f = \{ (x, y, z) \in \mathbb{R}^3 : x + y + z \neq 0 \}$

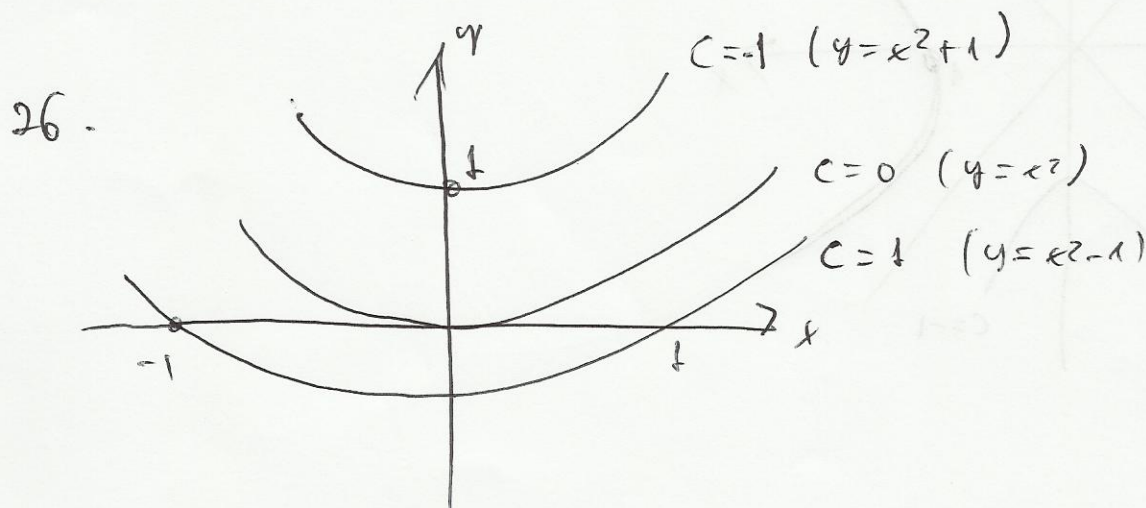
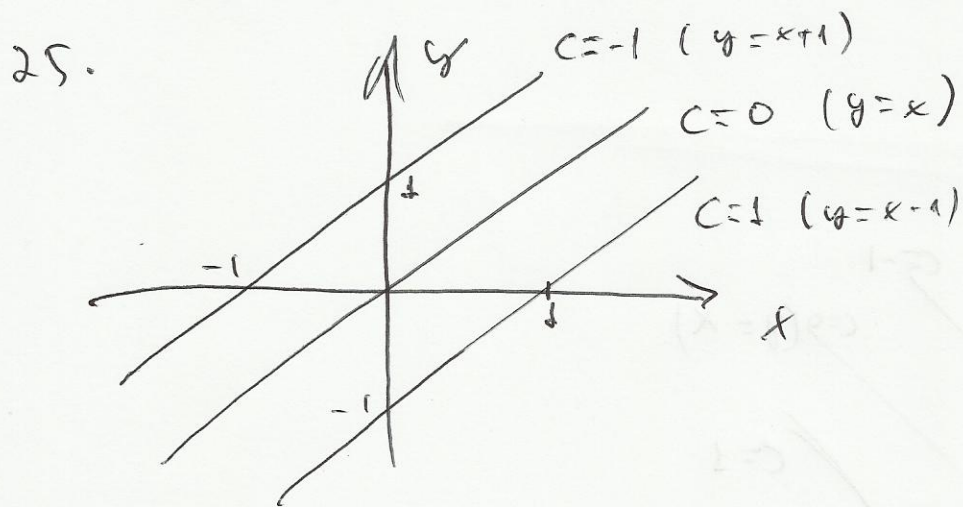
20. Dom $f = \{ (x, y, z) \in \mathbb{R}^3 : y \neq x, y \neq -x \}$

21. Dom $f = \{ (x, y, z) \in \mathbb{R}^3 : x^2 - y^2 > 0 \}$

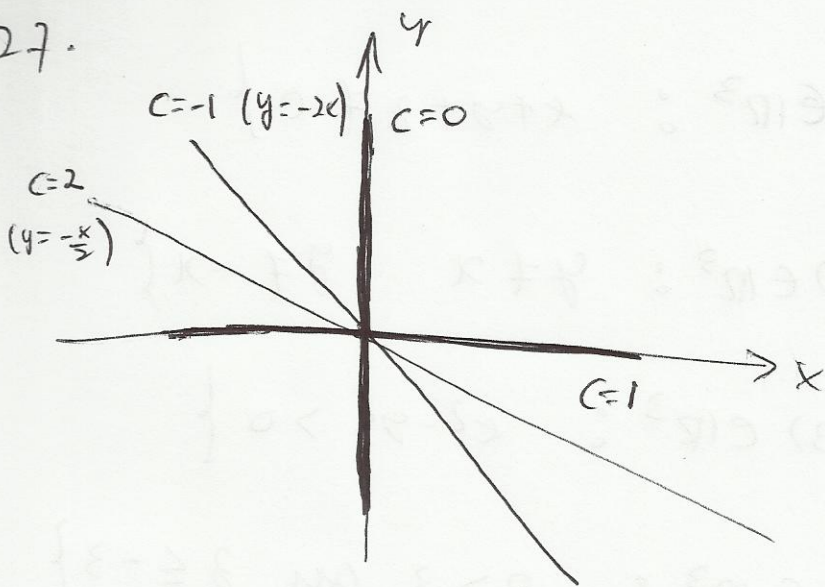
22. Dom $f = \{ (x, y, z) \in \mathbb{R}^3 : z > 3 \text{ or } z \leq -3 \}$

23. Dom $f = \{ (x, y, z) \in \mathbb{R}^3 : x + 2y + 3z > 0 \}$

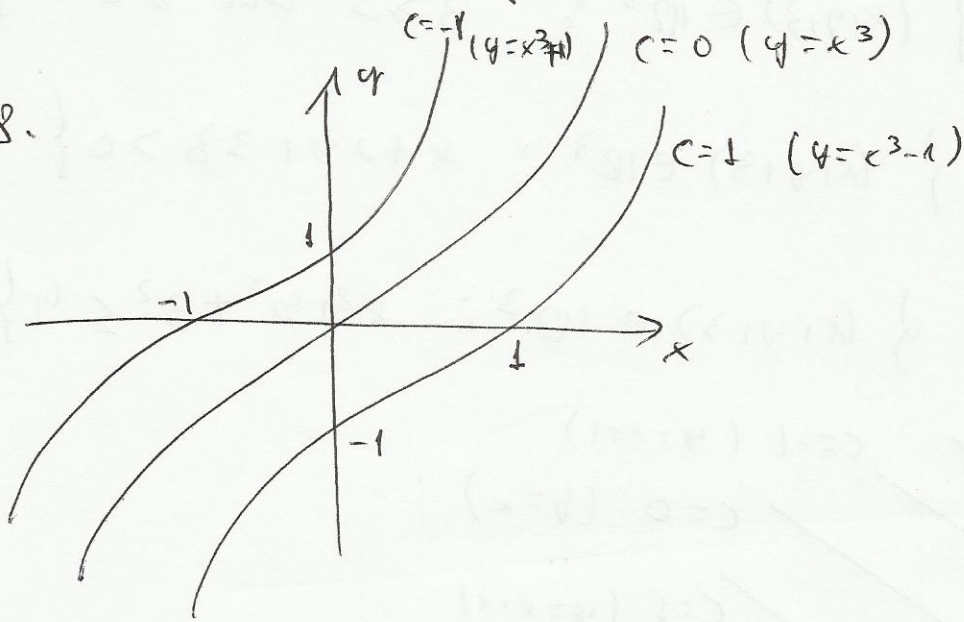
24. Dom $f = \{ (x, y, z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 \leq 4 \}$



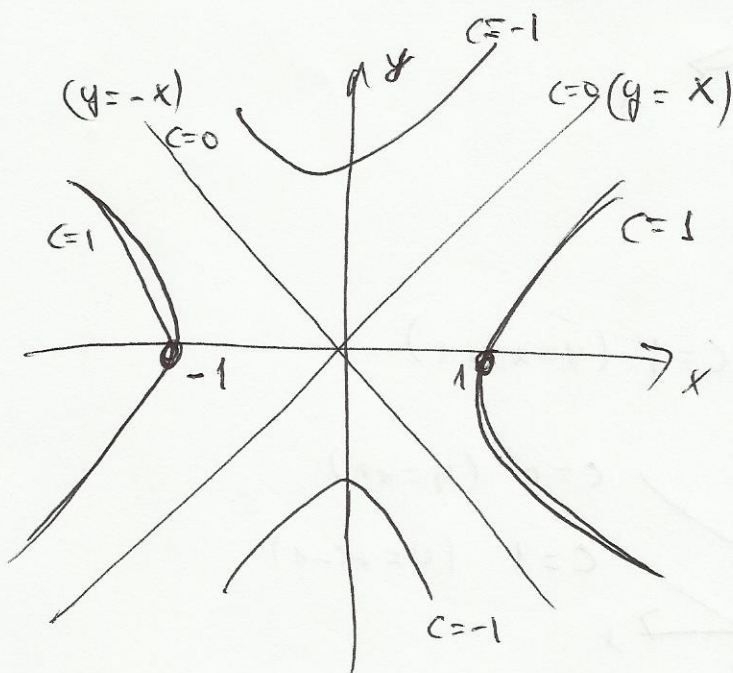
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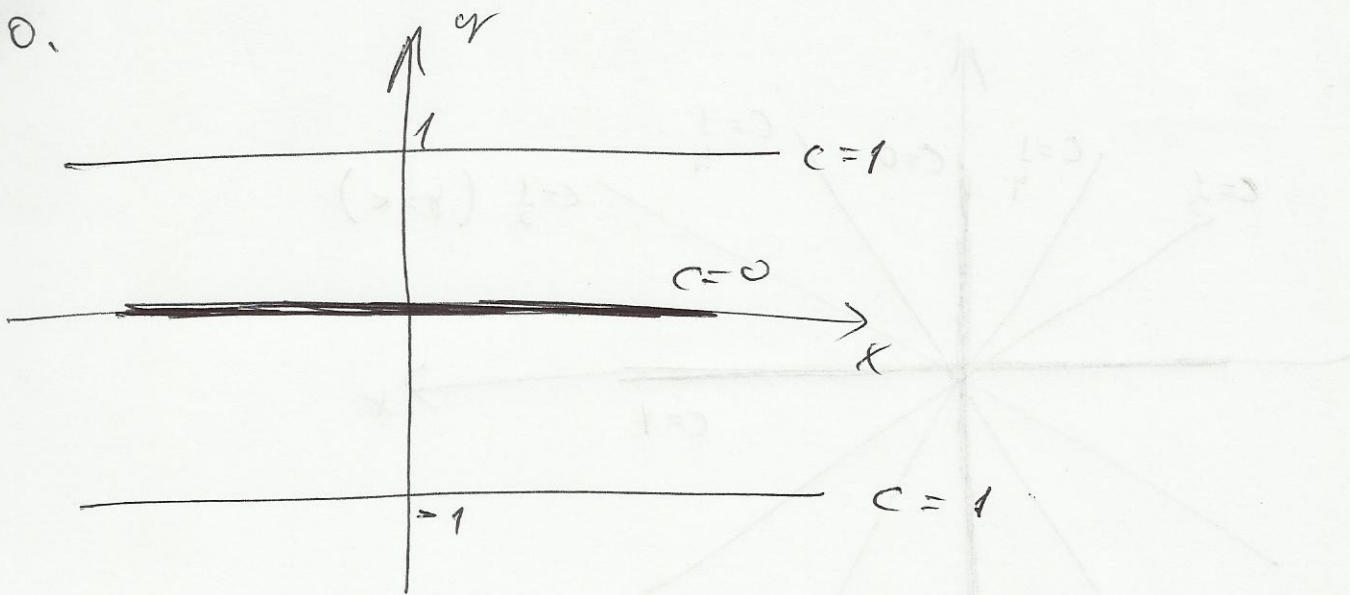
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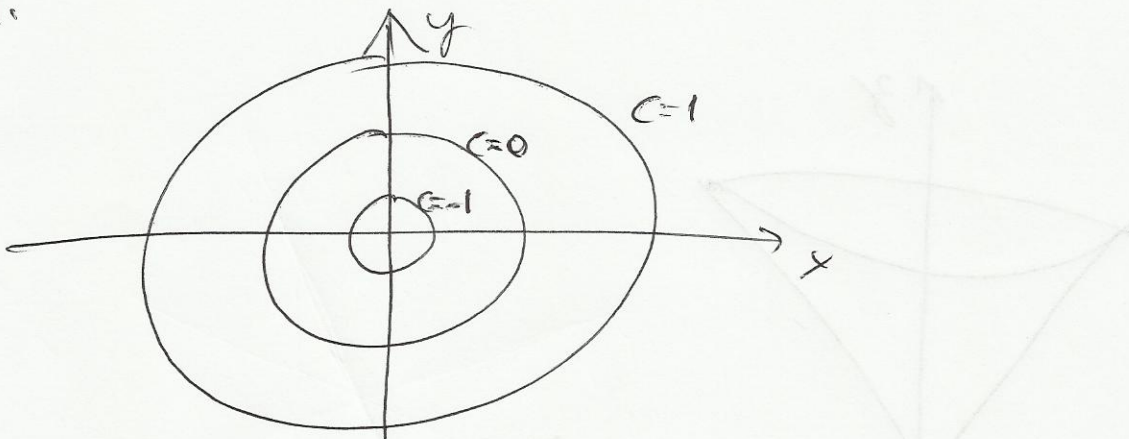
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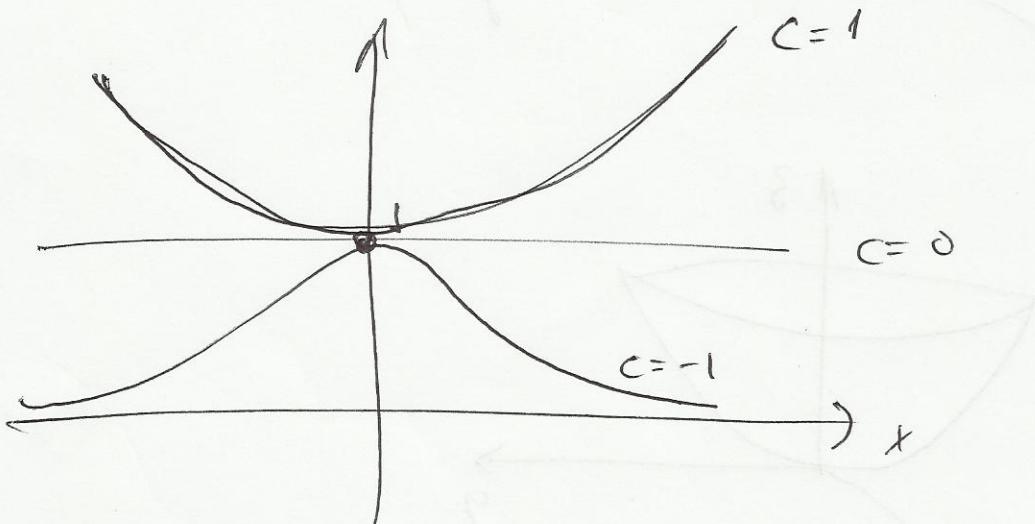
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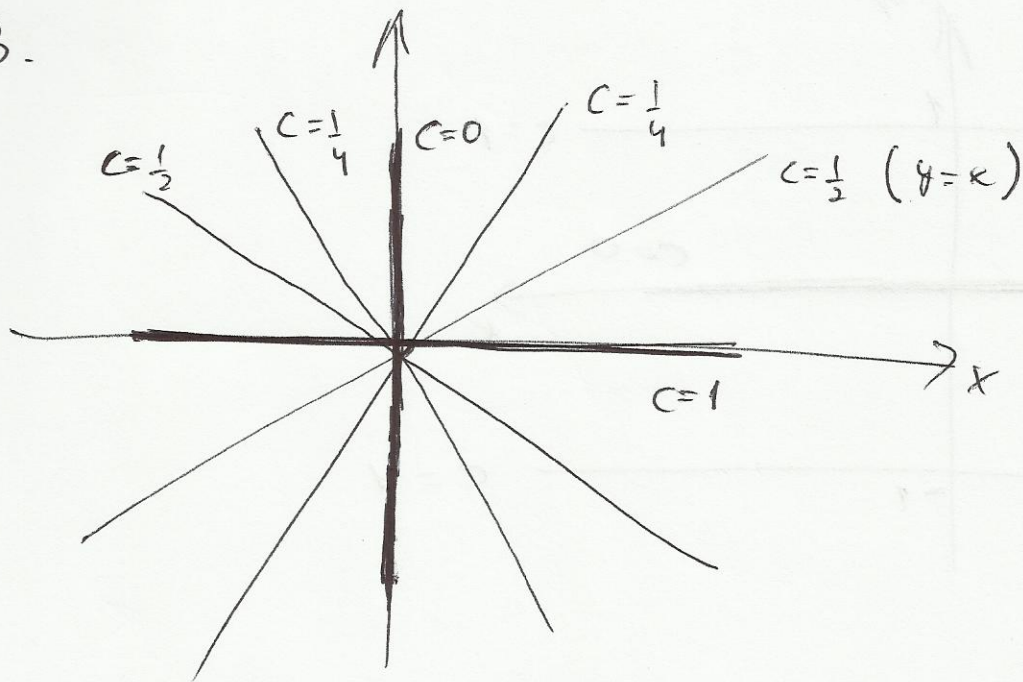
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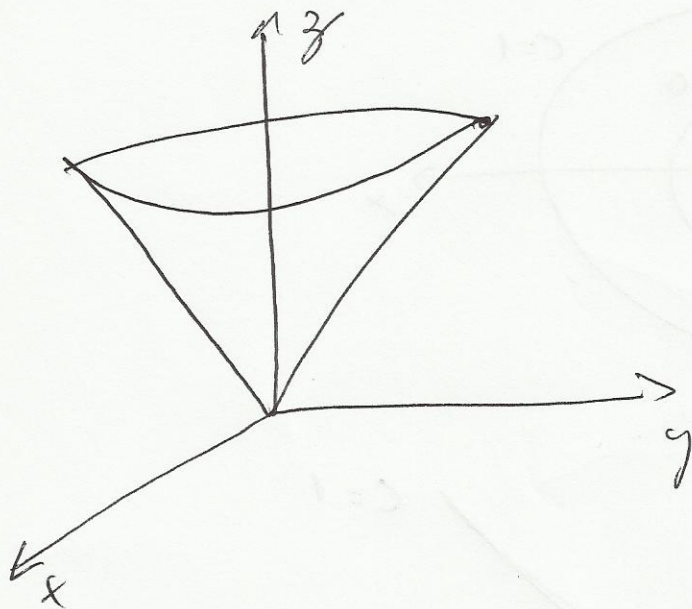
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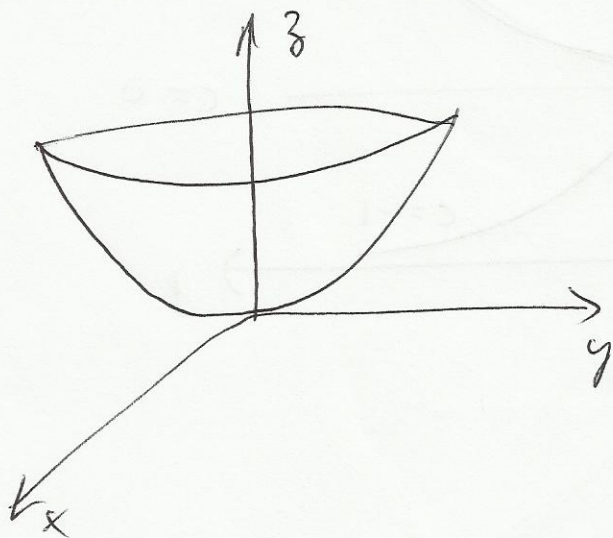
33.



34.

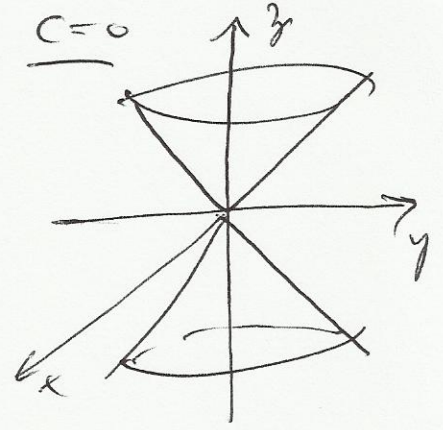
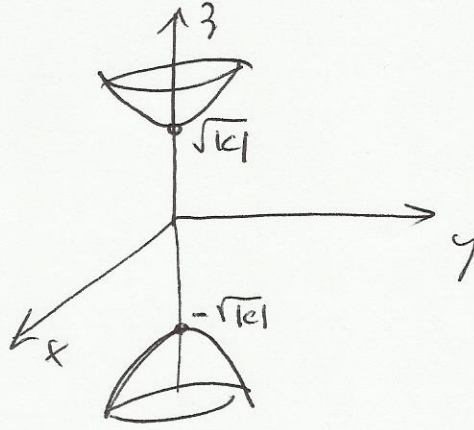


35.

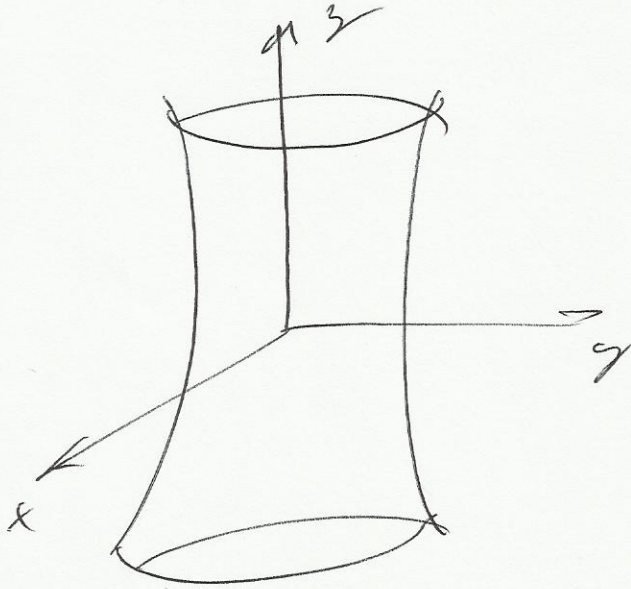


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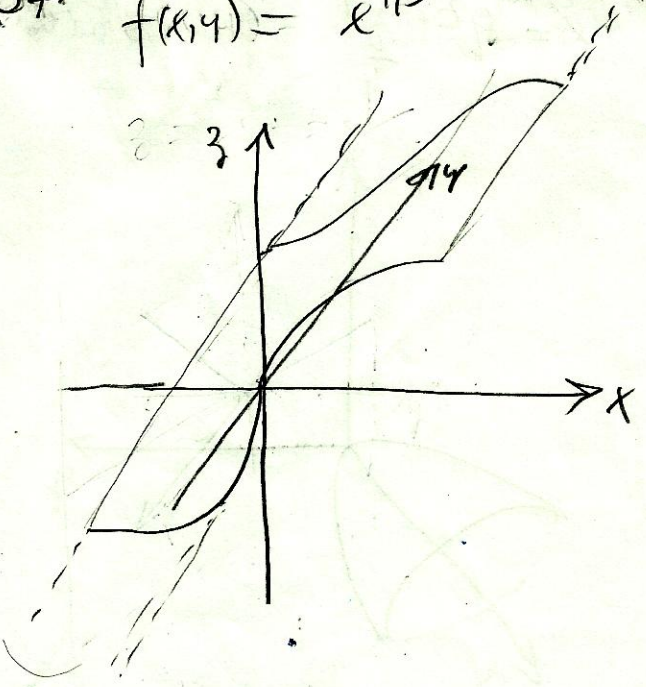
$c < 0$



$c > 0$



37. $f(x,y) = x^{1/3}$

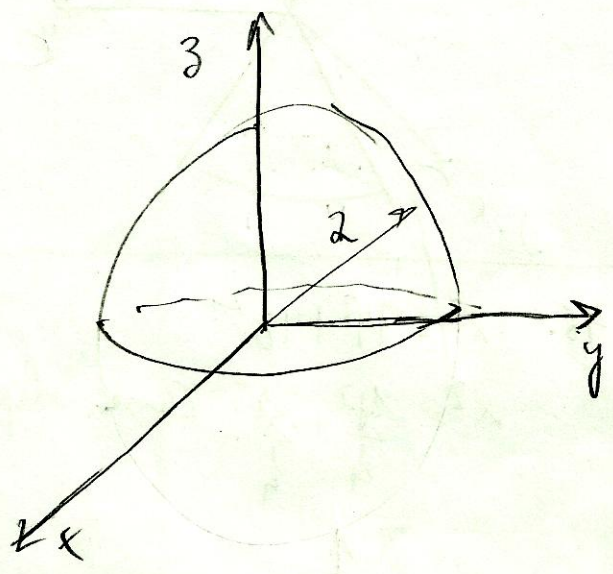


38. $f(x,y) = \sqrt{4-x^2-y^2}$

$z = \sqrt{4-x^2-y^2}$

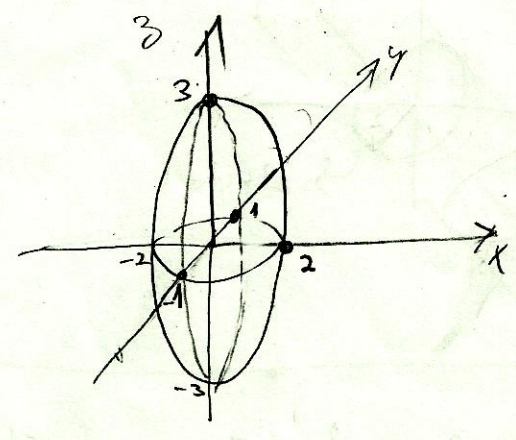
$z^2 = 4-x^2-y^2$

$x^2+y^2+z^2 = 4$



39. $\frac{x^2}{4} + \frac{y^2}{1} + \frac{z^2}{9} = 1$

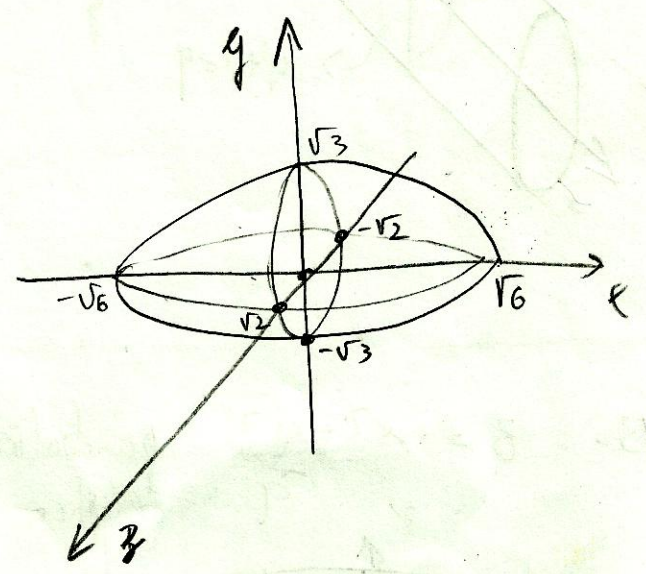
Ellipsoide



40.

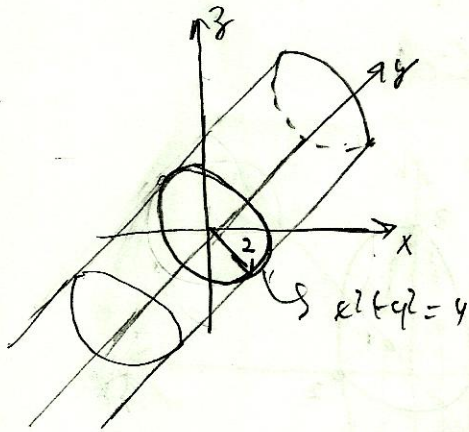
$x^2 + 2y^2 + 3z^2 = 6$

$\frac{x^2}{6} + \frac{y^2}{3} + \frac{z^2}{2} = 1$



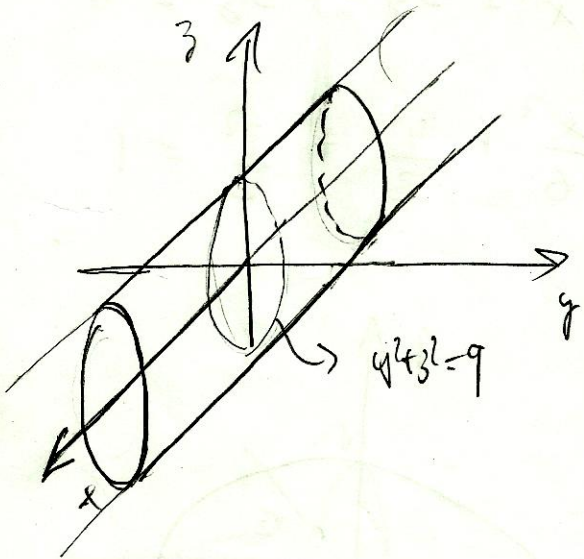
41. $x^2 + z^2 = 4$

cilindro
ao longo
do eixo y



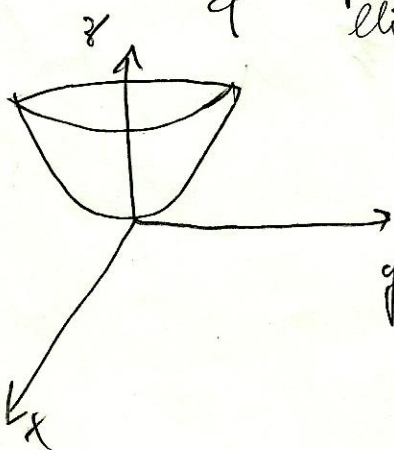
42. $y^2 + z^2 = 9$

cilindro
ao longo
do eixo x



43. $z = x^2 + \frac{y^2}{9}$

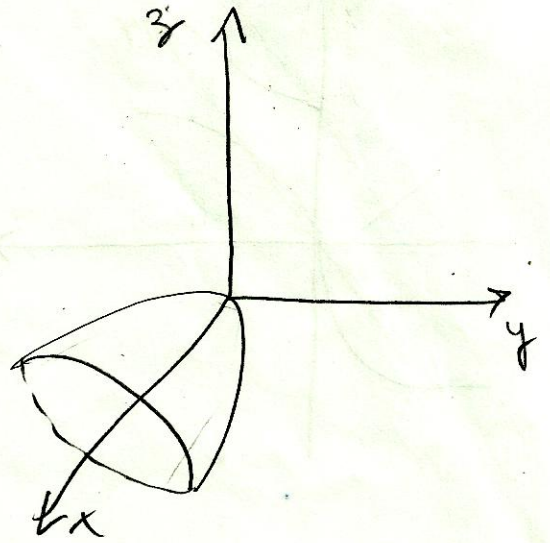
parabolóide
elíptico



44.

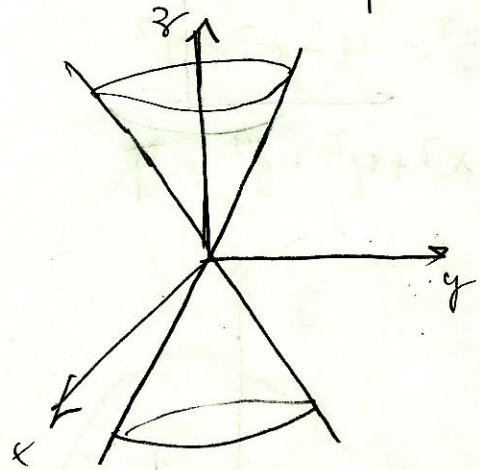
$x = y^2 + \frac{z^2}{9}$

parabolóide
elíptico



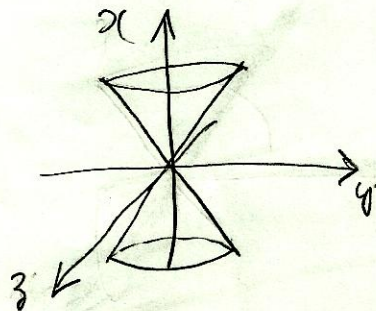
45. $z^2 = x^2 + 4y^2$

cone
elíptico



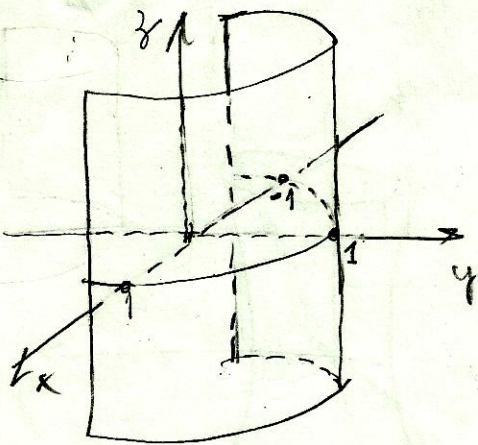
46. $x^2 = 9y^2 + 9z^2$

cone elíptico



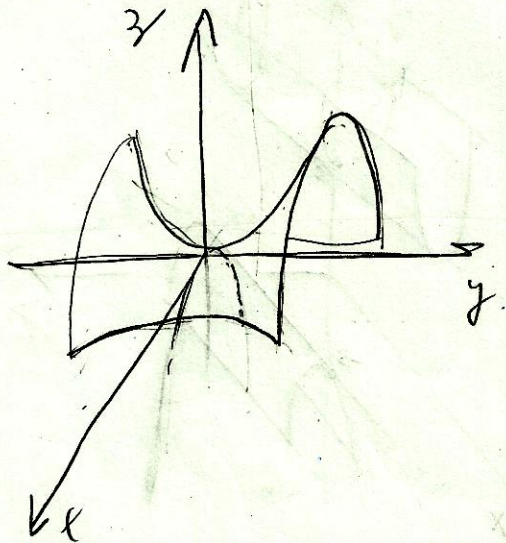
47. $y = 1 - x^2$

Cilindro parabólico



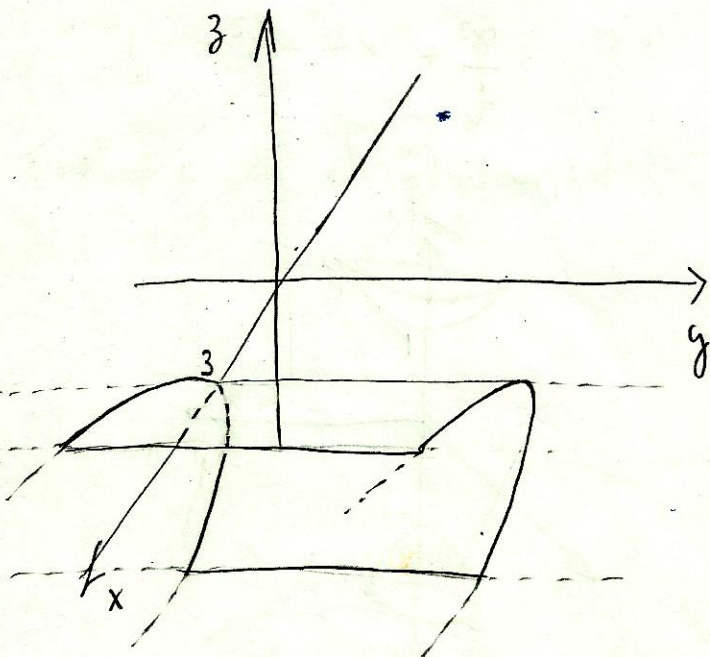
49. $z = y^2 - 4x^2$

Parabolóide hiperbólico

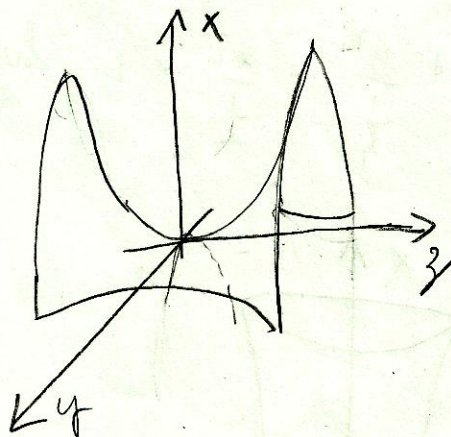


48. $x = z^2 + 3$

Cilindro parabólico

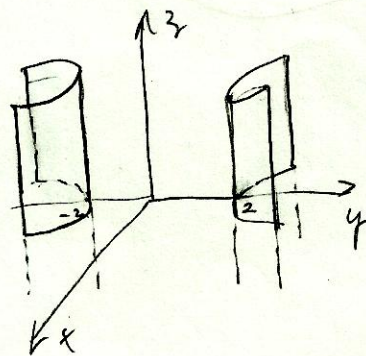


50. $x = 4z^2 - y^2$



51. $y^2 - x^2 = 4$

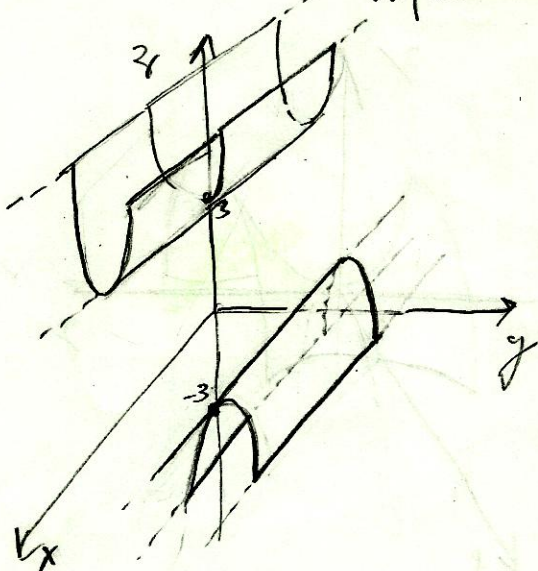
Cilindro hiperbólico



52.

$$z^2 - y^2 = 9$$

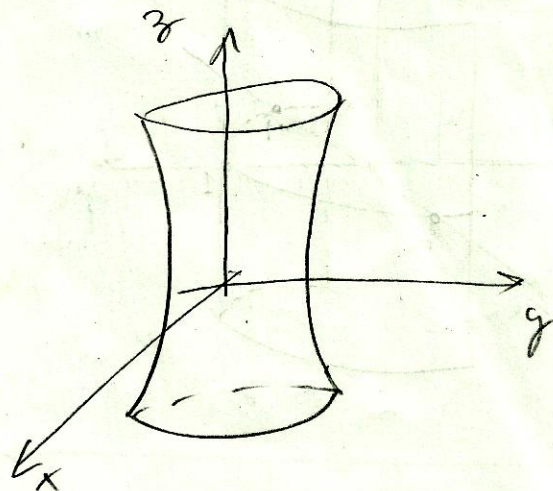
Cilindro
hiperbólico



$$54. 4x^2 + y^2 - z^2 = 16$$

$$\frac{x^2}{4} + \frac{y^2}{16} - \frac{z^2}{16} = 1$$

hiperboloide
de 1 folha



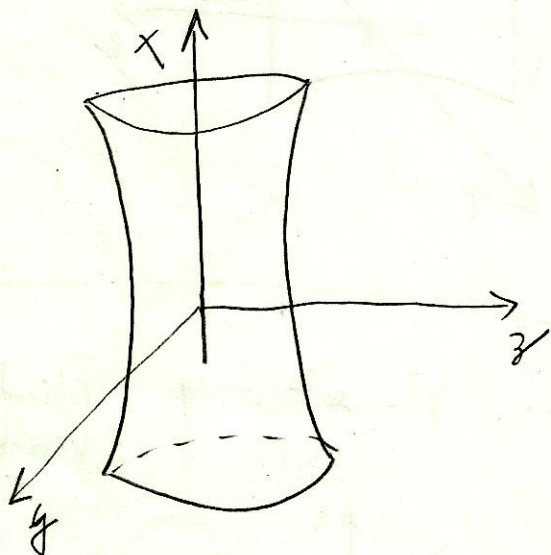
53.

$$z^2 + 4y^2 - 2x^2 = 1$$

$$\therefore$$

$$z^2 + \frac{y^2}{\frac{1}{4}} - \frac{x^2}{\frac{1}{2}} = 1$$

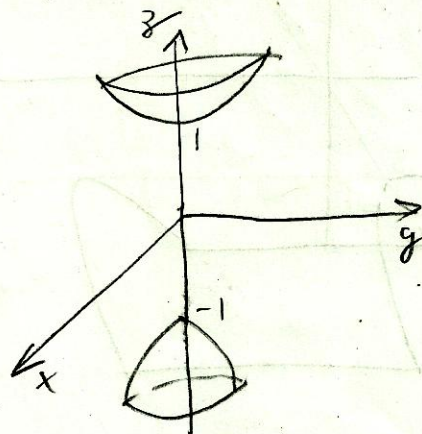
hiperboloide
de 1 folha



$$55. z^2 - 4y^2 - x^2 = 1$$

$$\therefore \frac{y^2}{\frac{1}{4}} + x^2 - z^2 = -1$$

hiperboloide de 2 folhas



$$56. x^2 - 9y^2 - 4z^2 = 36$$

$$\therefore \frac{y^2}{4} + \frac{z^2}{9} - \frac{x^2}{36} = -1$$

hiperboloide de 2 folhas

