Cálculo B - Lista 13

Derivadas parciais

Encontre as derivadas parciais de primeira ordem das funções

1. $f(x,y) = \frac{2}{3}x^{\frac{3}{2}}$
2. $f(x,y) = 2x + 3x^2y^4$
3. $g(u,v) = \frac{u^3+v^3}{u^2+v^2}$
4. $f(x,y) = \sqrt{4 - x^2 - 9y^2}$
5. $z = \sqrt{(1 - x^{\frac{2}{3}})^3 - y^2}$
6. $z = (\sin(x^2y))^3$
7. $f(x,y,z) = x^2y^5 + xz^2$
8. $f(x,y,z) = \frac{x+y+z}{xy+yz+zx}$
9. $w = e^x(\cos y + \sin z)$
10. $w = \arcsin \frac{1}{1+xyz^2}$

Nos exercícios a seguir encontre as derivadas parciais $f_{xx}, \ f_{xy}, \ f_{yy}$ da função $f(x,y)$.

11. $f(x,y) = 3x^2 - \sqrt{2}\, xy^2 + y^5 - 2$
12. $f(x,y) = x^2 - y^2$
13. $f(x,y) = \sqrt{x^2 + y^2}$
14. $f(x,y) = \frac{x}{x^2+y^2}$

Regra da cadeia

Nos exercícios a seguir use a regra da cadeia para calcular $\frac{dz}{dt}, \ \frac{dw}{dt}$

15. $z = 2x^2 - 3y^3, \quad x = \sqrt{t}, \ y = e^{2t}$
16. $z = \sin x + \cos xy, \quad x = t^2, \ y = t$
17. $z = \sqrt{2x - 4y}, \quad x = \ln t, \ y = 1 - 3t^2$
18. $w = \frac{x}{y} - \frac{z}{x}, \quad x = \sin t, \ y = \cos t, \ z = \tan t$
19. $w = \sqrt{x^2 + y^2 + z^2}, \quad x = e^t, \ y = e^{-t}, \ z = 2t$

20. $w = \sin xy^2z^3, \quad x = 3t,\ y = \sqrt{t},\ z = \sqrt[3]{t}$

21. $w = e^{-x^2-y^2}, \quad x = t,\ y = \sqrt{t}$

22. $w = \sin xyz, \quad x = t,\ y = t^2,\ z = t^3$

Nos exercícios a seguir calcule $\frac{\partial z}{\partial u}$, $\frac{\partial z}{\partial v}$

23. $z = \frac{4}{xy} - \frac{x}{y}, \quad x = u^2,\ y = uv$

24. $z = \ln(x^2 - y^2), \quad x = u - v,\ y = u^2 + v^2$

25. $z = \sin 2x\ \cos 3y, \quad x = (u+v)^2,\ y = (u-v)^2$

26. $z = xe^y + ye^{-x}, \quad x = \ln u,\ y = v \ln u$

Nos exercícios a seguir dado $w = w(x, y, z)$ com $x = x(u, v),\ y = y(u, v),\ z = z(u, v)$, calcule $\frac{\partial w}{\partial u}$ e $\frac{\partial w}{\partial v}$

27. $w = \ln(x^2 + y^2 + z^2), \quad x = u - v,\ y = u + v,\ z = 2\sqrt{uv}$

28. $w = \sqrt{x^2 + y^2 + z^2}, \quad x = 3e^v \sin u,\ y = 3e^v \cos u,\ z = 4e^v$

Derivada direcional

Para cada função $f(x, y)$ calcule a derivada direcional de f ao longo do vetor dado e avalie a derivada direcional no ponto P.

29. $f(x, y) = x^2 + 2xy + 3y^2;\ \ \vec{u} = (1, 1);\ \ P = (2, 1)$

30. $f(x, y) = e^x \sin y;\ \ \vec{u} = (1, -1);\ \ P = (0, \pi/4)$

31. $f(x, y) = x^3 - x^2y + xy^2 + y^3;\ \ \vec{u} = (2, 3);\ \ P = (1, -1)$

32. $f(x, y) = \cot \frac{y}{x};\ \ \vec{u} = (3, 4);\ \ P = (2, \pi)$

33. $f(x, y) = \sin x \cos y;\ \ \vec{v} = (4, -3);\ \ P = (\frac{\pi}{3}, -\frac{2\pi}{3})$

34. $f(x, y) = 2x^2 - 3xy + y^2 + 15;\ \ \vec{a} = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}});\ \ P = (1, 1)$

35. $f(x, y) = \frac{x^2-y^2}{x^2+y^2};\ \ \vec{a} = (\frac{1}{2}, -\frac{\sqrt{3}}{2});\ \ P = (3, 4)$

36. $f(x, y) = e^{4y};\ \ \vec{a} = (4, 0);\ \ P = (\frac{1}{2}, \frac{1}{4})$

Respostas

1. $f_x = \sqrt{x}, \qquad f_y = 0$

2. $f_x = 2 + 6xy^4, \qquad f_y = 12x^2y^3$

3. $g_u = \frac{u^4+3u^2v^2-2uv^3}{(u^2+v^2)^2}$, $\quad g_v = \frac{v^4+3u^2v^2-2vu^3}{(u^2+v^2)^2}$

4. $f_x = \frac{-x}{\sqrt{4-x^2-9y^2}}$, $\quad f_y = \frac{-9y}{\sqrt{4-x^2-9y^2}}$

5.

$$z_x = \frac{-(1-x^{\frac{2}{3}})^2}{x^{\frac{1}{3}}\sqrt{(1-x^{\frac{2}{3}})^3-y^2}}, \qquad z_y = \frac{-y}{\sqrt{(1-x^{\frac{2}{3}})^3-y^2}}$$

6. $z_x = 6xy\sin^2(x^2y)\cos(x^2y)$, $\quad z_y = 3x^2\sin^2(x^2y)\cos(x^2y)$

7. $f_x = 2xy^5 + z^2$, $\quad f_y = 5x^2y^4$, $\quad f_z = 2xz$

8.

$$f_x = \frac{-y^2-yz-z^2}{(xy+yz+xz)^2}, \qquad f_y = \frac{-x^2-xz-z^2}{(xy+yz+xz)^2}, \qquad f_z = \frac{-x^2-xy-y^2}{(xy+yz+xz)^2}$$

9. $w_x = e^x(\cos y + \sin z)$, $\quad w_y = -e^x\sin y$, $\quad w_z = e^x\cos z$

10.

$$w_x = \frac{yz^2}{\sqrt{1-(xyz^2)^2}}, \qquad w_y = \frac{xz^2}{\sqrt{1-(xyz^2)^2}}, \qquad w_z = \frac{2xyz}{\sqrt{1-(xyz^2)^2}}$$

11. $f_x = 6x - \sqrt{2}y^2$, $\quad f_y = -2\sqrt{2}xy + 5y^4$, $\quad f_{xy} = f_{yx} = -2\sqrt{2}y$
$f_{xx} = 6$, $\quad f_{yy} = -2\sqrt{2}x + 20y^3$

12. $f_x = 2x$, $\quad f_y = -2y$, $\quad f_{xy} = f_{yx} = 0$, $\quad f_{xx} = 2$, $\quad f_{yy} = -2$

13.

$$f_x = \frac{x}{\sqrt{x^2+y^2}}, \qquad f_y = \frac{y}{\sqrt{x^2+y^2}}, \qquad f_{xy} = f_{yx} = \frac{-xy}{(x^2+y^2)^{\frac{3}{2}}}$$
$$f_{xx} = \frac{y^2}{(x^2+y^2)^{\frac{3}{2}}}, \qquad f_{yy} = \frac{x^2}{(x^2+y^2)^{\frac{3}{2}}}$$

14.

$$f_x = \frac{y^2-x^2}{(x^2+y^2)^2}, \qquad f_y = \frac{-2xy}{(x^2+y^2)^2}, \qquad f_{xy} = f_{yx} = \frac{2y(3x^2-y^2)}{(x^2+y^2)^3}$$
$$f_{xx} = \frac{2x(x^2-3y^2)}{(x^2+y^2)^3}, \qquad f_{yy} = \frac{-2x(x^2-3y^2)}{(x^2+y^2)^3}$$

15. $\frac{dz}{dt} = 2 - 18e^{6t}$

16. $\frac{dz}{dt} = 2t\cos t^2 - 3t^2 \sin t^3$

17.

$$\frac{dz}{dt} = \frac{1+12t^2}{t\sqrt{2\ln t - 4 + 12t^2}}$$

18.

$$\frac{dw}{dt} = \frac{1-\sin t}{\cos^2 t}$$

19.

$$\frac{dw}{dt} = \frac{e^{2t} - e^{-2t} + 4t}{\sqrt{e^{2t} + e^{-2t} + 4t^2}}$$

20. $\frac{dw}{dt} = 9t^2\cos(3t^3)$

21. $\frac{dw}{dt} = -(2t+1)e^{-t^2-t}$

22. $\frac{dw}{dt} = 6t^5 \cos t^6$

23.

$$\frac{\partial z}{\partial u} = \frac{-12}{u^4 v} - \frac{1}{v}, \qquad \frac{\partial z}{\partial v} = \frac{-4}{u^3 v^2} + \frac{u}{v^2}$$

24.

$$\frac{\partial z}{\partial u} = \frac{2(u-v) - 4u(u^2+v^2)}{(u-v)^2 - (u^2+v^2)^2}$$
$$\frac{\partial z}{\partial v} = \frac{-2(u-v) - 4v(u^2+v^2)}{(u-v)^2 - (u^2+v^2)^2}$$

25. $\frac{\partial z}{\partial u} = 4(u+v)\cos 2(u+v)^2 \cos 3(u-v)^2 - 6(u-v)\sin 2(u+v)^2 \sin 3(u-v)^2$
$\frac{\partial z}{\partial v} = 4(u+v)\cos 2(u+v)^2 \cos 3(u-v)^2 + 6(u-v)\sin 2(u+v)^2 \sin 3(u-v)2$

26. $\frac{\partial z}{\partial u} = u^{v-1}(1 + v\ln u) + \frac{v}{u^2}(1 - \ln u)$
$\frac{\partial z}{\partial v} = (u^v \ln u + \frac{1}{u}) \ln u$

27. $\frac{\partial w}{\partial u} = \frac{2}{u+v}, \qquad \frac{\partial w}{\partial v} = \frac{2}{u+v}$

28. $\frac{\partial w}{\partial u} = 0, \qquad \frac{\partial w}{\partial v} = 5e^v$

29. $(D_{\widehat{u}} f)(2,1) = \frac{16}{\sqrt{2}}$

30. $(D_{\widehat{u}} f)(0, \frac{\pi}{4}) = 0$

31. $(D_{\widehat{u}}f)(1,-1) = \frac{12}{\sqrt{13}}$

32. $(D_{\widehat{u}}f)(2,\pi) = \frac{3\pi-8}{20}$

33. $(D_{\widehat{v}}f)(\frac{\pi}{3},-\frac{2\pi}{3}) = -\frac{13}{20}$

34. $(D_{\widehat{a}}f)(1,1) = 0$

35. $(D_{\widehat{a}}f)(3,4) = \frac{96+72\sqrt{3}}{625}$

36. $(D_{\widehat{a}}f)(\frac{1}{2},\frac{1}{4}) = 0$