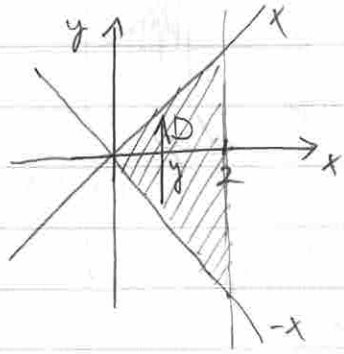
Lista 13 : Solução

1. $\int_D x^3y^2\, dA = \int_0^2\left(\int_{-x}^{x} x^3y^2\, dy\right) dx$



$$= \int_0^2 \frac{x^3y^3}{3}\Big]_{y=-x}^{x} dx$$

$$= \int_0^2 \frac{x^6}{3} - \frac{x^3(-x)^3}{3}\, dx$$

$$= \int_0^2 \frac{2x^6}{3}\, dx = \frac{2}{3}\frac{x^7}{7}\Big]_0^2 = \frac{2^8}{21} = \frac{256}{21}$$

2. $\int_D \frac{2y}{x^2+1}\, dA = \int_0^1\left(\int_0^{\sqrt{x}} \frac{2y}{x^2+1}\, dy\right) dx$

$$= \int_0^1 \frac{\frac{\cancel{2}y^2}{\cancel{2}}}{x^2+1}\Big]_0^{\sqrt{x}} dx$$

$$= \int_0^1 \frac{x}{x^2+1}\, dx = \frac{1}{2}\ln(x^2+1)\Big]_{x=0}^{1}$$

$$= \frac{1}{2}\ln 2 - \frac{1}{2}\underbrace{\ln 1}_{0}$$

$$= \frac{1}{2}\ln 2$$

/ /

3. $\int_D e^{x/y}\, dA = \int_1^2 \left(\int_y^{y^3} e^{x/y}\, dx \right) dy$

$= \int_1^2 y e^{x/y} \Big]_{x=y}^{x=y^3} dy$

$= \int_1^2 y e^{y^3/y} - y e^{y/y}\, dy$

$= \int_1^2 y(e^{y^2} - e)\, dy$

$= \int_1^2 y e^{y^2} dy - \int_1^2 y e\, dy$

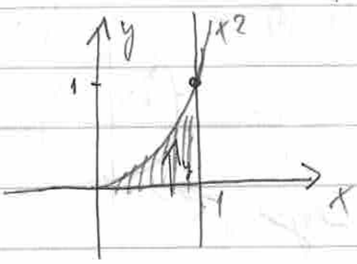
$= \frac{1}{2} e^{y^2} \Big]_1^2 - \frac{y^2}{2} e \Big]_1^2$

$= \frac{1}{2}(e^4 - e) - \left(\frac{4}{2} - \frac{1}{2}\right) e$

$= \frac{1}{2}(e^4 - e) - \frac{3}{2} e$

$= \frac{e^4}{2} - 2e$ //

4. $\int_D x\cos y \, dA = \int_0^1 \left(\int_0^{x^2} x\cos y \, dy \right) dx$



$= \int_0^1 + x \sin y \Big|_{y=0}^{y=x^2} dx$

$= \int_0^1 x \sin x^2 \, dx$

$0 \leq x \leq 1$

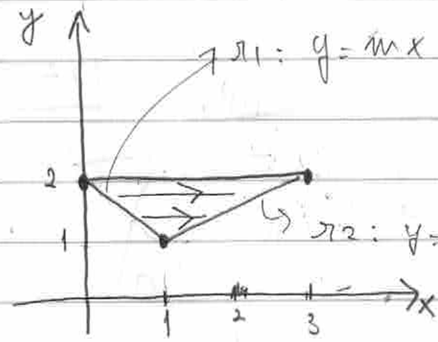
$0 \leq y \leq x^2$

$= -\frac{1}{2} \cos x^2 \Big]_0^1$

$= -\frac{1}{2}\cos 1 + \frac{1}{2}\underbrace{\cos 0}_{1}$

$= \frac{1}{2} - \frac{1}{2}\cos 1 = \frac{1}{2}(1 - \cos 1)$ //

5. $\int_D y^3 \, dA$.



$r_1: y = mx + q$: $x = 0 \Rightarrow y = 2$: $2 = q$

$x = 1 \Rightarrow y = 1$: $1 = m \cdot 1 + 2 \Rightarrow m = -1$.

$\Rightarrow r_1: y = -x + 2 \Leftrightarrow x = 2 - y$

$r_2: y = mx + q$

: $x = 1, y = 1$: $1 = m + q$; $x = 3, y = 2$: $2 = m \cdot 3 + q$ $\Big\}$ $1 = 2m \Rightarrow m = \frac{1}{2}$

$1 = \frac{1}{2} + q \Rightarrow q = +\frac{1}{2}$

$D: \begin{cases} 1 \leq y \leq 2 \\ 2 - y \leq x \leq 2y - 1 \end{cases}$

$r_2: y = \frac{1}{2}x + \frac{1}{2} \Leftrightarrow x = 2y - 1$

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$$\therefore \int_D y^3 \, dA = \int_1^2 \left(\int_{2-y}^{2y-1} y^3 \, dx \right) dy$$

$$= \int_1^2 y^3 x \Big]_{x=2-y}^{x=2y-1} dy$$

$$= \int_1^2 y^3(2y-1) - y^3(2-y) \, dy$$

$$= \int_1^2 2y^4 - y^3 - 2y^3 + y^4 \, dy$$

$$= \int_1^2 3y^4 - 3y^3 \, dy$$

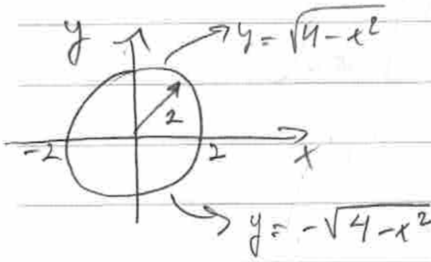
$$= \frac{3y^5}{5} - \frac{3y^4}{4} \Big]_1^2 = \frac{3}{5}2^5 - \frac{3 \cdot 2^4}{4} - \left(\frac{3}{5} - \frac{3}{4}\right)$$

$$= \frac{3 \cdot 32}{5} - \frac{3 \cdot 16}{4} - \frac{3}{5} + \frac{3}{4}$$

$$= \frac{93}{5} - \frac{45}{4} = \frac{372 - 225}{20}$$

$$= \frac{147}{20} \quad //$$

6. $\int_D (2x-y)\,dA = \int_{-2}^{2}\left(\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} (2x-y)\,dy\right)dx$



$= \int_{-2}^{2} 2xy - \frac{y^2}{2}\Big]_{y=-\sqrt{4-x^2}}^{y=\sqrt{4-x^2}}\,dx$

$= \int_{-2}^{2}\left[2x\sqrt{4-x^2} - \cancel{\frac{(4-x^2)}{2}} - \left(2x(-)\sqrt{4-x^2} - \cancel{\frac{(4-x^2)}{2}}\right)\right]dx$

$D: \begin{cases} -2 \le x \le 2 \\ -\sqrt{4-x^2} \le y \le \sqrt{4-x^2} \end{cases}$

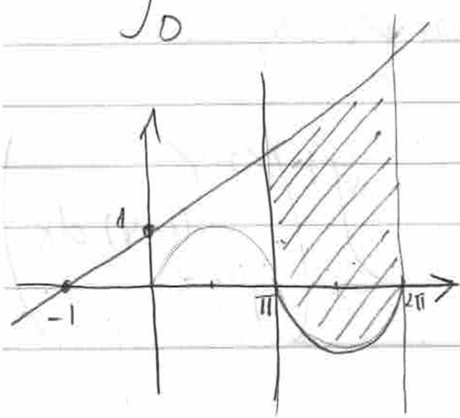
$= \int_{-2}^{2} 4x\sqrt{4-x^2}\,dx$

$= \frac{4\cancel{2}}{3\cancel{2}}\cdot\frac{1}{}(4-x^2)^{3/2}\Big]_{-2}^{2} = -\frac{4}{3}(4-x^2)^{3/2}\Big]_{-2}^{2}$

$= -\frac{4}{3}(4-4)^{3/2} + \frac{4}{3}(4-(-2)^2)^{3/2}$

$= 0$ //

7. $\int_D 1\,dA$



$D: \begin{cases} \pi \le x \le 2\pi \\ \sin x \le y \le x+1 \end{cases}$

$$\int_D 1\, dA = \int_{\pi}^{2\pi}\left(\int_{\sin x}^{x+1} dy\right)dx$$

$$= \int_{\pi}^{2\pi} y\Big]_{y=\sin x}^{y=x+1} dx$$

$$= \int_{\pi}^{2\pi}(x+1-\sin x)\,dx$$

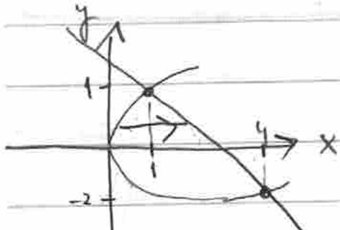
$$= \frac{x^2}{2} + x + \cos x\Big]_{\pi}^{2\pi}$$

$$= \frac{4\pi^2}{2} + 2\pi + 1 - \left(\frac{\pi^2}{2} + \pi - 1\right)$$

$$= 2\pi^2 + 2\pi + 1 - \frac{\pi^2}{2} - \pi + 1$$

$$= \frac{3\pi^2}{2} + \pi + 2 \quad //$$

8. $\int_D (1-y)\, dA$



$$D: \begin{cases} -2 \le y \le 1 \\ y^2 \le x \le 2-y \end{cases}$$

$\left.\begin{array}{l} x=y^2 \\ x=2-y \end{array}\right\}$ $y^2 = 2-y$

$y^2+y-2=0$

$\begin{cases} y=1 \to x=1 \\ y=-2 \to x=4 \end{cases}$

$$\therefore \int_D (1-y)\, dA = \int_{-2}^{1}\left(\int_{y^2}^{2-y}(1-y)\,dx\right)dy$$

8. Cont.

$$= \int_{-2}^{1} \left(\int_{y^2}^{2-y} (1-y)\, dx \right) dy$$

$$= \int_{-2}^{1} (1-y)x \Big]_{x=y^2}^{x=2-y} dy$$

$$= \int_{-2}^{1} (1-y)(2-y) - (1-y)y^2 \; dy$$

$$= \int_{-2}^{1} 2-2y-y+\cancel{y^2}-\cancel{y^2}+y^3 \; dy$$

$$= \int_{-2}^{1} y^3-3y+2 \; dy$$

$$= \frac{y^4}{4} - \frac{3y^2}{2} + 2y \Big]_{-2}^{1}$$

$$= \frac{1}{4} - \frac{3}{2} + 2 - \left(\frac{16}{4} - \frac{3 \cdot \cancel{4}}{\cancel{2}} - 4 \right)$$

$$= \frac{1}{4} - \frac{3}{2} + 2 - (\cancel{4} - 6 - \cancel{4})$$

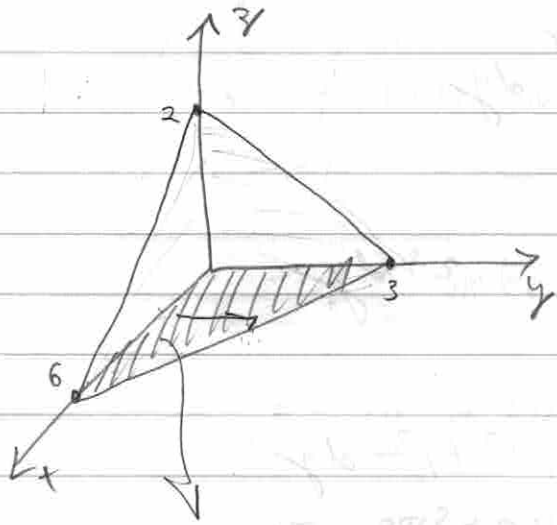
$$= \frac{1}{4} - \frac{3}{2} + 2 + 6$$

$$= \frac{1}{4} - \frac{3}{2_{/2}} + 8_{/4} = \frac{1-6+32}{4} = \frac{27}{4}$$

9.

$x + 2y + 3z = 6 \Leftrightarrow 3z = 6 - x - 2y$

$z = \dfrac{6 - x - 2y}{3} = 2 - \dfrac{x}{3} - \dfrac{2}{3}y$



$x + 2y + 3z = 6$

$x = 0: \quad 2y + 3z = 6$

$z = 2 - \dfrac{2y}{3}$

$y = 0: \quad x + 3z = 6$

$z = 2 - \dfrac{x}{3}$

$z = 0: \quad x + 2y = 6$

$y = 3 - \dfrac{x}{2}$

$D: \begin{cases} 0 \le x \le 6 \\ 0 \le y \le 3 - \frac{x}{2} \end{cases}$

$\therefore \quad V = \int_D z(x,y)\, dA = \int_0^6 \left(\int_0^{3-\frac{x}{2}} \left(2 - \frac{x}{3} - \frac{2}{3}y\right) dy \right) dx$

$= \int_0^6 \left. 2y - \frac{x}{3}y - \frac{\cancel{2}}{3}\frac{y^2}{\cancel{2}} \right]_0^{3-\frac{x}{2}} dx$

$= \int_0^6 2\left(3 - \frac{x}{2}\right) - \frac{x}{3}\left(3 - \frac{x}{2}\right) - \frac{1}{3}\left(3 - \frac{x}{2}\right)^2 dx$

$= \int_0^6 6 - x - x + \frac{x^2}{6} - \frac{1}{3}\left(9 - 3x + \frac{x^2}{4}\right) dx$

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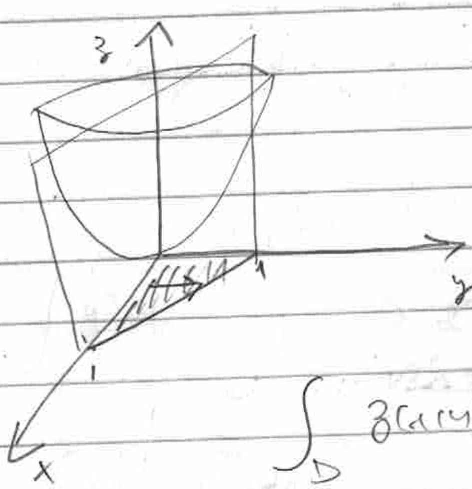
$$= \int_0^6 \left(6 - 2x + \frac{x^2}{6} - 3 + x - \frac{x^2}{12}\right) dx$$

$$= \int_0^6 \left(3 - x + \frac{x^2}{12}\right) dx$$

$$= 3x - \frac{x^2}{2} + \frac{x^3}{36} \Big]_0^6$$

$$= 18 - \frac{36}{2} + \frac{6^3}{36} = \cancel{18} - \cancel{18} + 6 = 6$$

10. $z = x^2 + y^2$



$$D: \begin{cases} 0 \le x \le 1 \\ 0 \le y \le 1 - x \end{cases}$$

$$\int_D z(x,y)\, dA = \int_0^1 \left(\int_0^{1-x} (x^2 + y^2)\, dy \right) dx$$

$$= \int_0^1 x^2 y + \frac{y^3}{3} \Big]_{y=0}^{y=1-x} dx$$

$$= \int_0^1 x^2(1-x) + \frac{(1-x)^3}{3}\, dx$$

$$= \int_0^1 x^2 - x^3 + \frac{1}{3} + x^2 - x - \frac{x^3}{3}\, dx$$

$$= \int_0^1 2x^2 - \frac{4x^3}{3} - x + \frac{1}{3} \, dx$$

$$= \frac{2x^3}{3} - \frac{4}{3}\frac{x^4}{4} - \frac{x^2}{2} + \frac{1}{3}x \Big]_0^1$$

$$= \frac{2}{3} \cancel{-\frac{1}{3}} - \frac{1}{2} \cancel{+\frac{1}{3}} = \frac{1}{6}$$