Cálculo B – Recuperação

1. Resolva a integral

$$\int \sin^4 z \, dz$$

2. Calcule a área da superfície de revolução gerada pela rotação da curva

$$y = x^3 \;, \quad 1 \leq x \leq 2$$

em torno do eixo x.

3. Encontre os valores extremos de
$f(x,y) = 5 - 3x + 4y$
definida na região triangular fechada com vértices $(0,0)$, $(4,0)$ e $(4,5)$.
$(4,5)$

4. Encontre as dimensões de uma caixa retangular com volume máximo e que tenha área superficial 600.

5. Calcule a integral tripla $\int_\Omega 3y^2 \, dV$ onde Ω é a região sólida limitada acima pelo plano $z = 1$ e abaixo pelo cone $z = \sqrt{x^2 + y^2}$.

Cálculo B - Doc.

1. $\int \sin^4 z \, dz = \int (\sin^2 z)^2 dz$

$$= \int \left(\frac{1-\cos 2z}{2}\right)^2 dz$$

$$= \int \left(\frac{1}{4} - \frac{\cos 2z}{2} + \frac{\cos^2 2z}{4}\right) dz$$

$$= \frac{1}{4}z - \frac{1}{4}\sin 2z + \frac{1}{4}\int \cos^2 2z \, dz$$

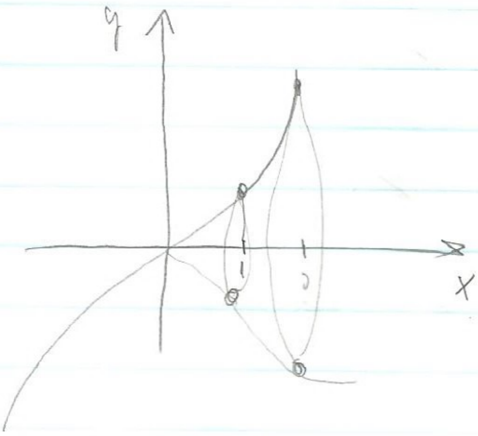
$$= \frac{1}{4}z - \frac{1}{4}\sin 2z + \frac{1}{4}\int \frac{1+\cos 4z}{2} dz$$

$$= \frac{1}{4}z - \frac{1}{4}\sin 2z + \frac{1}{8}\int (1+\cos 4z) \, dz$$

$$= \frac{1}{4}z - \frac{1}{4}\sin 2z + \frac{1}{8}z + \frac{1}{32}\sin 4z$$

$$= \frac{3}{8}z - \frac{1}{4}\sin 2z + \frac{1}{32}\sin 4z \quad //$$

2. $y = x^3 \quad , \quad 1 \leq x \leq 2$



$$S = \int_1^2 2\pi\, y(x) \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$= \int_1^2 2\pi x^3 \sqrt{1 + (3x^2)^2}\, dx$$

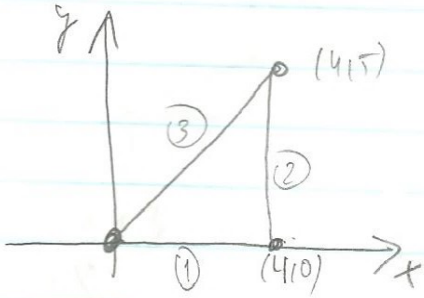
$$= \int_1^2 2\pi x^3 \sqrt{1 + 9x^4}\, dx$$

$$= \cancel{2\pi}\, \frac{\cancel{2}}{3} \frac{1}{\cancel{36}\,9} (1 + 9x^4)^{3/2} \Big]_1^2$$

$$= \frac{\pi}{27} \left((1 + 9 \cdot 16)^{3/2} - (1 + 9)^{3/2} \right)$$

$$= \frac{\pi}{27} \left(145^{3/2} - 10^{3/2} \right)$$

3. $f(x,y) = 5 - 3x + 4y$



Pontos críticos

$f_x = 0 \therefore -3 = 0$

$f_y = 0 \therefore 4 = 0$

não é satisfeita

Fronteira

- l_1: $y = 0$, $0 \leq x \leq 4$

$h(x) = f(x, y=0) = 5 - 3x$

$h'(x) = -3 \neq 0$

Só temos que considerar os extremos

$x = 0, x = 4 \therefore (0,0), (4,0)$

• ℓ_2 : $x = 4$, $0 \le y \le 5$

$h(y) = f(x=4, y) = -7 + 4y$

$h'(y) = 4 \neq 0$

Extremos do intervalo : $y = 0$ ou $y = 5$.

$\underline{(0,0), (4,5)}$

• ℓ_3 : $y = \frac{5}{4}x$, $0 \le x \le 4$

$h(x) = f(x, y = \frac{5}{4}x) = 5 - 3x + \cancel{4}\frac{5}{\cancel{4}}x = 5 + 2x$

$h'(x) = 2 \neq 0$.

Extremos do intervalo : $\underline{(0,0), (4,5)}$

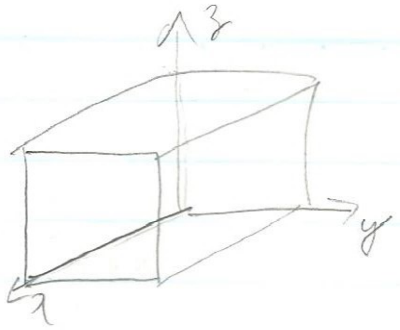
Identificação dos extremos :

(x,y)	$f(x,y)$
$(0,0)$	5
$(4,0)$	-7
$(4,5)$	13

Máximo $f = 13$

Mínimo $f = -7$

4.



$$\begin{cases} V = xyz \\ S = 2xy + 2xz + 2yz = 600 \end{cases}$$

$$g(x,y,z) = xy + xz + yz - 300$$

$$\begin{cases} V_x = \lambda g_x \longrightarrow yz = \lambda(y+z) \\ V_y = \lambda g_y \longrightarrow xz = \lambda(x+z) \\ V_z = \lambda g_z \longrightarrow xy = \lambda(x+y) \\ g(x,y,z) = 0 \longrightarrow xy + xz + yz = 300 \end{cases}$$

$$yz = \lambda y + \lambda z \Leftrightarrow y(z-\lambda) = \lambda z$$

$$y = \frac{\lambda z}{z-\lambda}$$

$$xz = \lambda(x+z) \Leftrightarrow x = \frac{\lambda z}{z-\lambda}$$

$$xy = \lambda(x+y) \Leftrightarrow$$

$$\cancel{\frac{\lambda^2 z^2}{(z-\lambda)^2}} = \cancel{\lambda\left(\frac{\lambda z}{z-\lambda} + \frac{\lambda z}{z-\lambda}\right)}$$

$$\frac{z^{\cancel{2}}}{z-\lambda} = 2\cancel{z} \Rightarrow z = 2z - 2\lambda \Rightarrow \boxed{z = 2\lambda}$$

$$x = \frac{\lambda z}{z-\lambda} = \frac{\lambda 2\lambda}{2\lambda-\lambda} = \frac{2\lambda^2}{\lambda} = 2\lambda$$

$$\therefore \quad // x = y = z = 2\lambda //$$

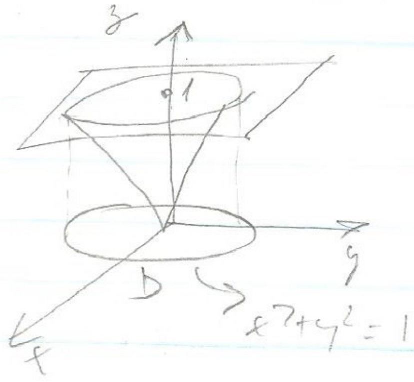
$$xy + xz + yz = 300$$
$$4\lambda^2 + 4\lambda^2 + 4\lambda^2 = 300$$
$$12\lambda^2 = 300$$
$$\lambda^2 = 25 \implies // \lambda = 5 //$$

300 | 12
60 | 25
00

$$\therefore \quad \boxed{x = y = z = 10}$$

As dimensões da caixa devem ser $x = y = z = 10$

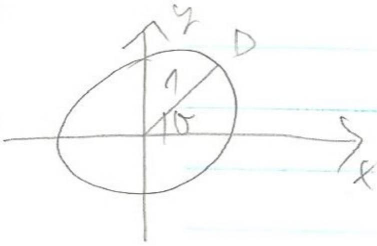
5. $\displaystyle\int_\Omega z y^2 \, dV$



$$\Omega : \begin{cases} \sqrt{x^2+y^2} \le z \le 1 \\ x^2+y^2=1 \end{cases}$$

$$\int_\Omega z y^2 \, dV = \int_D \int_{\sqrt{x^2+y^2}}^{1} z y^2 \, dz \, dA$$

$$= \int_D \left. \frac{z^2 y^2}{2} \right]_{\sqrt{x^2+y^2}}^{1} dA$$



$$= \int_D \left(\frac{y^2}{2} - \frac{(x^2+y^2)y^2}{2} \right) dA$$

$x = r\cos\theta$
$y = r\sin\theta$

$$= \int_D \frac{1}{2}\left(1-(x^2+y^2)\right) y^2 \, dA$$

$$\begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \theta} \end{vmatrix} = \begin{vmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{vmatrix}$$

$= r\cos^2\theta + r\sin^2\theta$
$= r$

$$= \int_0^1 \int_0^{2\pi} \frac{1}{2}(1-r^2) r^2 \sin^2\theta \; r \, dr \, d\theta$$

$$= \int_0^1 \int_0^{2\pi} \frac{1}{2}(1-r^2) r^3 \sin^2\theta \, d\theta \, dr$$

$$= \int_0^1 \frac{1}{2}(1-r^2) r^3 \left(\int_0^{2\pi} \underbrace{\sin^2\theta}_{\frac{1-\cos 2\theta}{2}} \, d\theta \right) dr$$

$$= \int_0^1 \frac{1}{2}(1-r^2)\,r^3 \cdot \left(\frac{1}{2}\theta - \frac{\sin 2\theta}{4}\right)\Bigg|_0^{2\pi} dr$$

$$= \int_0^1 \frac{1}{2}(1-r^2)\,r^3 \cdot \pi \; dr$$

$$= \frac{\pi}{2}\int_0^1 (r^3 - r^5)\,dr$$

$$= \frac{\pi}{2}\left(\frac{r^4}{4} - \frac{r^6}{6}\right)\Bigg|_0^1$$

$$= \frac{\pi}{2}\left(\frac{1}{4} - \frac{1}{6}\right) = \frac{\pi}{\cancel{2}}\,\frac{\cancel{2}}{24} = \frac{\pi}{24}$$