Cálculo B - Recuperação

1.0 1. Calcule

$$\int \frac{2\sin x - 1}{\cos^2 x}\, dx$$

1.0 2. Calcule

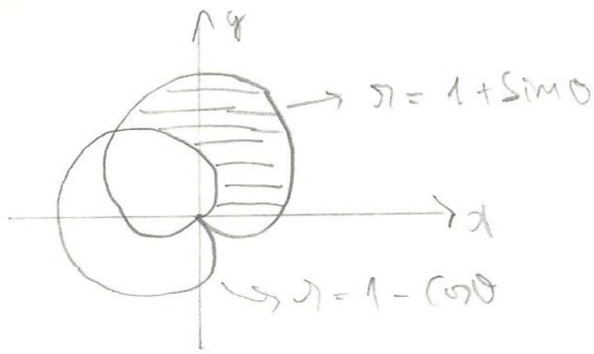
$$\int \frac{\sqrt{16 - e^{2x}}}{e^x}\, dx$$

0.5 3. (i) Faça um esboço do domínio de $f(x, y) = \sqrt{y - x}$

0.5 (ii) Faça um esboço do gráfico de $f(x, y) = 3 - x^2 - y^2$

1.0 (iii) Represente as curvas de nível de $f(x, y) = \sqrt{x^2 - y^2}$ para $c = 0$ e $c = 1$.

2.0 4. Calcule a área da região mostrada na figura



1.5 5. Determine o volume da maior caixa retangular no primeiro octante e que tem três faces nos planos coordenados e um vértice no plano $x + 2y + 3z = 6$.

1.5 6. Calcule $\int_\Omega x^2\, dV$ onde Ω é a região no interior da superfície $x^2 + y^2 = 1$, acima do plano $z = 0$ e abaixo da superfície $z^2 = 4x^2 + 4y^2$.

1. $\displaystyle\int \frac{2\sin x - 1}{\cos^2 x}\,dx$

$\displaystyle = \int 2\,\frac{\sin x}{\cos x}\,\frac{1}{\cos x}\,dx - \int \frac{1}{\cos^2 x}\,dx$

$\displaystyle = 2\int \operatorname{tg} x \sec x\,dx - \int \sec^2 x\,dx$

$= 2\sec x - \operatorname{tg} x$

2. $\displaystyle\int \frac{\sqrt{16-e^{2x}}}{e^x}\,dx$

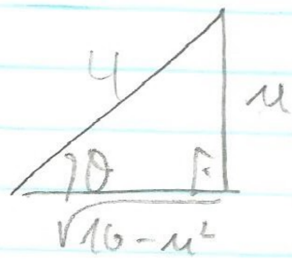
$u = e^x \quad , \quad du = e^x\,dx \quad \therefore \quad dx = \frac{1}{u}\,du$

$\displaystyle\therefore \int \frac{\sqrt{16-e^{2x}}}{e^x}\,dx = \int \frac{\sqrt{16-u^2}}{u}\,\frac{1}{u}\,du$

$\displaystyle = \int \frac{\sqrt{16-u^2}}{u^2}\,du$

Seja $\quad u = 4\sin\theta$

$\quad du = 4\cos\theta\,d\theta$



$\displaystyle = \int \frac{\sqrt{16-16\sin^2\theta}}{16\sin^2\theta}\,4\cos\theta\,d\theta$

$\displaystyle = \int \frac{\cancel{4}\sqrt{1-\sin^2\theta}\;\cancel{4}\cos\theta}{\cancel{16}\sin^2\theta}\,d\theta$

$\displaystyle = \int \frac{\cos\theta\cos\theta}{\sin^2\theta}\,d\theta$

$\displaystyle = \int \cot^2\theta\,d\theta = \int (\csc^2\theta - 1)\,d\theta$

$\displaystyle = -\cot\theta - \theta = -\frac{\sqrt{16-u^2}}{u} - \arcsin\frac{u}{4}$

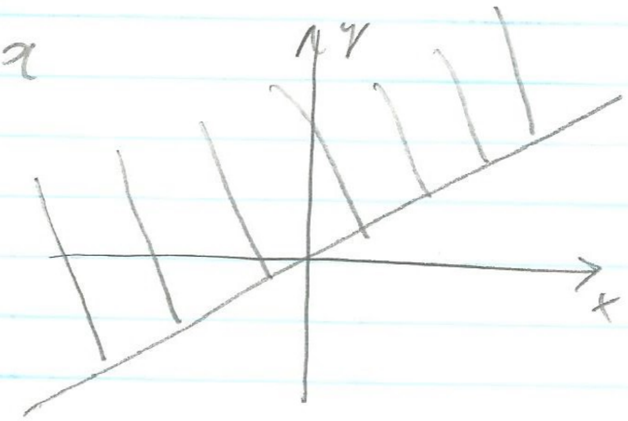
$$= -\frac{\sqrt{16-e^{2x}}}{e^{x}} - \arcsin \frac{e^{x}}{4}$$

3.

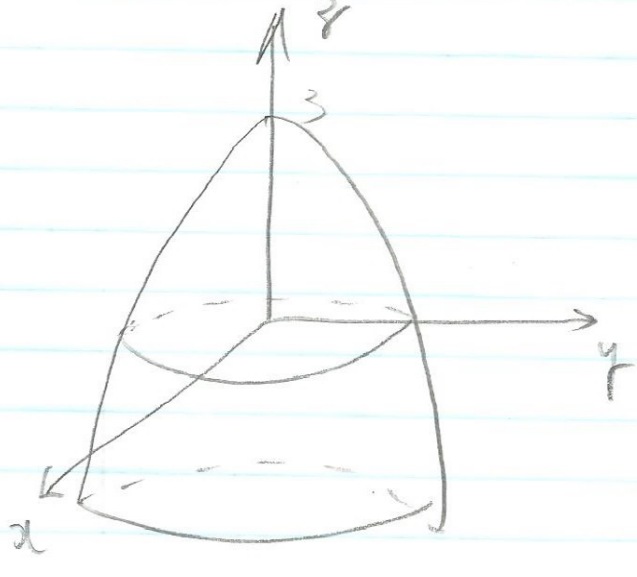
(i) $f(x,y) = \sqrt{y - x}$

$y - x \geq 0$

$\therefore \quad y \geq x$



(ii) $f(x,y) = 3 - x^2 - y^2$

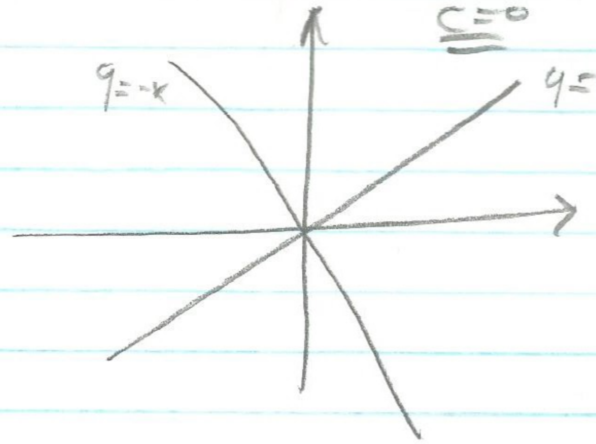


(iii) $f(x,y) = \sqrt{x^2-y^2}$

$\underline{\underline{c=0}}$; $0 = \sqrt{x^2-y^2}$

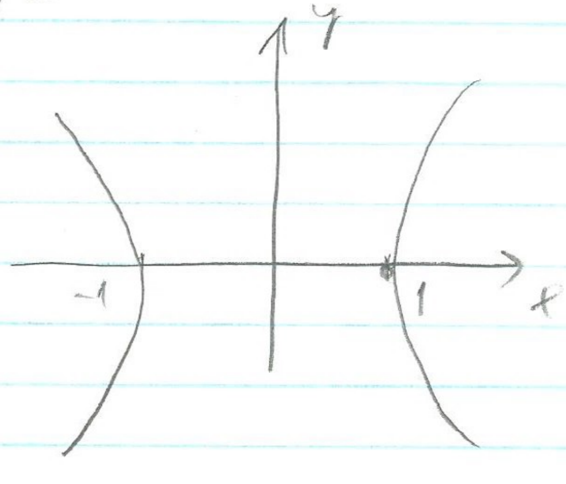
$\therefore \quad x^2 = y^2$

$\therefore \quad x = \pm y$

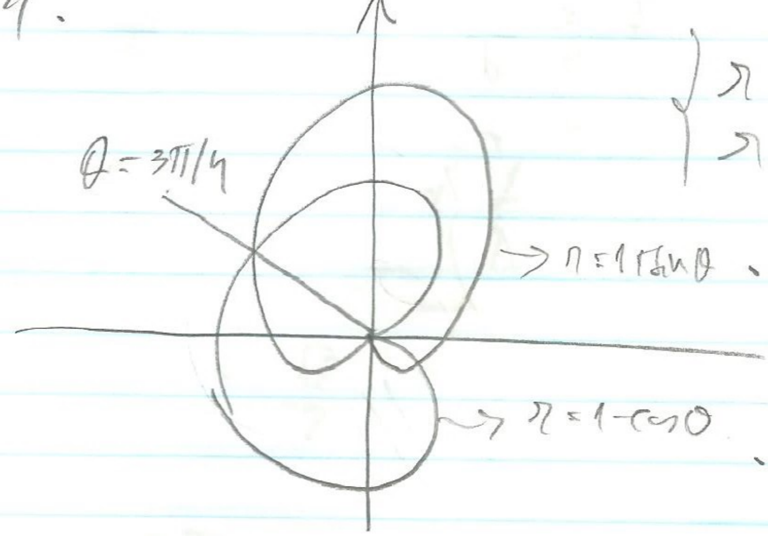


$\underline{\underline{c=1}}$; $1 = \sqrt{x^2-y^2}$

$1 = x^2-y^2$



4.

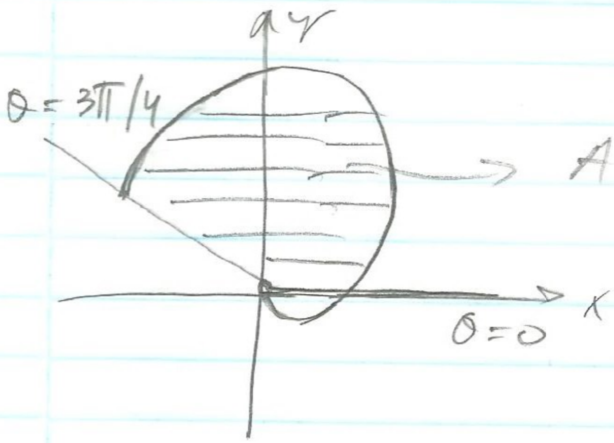


$$\begin{cases} r = 1+\sin\theta \\ r = 1-\cos\theta \end{cases}$$

$\therefore\ 1+\sin\theta = 1-\cos\theta$

$\therefore\ \sin\theta = -\cos\theta$

$\therefore\ \begin{cases} \theta = \frac{3\pi}{4} + 2n\pi \\ \text{or} \\ \theta = -\frac{\pi}{4} + 2n\pi \end{cases}$



$$A1 = \int_0^{\frac{3\pi}{4}} \frac{1}{2}(1+\sin\theta)^2\,d\theta$$

$$= \frac{1}{2}\int_0^{\frac{3\pi}{4}} (1+2\sin\theta+\sin^2\theta)\,d\theta$$

$$= \frac{1}{2}\left(\theta\Big]_0^{\frac{3\pi}{4}} - 2\cos\theta\Big]_0^{\frac{3\pi}{4}} + \int_0^{\frac{3\pi}{4}} \sin^2\theta\,d\theta\right)$$

$\therefore$

$$A1 = \frac{1}{2}\left(\frac{3\pi}{4} - 2\cos\frac{3\pi}{4} + 2\cos 0 + \int_0^{\frac{3\pi}{4}} \frac{1-\cos 2\theta}{2}\,d\theta\right)$$

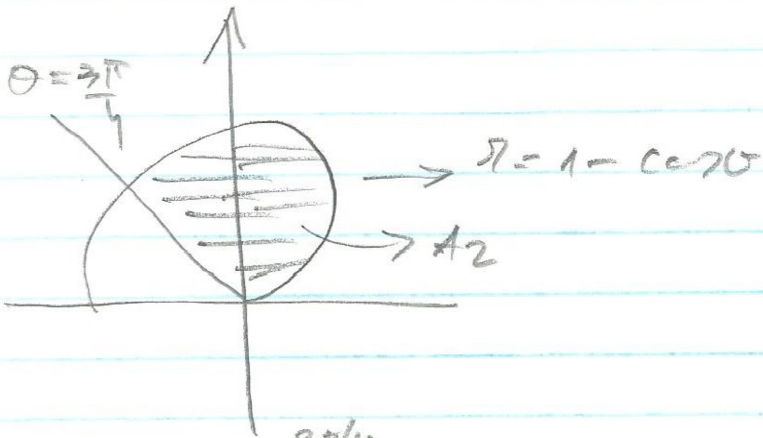
$$= \frac{1}{2}\left(\frac{3\pi}{4} - 2\left(-\frac{\sqrt{2}}{2}\right) + 2 + \frac{1}{2}\theta\Big]_0^{\frac{3\pi}{4}} - \frac{1}{4}\sin 2\theta\Big]_0^{\frac{3\pi}{4}}\right)$$

$$= \frac{1}{2}\left(\frac{3\pi}{4} + \sqrt{2} + 2 + \frac{3\pi}{8} - \frac{1}{4} \ln \frac{3\pi}{2} \right)$$

$$= \frac{1}{2}\left(\frac{9\pi}{8} + \sqrt{2} + 2 + \frac{1}{4} \right)$$

$$= \frac{1}{2}\left(\frac{9\pi}{8} + \sqrt{2} + 2 + \frac{1}{4} \right)$$

$$= \frac{1}{2}\left(\frac{9\pi}{8} + \sqrt{2} + \frac{9}{4} \right) //$$



$$A_2 = \int_0^{3\pi/4} \frac{1}{2}(1-\cos\theta)^2 \, d\theta$$

$$= \int_0^{\frac{3\pi}{4}} \left(\frac{1}{2} - \cos\theta + \frac{1}{2}\cos^2\theta \right) d\theta$$

$$= \frac{1}{2}\theta \Big]_0^{\frac{3\pi}{4}} - \sin\theta \Big]_0^{\frac{3\pi}{4}} + \frac{1}{2}\int_0^{\frac{3\pi}{4}} \cos^2\theta \, d\theta$$

$$= \frac{3\pi}{8} - \sin\frac{3\pi}{4} + \cancel{\sin\theta}^{\,0} + \frac{1}{2}\int_0^{3\pi/4} \frac{1+\cos 2\theta}{2} \, d\theta$$

$$= \frac{3\pi}{8} - \frac{\sqrt{2}}{2} + \frac{1}{4}\left(\theta\Big]_0^{\frac{3\pi}{4}} + \frac{\sin 2\theta}{2}\Big]_0^{\frac{3\pi}{4}}\right)$$

$$= \frac{3\pi}{8} - \frac{\sqrt{2}}{2} + \frac{1}{4}\left(\frac{3\pi}{4} + \frac{1}{2}\underbrace{\sin\frac{3\pi}{2}}_{-1} - \frac{1}{2}\overbrace{\sin 0}^{0}\right)$$

$$= \frac{3\pi}{8} - \frac{\sqrt{2}}{2} + \frac{3\pi}{16} - \frac{1}{8}$$

$$A_2 = \frac{9\pi}{16} - \frac{\sqrt{2}}{2} - \frac{1}{8} \quad //$$

$$A = A_1 - A_2$$

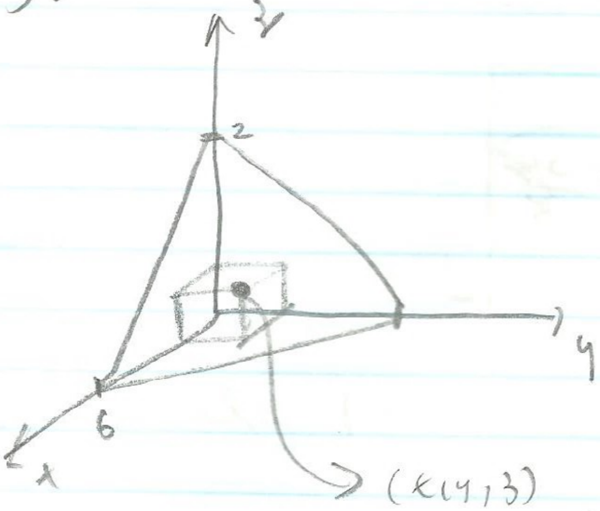
$$= \frac{9\pi}{16} + \frac{\sqrt{2}}{2} + \frac{9}{8} - \left(\frac{9\pi}{16} - \frac{\sqrt{2}}{2} - \frac{1}{8}\right)$$

$$= \cancel{\frac{9\pi}{16}} + \frac{\sqrt{2}}{2} + \frac{9}{8} - \cancel{\frac{9\pi}{16}} + \frac{\sqrt{2}}{2} + \frac{1}{8}$$

$$= \sqrt{2} + \frac{10}{8}$$

$$// \; A = \sqrt{2} + \frac{5}{4} \; //$$

5.



(x,y,z) ; $x+2y+3z=6$

$$\begin{cases} V = xyz & ; \quad x>0,\ y>0,\ z>0 \\ \phi = x+2y+3z-6 \end{cases}$$

$$\begin{cases} V_x = \lambda \phi_x \longrightarrow yz = \lambda & (*) \\ V_y = \lambda \phi_y \longrightarrow xz = \lambda 2 & (**) \\ V_z = \lambda \phi_z \longrightarrow xy = \lambda 3 & (***) \\ \phi = 0 \longrightarrow x+2y+3z = 6 & (4*) \end{cases}$$

$(*)$, $(**)$: $\quad y\cancel{z} = \frac{x\cancel{z}}{2} \quad \therefore \quad y = \frac{x}{2} \quad (5*)$

$(*)$, $(***)$: $\quad \cancel{y}z = \frac{x\cancel{y}}{3} \quad \therefore \quad z = \frac{x}{3} \quad (6*)$

$(**)$, $(***)$: $\quad \frac{\cancel{x}z}{2} = \frac{\cancel{x}y}{3} \quad \therefore \quad z = \frac{2y}{3}$

Temos a consistência das eqs. pois:

$$\left.\begin{array}{l} z = \frac{x}{3} \\ z = \frac{2y}{3} \end{array}\right\} \therefore \frac{x}{\cancel{3}} = \frac{2y}{\cancel{3}} \therefore x = 2y \quad \text{ok!}$$

Subst. (5*), (6*) em (4*) temos:

$$x + 2y + 3z = 6$$

$$\therefore x + x + x = 6$$

$$3x = 6 \therefore \underline{\underline{x = 2}}$$

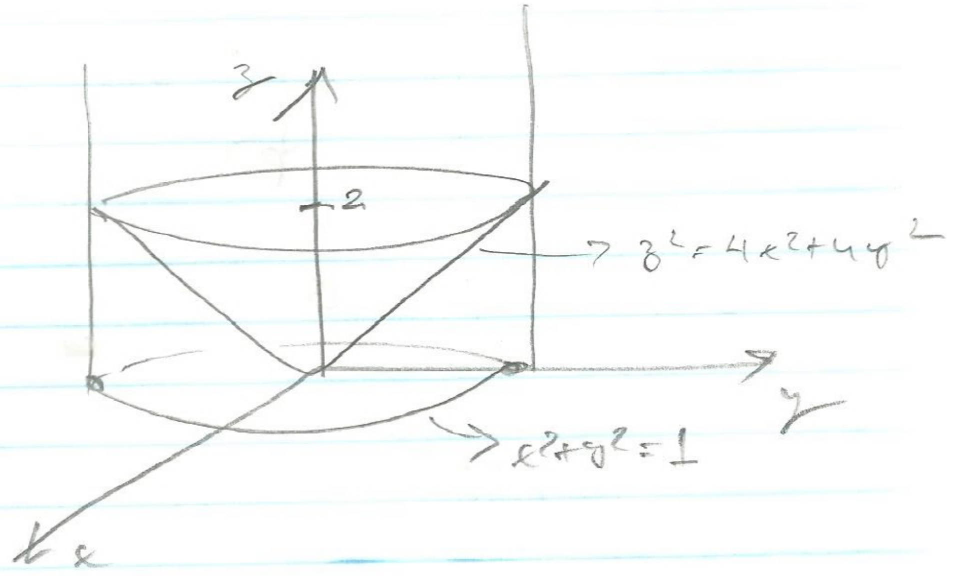
$$y = \frac{x}{2} = 1 \therefore \underline{\underline{y = 1}}$$

$$z = \frac{x}{3} \therefore \underline{\underline{z = \frac{2}{3}}}$$

$$V = xyz = 2 \cdot 1 \cdot \frac{2}{3} = \frac{4}{3}$$

$$\therefore \boxed{V = \frac{4}{3}}$$

6. $\displaystyle\int_\Omega x^2\,dV$



$$\Omega = \begin{cases} 0 \le z \le 2\rho \\ 0 \le \rho \le 1 \\ 0 \le \theta \le 2\pi \end{cases}$$

$$\int_{\Omega(x,y,z)} x^2\,dV = \int_{\Omega(\rho,\theta,z)} (\rho\cos\theta)^2\,\rho\,d\rho\,d\theta\,dz$$

$$= \int_{\theta=0}^{2\pi}\int_{\rho=0}^{1}\int_{z=0}^{2\rho} \rho^2\cos^2\theta\,dz\,d\rho\,d\theta$$

$$= \int_{\theta=0}^{2\pi}\int_{\rho=0}^{1} \rho^2\cos^2\theta\, z\Big]_0^{2\rho}\,d\rho\,d\theta$$

$$= \int_{\theta=0}^{2\pi}\int_{\rho=0}^{1} 2\rho^3\cos^2\theta\,d\rho\,d\theta$$

$$= \int_{\theta=0}^{2\pi} \cancel{2}\frac{\rho^4}{\cancel{4}2}\cos^2\theta \Big]_{\rho=0}^{1} d\theta$$

$$= \int_{\theta=0}^{2\pi} \frac{1}{2}\cos^2\theta \, d\theta$$

$$= \frac{1}{2}\int_0^{2\pi} \frac{1+\cos 2\theta}{2} \, d\theta$$

$$= \frac{1}{4}\left(\theta\Big|_0^{2\pi} + \frac{\sin 2\theta}{2}\Big|_0^{2\pi} \right)$$

$$= \frac{1}{4}\left(2\pi \right) = \frac{\pi}{2}$$