## Cálculo B - Recuperação

1. Resolva $\int \frac{1}{x^3+x}\,dx$. (1.5)

2. Resolva $\int_{-8}^{1} \frac{1}{\sqrt[3]{x}}\,dx$ (1.5)

3. Seja $f(x, y) = \frac{\sqrt{x^2-y^2}}{2}$.
   (i) Faça um esboço do domínio da $f$ 0.5
   (ii) Represente as curvas de nível de $f$ para $k = 0$, $k = 1$ 1.0
   (iii) Faça um esboço do gráfico da $f$ 0.5

   2.0

4. Calcule, caso exista, o limite

$$\lim_{(x,y)\to(0,0)} |y|^{|x|}$$

1.0

5. Encontre o volume máximo de uma caixa retangular que tem um de seus vértices na origem e o vértice oposto situado no plano $x + 2y + z = 3$. (1.5)

6. Usando coordenadas cilíndricas, determine o volume da região no interior de $x^2 + y^2 = 1$, no exterior de $z = \sqrt{x^2 + y^2}$ e acima de $z = 0$. (1.5)

1. $\int \frac{1}{x^3+x}\,dx = \int \frac{1}{x(x^2+1)}\,dx$

$$\frac{1}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1} \qquad (0.5)$$

$$= \frac{Ax^2+A+Bx^2+Cx}{x(x^2+1)}$$

$$\frac{1}{x(x^2+1)} = \frac{(A+B)x^2+Cx+A}{x(x^2+1)}$$

$\therefore$ $A+B=0 \Rightarrow // B=-A=-1 //$

$// C=0 //$

$// A=1 //$ (1.0)

$\therefore$

$$\frac{1}{x(x^2+1)} = \frac{1}{x} + \frac{-x}{x^2+1}$$

$\therefore$

$$\int \frac{1}{x^3+x}\,dx = \int \frac{1}{x}\,dx - \int \frac{x}{x^2+1}\,dx$$

$$= \ln|x| - \frac{1}{2}\ln|x^2+1| + C$$

(1.5)

2. $$\int_{-8}^{1} \frac{1}{\sqrt[3]{x}}\, dx =$$

(0.5)

$$= \lim_{t \to 0^-} \int_{-8}^{t} \frac{1}{\sqrt[3]{x}}\, dx + \lim_{t \to 0^+} \int_{t}^{1} \frac{1}{\sqrt[3]{x}}\, dx$$

$$= \lim_{t \to 0^-} \frac{3}{2} x^{2/3} \Big]_{-8}^{t} + \lim_{t \to 0^+} \frac{3}{2} x^{2/3} \Big]_{t}^{1}$$

$$= \lim_{t \to 0^-} \left\{ \frac{3}{2} t^{2/3} - \frac{3}{2} (-8)^{2/3} \right\} +$$

$$+ \lim_{t \to 0^+} \frac{3}{2} \left( 1 - t^{2/3} \right)$$

$$= -\frac{3}{2} (-8)^{2/3} + \frac{3}{2}$$

$$= -\frac{3}{2} (-2^3)^{2/3} + \frac{3}{2}$$

$$= -\frac{3}{2}\, 2^2 + \frac{3}{2} = -6 + \frac{3}{2} = -\frac{9}{2}$$ (1.5)

3

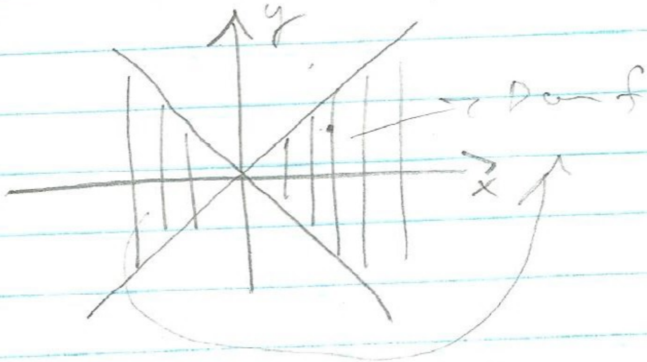
$$f(x,y) = \frac{\sqrt{x^2-y^2}}{2}$$

Dom

$$x^2 - y^2 \geq 0$$

$$x^2 - y^2 = 0$$

$$\therefore \; x = \pm y$$




$$\text{Dom } f = \{(x,y) \in \mathbb{R}^2 : x^2 - y^2 \geq 0\}$$

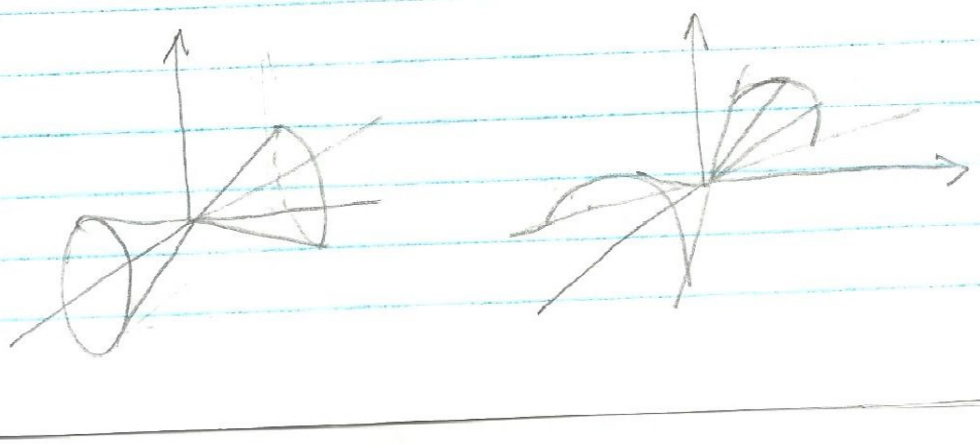
graf

$$z = \frac{\sqrt{x^2-y^2}}{2} \; ; \; z \geq 0$$

$$2z = \sqrt{x^2-y^2}$$

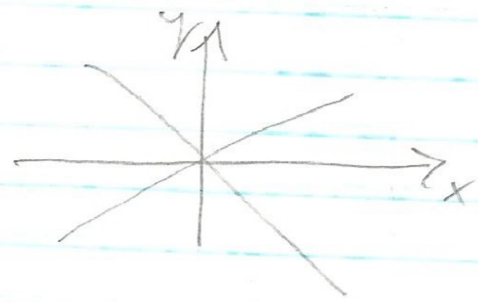
$$4z^2 = x^2 - y^2$$

$$\therefore \; x^2 = 4z^2 + y^2 \; ; \text{ Cone on } z \geq 0$$



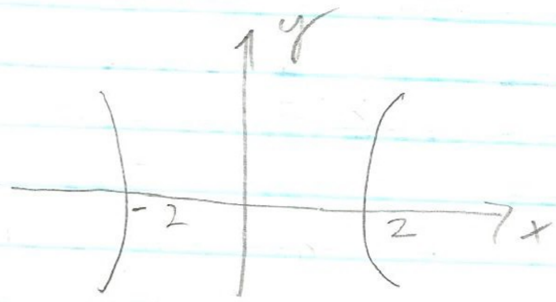
ii) $k=0$ : $\frac{\sqrt{x^2-y^2}}{2}=0$

$x^2-y^2=0 \quad \therefore \quad x=\pm y$




$k=1$ : $\frac{\sqrt{x^2-y^2}}{2}=1$

$x^2-y^2=4$



4

$$\lim_{(x,y)\to(0,0)} |y|^{|x|}$$

Seja o caminho:

$x=0,\ y\neq 0$ :

$$\lim_{(x,y)\to(0,0)} |y|^{|x|} = \lim_{y\to 0} |y|^{0} = \lim_{y\to 0} 1 = 1 \quad (*)$$

0,5

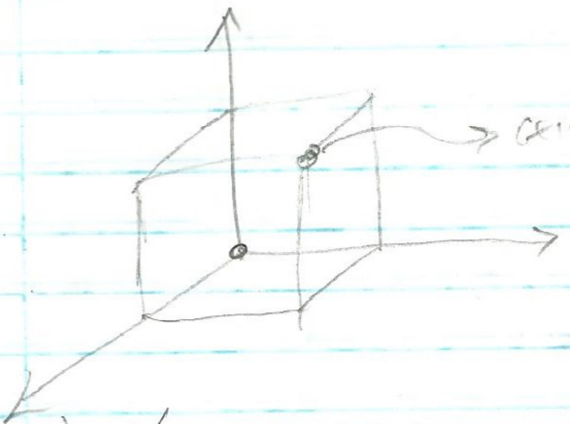
$y=0,\ x\neq 0$ :

$$\lim_{(x,y)\to(0,0)} |y|^{|x|} = \lim_{x\to 0} 0^{|x|} = 0 \quad (**)$$

0,5

De $(*)$ e $(**)$ temos que $\lim_{(x,y)\to(0,0)} |y|^{|x|} \nexists$.

5.



$(x,y,z)\ :\ x+2y+z=3$

$$\begin{cases} V = xyz & ; \ x \neq 0,\ y \neq 0,\ z \neq 0 \\ \phi(x,y,z) = x+2y+z-3 \end{cases}$$

$$\begin{cases} V_x = \lambda \phi_x & \rightarrow\ yz = \lambda & (*) \\ V_y = \lambda \phi_y & \rightarrow\ xz = 2\lambda & (2*) \\ V_z = \lambda \phi_z & \rightarrow\ xy = \lambda & (3*) \\ \phi = 0 & \rightarrow\ x+2y+z=3 & (4*) \end{cases}$$

(0.5)

$(*)$, $(2*)$ : $\quad y\cancel{z} = \frac{1}{2} x \cancel{z}$

$\therefore\ // x = 2y //$ $\quad (5*)$

$(*)$, $(3*)$ : $\quad \cancel{y}z = x\cancel{y}$ $\quad \therefore\ // z = x //$ $\quad (6*)$

$\left[ (2*), (3*) : \quad \cancel{x}z = 2\cancel{x}y \quad \therefore\ // z = 2y // \right]$

(consistent)

Subst. (5*), (6*) in (4*) :

$$x + x + x = 3$$

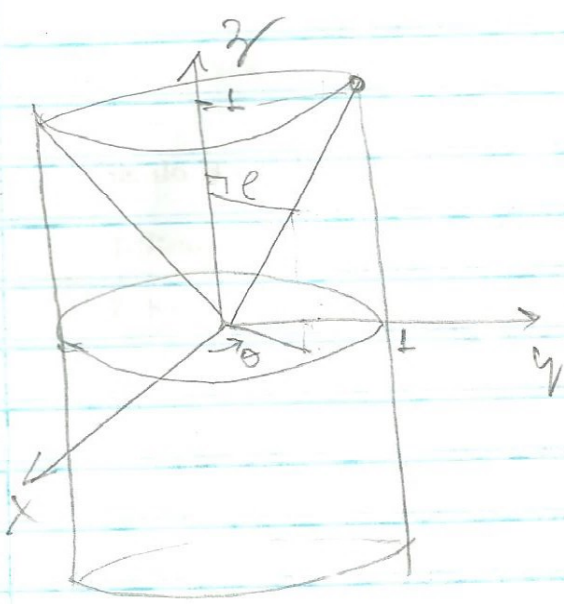
$$\therefore \quad \underline{\underline{x = 1}}$$

$$// \; y = \frac{x}{2} = \frac{1}{2} \; //$$

$$// \; z = x = 1 \; //$$

$$\boxed{V(x,y,z) = \frac{1}{2}} \qquad (1.5)$$

$$// \; x = 1, \; y = \frac{1}{2}, \; z = 1 \; //$$




$x^2+y^2=1$
$z=\sqrt{x^2+y^2}$ $\Big\}$ $z=1$

$\Omega : \begin{cases} 0 \le z \le \rho & (0.5) \\ 0 \le \theta \le 2\pi \\ 0 \le \rho \le 1 \end{cases}$

(0.75)

$$\int dV = \int_{\theta=0}^{2\pi}\int_{\rho=0}^{1}\int_{z=0}^{\rho} \rho \, dz \, d\rho \, d\theta$$

$$= \int_{\theta=0}^{2\pi}\int_{\rho=0}^{1} \rho z \Big]_{z=0}^{\rho} d\rho \, d\theta$$

$$= \int_{\theta=0}^{2\pi}\int_{\rho=0}^{1} \rho^2 \, d\rho \, d\theta$$

$$= \int_{\theta=0}^{2\pi} \frac{\rho^3}{3}\Big]_0^1 d\theta = \int_{\theta=0}^{2\pi} \frac{1}{3} d\theta$$

$$= \frac{1}{3}\theta\Big]_0^{2\pi} = \frac{2\pi}{3}.$$ (1.5)