

$$28 \quad (1+x^2y^2)y + (xy-1)^2xy' = 0$$

$$xy = u \quad \rightarrow \quad y = \frac{u}{x}$$

$$dy = \frac{1}{x} du - \frac{1}{x^2} u dx$$

$$(1+x^2y^2)y dx + (xy-1)^2x dy = 0$$

$$(1+x^2y^2)y dx + (xy-1)^2x \left( \frac{1}{x} du - \frac{1}{x^2} u dx \right) = 0$$

$$(1+x^2y^2)y dx + (xy-1)^2 du - \frac{(xy-1)^2}{x} u dx = 0$$

$$\left( 1 + x^2 \frac{u^2}{x^2} \right) \frac{u}{x} dx + (u-1)^2 du - \frac{(u-1)^2}{x} u dx = 0$$

$$\left[ (1+u^2)u \frac{1}{x} - \frac{u(u-1)^2}{x} \right] dx + (u-1)^2 du = 0$$

$$\frac{1}{x} \left( u(1+u^2) - \frac{u(u-1)^2}{-u^2u^2 - 2u + 1} \right) dx + (u-1)^2 du = 0$$

$$\frac{1}{x} \left( u + u^3 - \cancel{u^3} + 2u^2 - u \right) dx + (u-1)^2 du = 0$$

$$\frac{2u^2}{x} dx + (u-1)^2 du = 0$$

$$\frac{2}{x} dx + \frac{(u-1)^2}{u^2} du = 0$$

$$\int \frac{(u-1)^2}{u^2} du = -2 \int \frac{dx}{x} + C$$

$$\int \left( 1 - \frac{2}{u} + \frac{1}{u^2} \right) du = -2 \ln|x| + C$$

$$u - 2 \ln|u| - \frac{1}{u} = +2 \ln|x|^{-2} + C$$

$$\frac{u^2-1}{u} - 2 \ln |u| = \ln |x|^{-2} + C$$

$$\frac{u^2-1}{u} = \ln |u|^2 + \ln |x|^{-2} + C$$

$$\equiv \ln \left(\frac{u}{x}\right)^2 + C$$

$$\ln \left(\frac{u}{x}\right)^2 = \frac{u^2-1}{u} - C$$

$$\left(\frac{u}{x}\right)^2 = e^{\frac{u^2-1}{u} - C} = e^{u - \frac{1}{u}} e^{-C}$$

$u = xy$   $\therefore$   $y^2 = e^{xy - \frac{1}{xy}} C_1$

$$C_2 y^2 = e^{xy - \frac{1}{xy}}$$