

Cálculo C - Lista 2

Curvas definidas parametricamente

Esboce o gráfico da curva representada pelas funções vetoriais a seguir

1. $\vec{r}(t) = t\vec{i}$, $-1 \leq t \leq \frac{1}{2}$
2. $\vec{r}(t) = \cos \pi t \vec{k}$, $-1 \leq t \leq \frac{1}{3}$
3. $\vec{r}(t) = t\vec{i} + t\vec{j} + t\vec{k}$
4. $\vec{r}(t) = 2t\vec{i} - 3t\vec{j} + \vec{k}$
5. $\vec{r}(t) = (2t + 1)\vec{i} + (t - 1)\vec{j} + 3t\vec{k}$
6. $\vec{r}(t) = -16t^2\vec{k}$, $t \geq 0$
7. $\vec{r}(t) = t\vec{j} + t^2\vec{k}$
8. $\vec{r}(t) = (t^4 + 1)\vec{i} + t\vec{j}$
9. $\vec{r}(t) = t^3\vec{i} + t^2\vec{j}$
10. $\vec{r}(t) = \cos t\vec{i} + \sin t\vec{j}$, $0 \leq t \leq \frac{\pi}{2}$
11. $\vec{r}(t) = \cos 3t\vec{i} + \sin 3t\vec{j}$, $0 \leq t \leq \frac{\pi}{2}$
12. $\vec{r}(t) = 2 \cos t\vec{i} - \sin t\vec{j} - 3\vec{k}$, $-\pi \leq t \leq 0$
13. $\vec{r}(t) = \cos t\vec{i} + \sin t\vec{j} + t^2\vec{k}$
14. $\vec{r}(t) = 3 \sin t\vec{i} + 3 \sin t\vec{j} - 3\sqrt{2} \cos t\vec{k}$
15. $\vec{r}(t) = t\vec{i} + \cos 2t\vec{j} + \sin 2t\vec{k}$

Encontre a função vetorial que representa a curva obtida pela interseção das superfícies dadas a seguir. [Sugestão: Identifique geometricamente a forma da curva pela análise da interseção das superfícies. Uma vez reconhecida a curva, parametrize-a de modo a respeitar a orientação dada. Note que mais de uma parametrização é possível.]

16. $x + 2y + 3z = 6$ e $y - 2z = 3$ orientada de modo que z aumenta ao longo da curva.
17. $x^2 + y^2 = 2$ e $z = 4$ orientada de modo que y aumenta no primeiro octante.
18. $z = x^2 + y^2$ e $x^2 + y^2 = 5$ orientada de modo que x aumenta no primeiro octante.
19. $z = \sqrt{x^2 + y^2}$ e $y = x$ orientada de modo que ao se parametrizar $(x(t), y(t))$ tem-se (x, y) se afastando da origem $(0, 0)$ para valores crescentes de t .

Determine quais das parametrizações a seguir é suave, suave por parte ou nenhuma das duas.

20. $\vec{r}(t) = t\vec{i} + t^2\vec{j} + t^3\vec{k}$
21. $\vec{r}(t) = |t|\vec{i} + t\vec{j} + t\vec{k}$
22. $\vec{r}(t) = (1 + t)^{\frac{3}{2}}\vec{i} + (1 - t)^{\frac{3}{2}}\vec{j} + \frac{3t}{2}\vec{k}$
23. $\vec{r}(t) = \cos^2 t\vec{i} + \sin^2 t\vec{j} + t^2\vec{k}$

24. $\vec{r}(t) = (e^t - t)\vec{i} + t^2\vec{j} + t^3\vec{k}$

Encontre parametrizações suaves para as curvas a seguir

25. A linha reta passando por $(-3, 2, 1)$ e $(4, 0, 5)$.

26. O círculo no plano xy centrado na origem e com raio 6.

Encontre parametrizações suaves por partes para as curvas a seguir

27. O quadrado no plano xy cujos vértices são $(3, 0)$, $(3, 3)$, $(0, 3)$ e $(0, 0)$.

28. O triângulo no plano xy cujos vértices são $(0, 0)$, $(2, 0)$, $(0, 2)$.

Determine o comprimento das curvas a seguir

29. $\vec{r}(t) = \cos^3 t \vec{i} + \sin^3 t \vec{j}$, $0 \leq t \leq 2\pi$

30. $\vec{r}(t) = 2t \vec{i} + t^2 \vec{j} + \ln t \vec{k}$, $1 \leq t \leq 2$

31. $\vec{r}(t) = \frac{1}{3}(1+t)^{3/2}\vec{i} + \frac{1}{3}(1-t)^{3/2}\vec{j} + \frac{1}{2}t\vec{k}$, $-1 \leq t \leq 1$

32. $\vec{r}(t) = e^t \vec{i} + e^{-t}\vec{j} + \sqrt{2}t\vec{k}$, $0 \leq t \leq 1$

33. $\vec{r}(t) = 2(t^2 - 1)^{3/2}\vec{i} + 3t^2\vec{j} + 3t^2\vec{k}$, $0 \leq t \leq \sqrt{8}$

Encontre para cada uma das curvas o vetor tangente, o vetor normal e a curvatura.

34. $\vec{r}(t) = (t^2 + 4)\vec{i} + 2t\vec{j}$

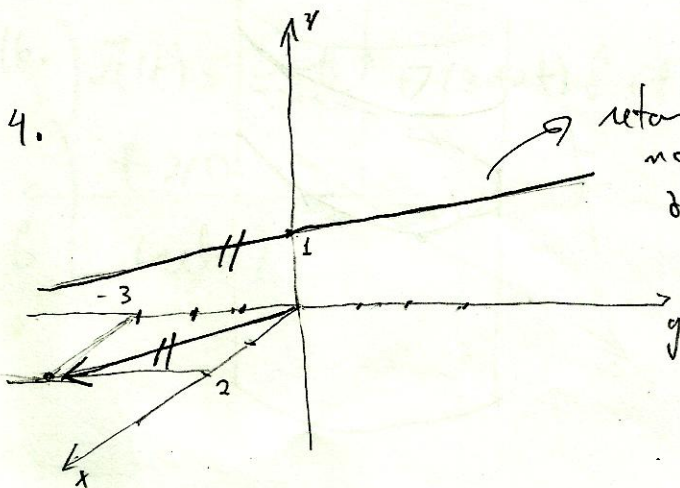
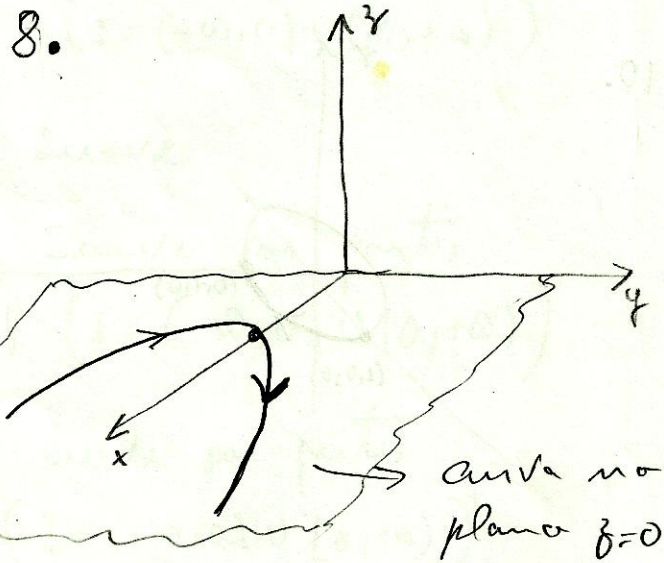
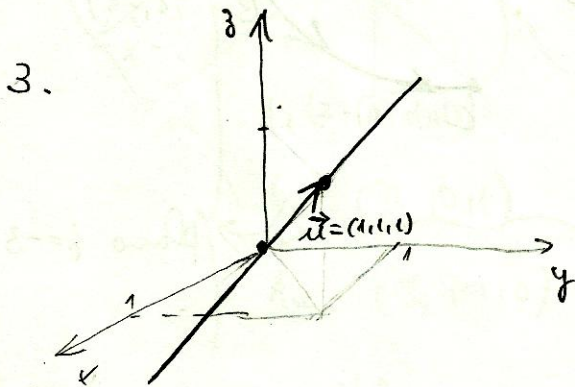
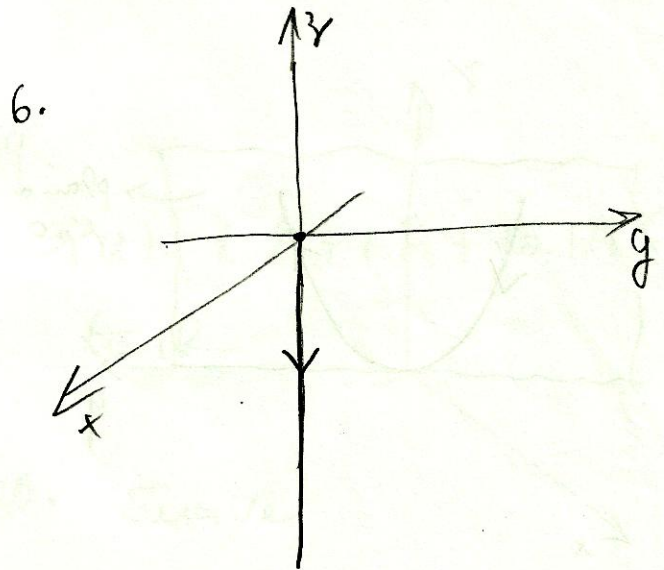
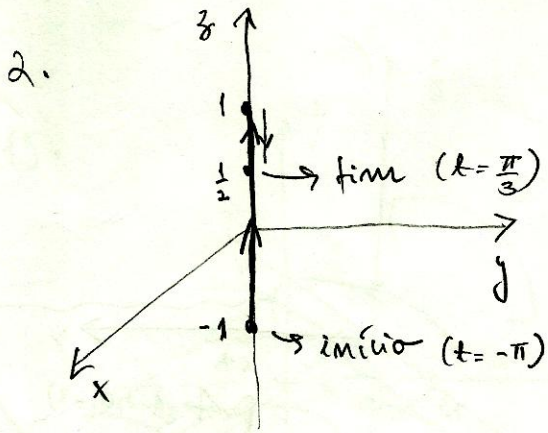
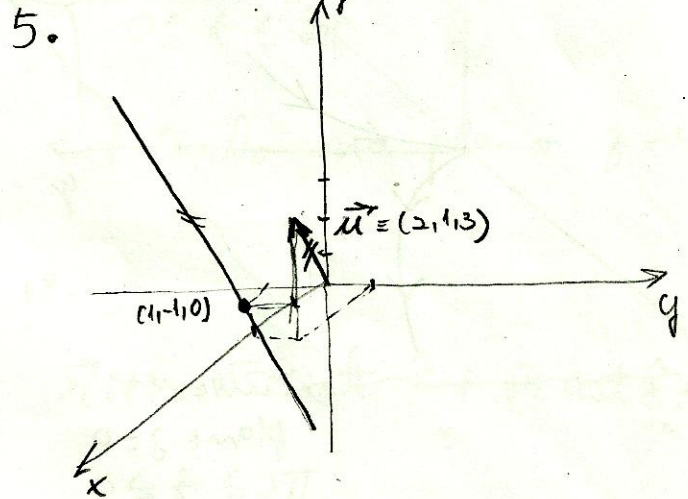
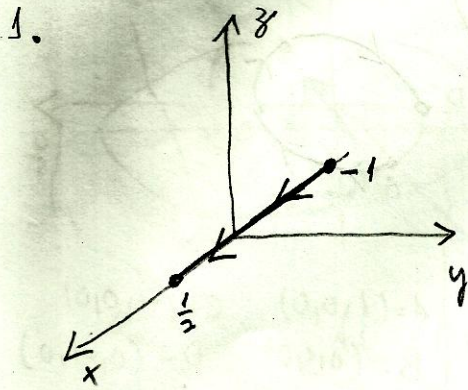
35. $\vec{r}(t) = \cos t \vec{i} + \cos t \vec{j} + \sqrt{2} \sin t \vec{k}$

36. $\vec{r}(t) = 2t \vec{i} + t^2 \vec{j} + \frac{1}{3}t^3 \vec{k}$

37. $\vec{r}(t) = e^t \vec{i} + e^{-t}\vec{j} + \sqrt{2}t\vec{k}$

38. $\vec{r}(t) = 2t\vec{i} + t^2\vec{j} + \ln t\vec{k}$

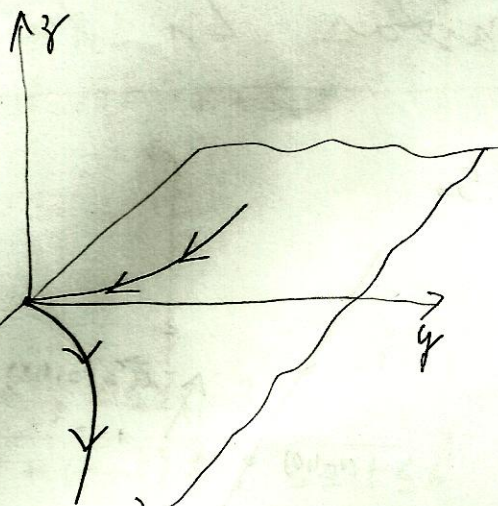
Cálculo C - Respostas Lista 2



reta está no plano $\delta=1$

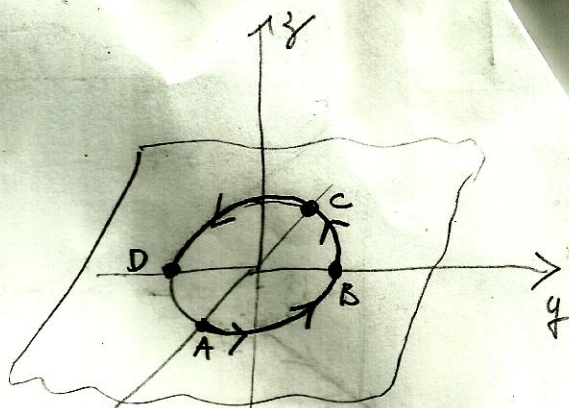
curva no plano $\delta=0$

9.



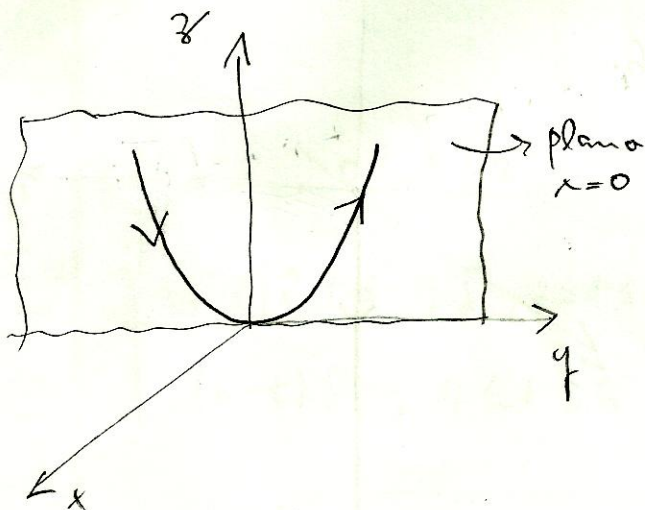
curva no plano $z=0$

11.



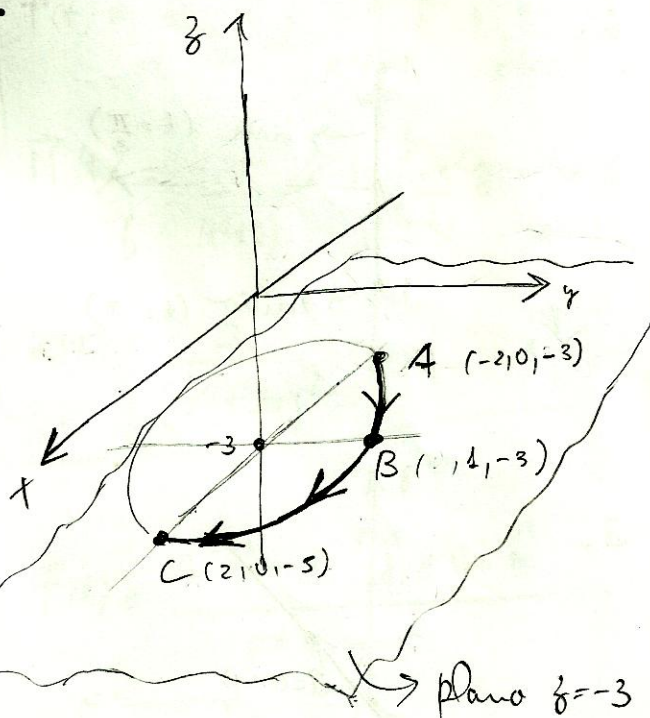
$A=(1,0,0)$ $C=(-1,0,0)$
 $B=(0,1,0)$ $D=(0,-1,0)$

7.



plano $x=0$

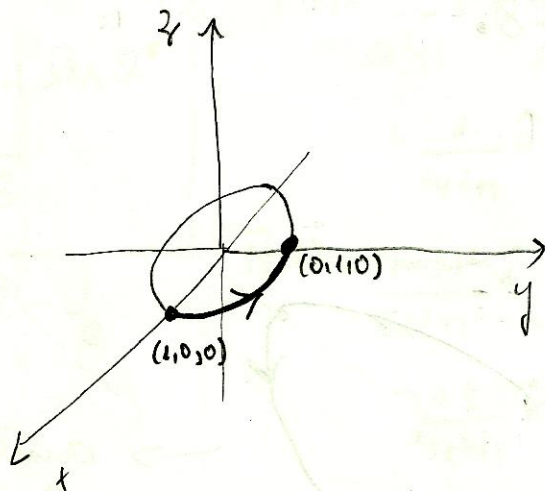
12.



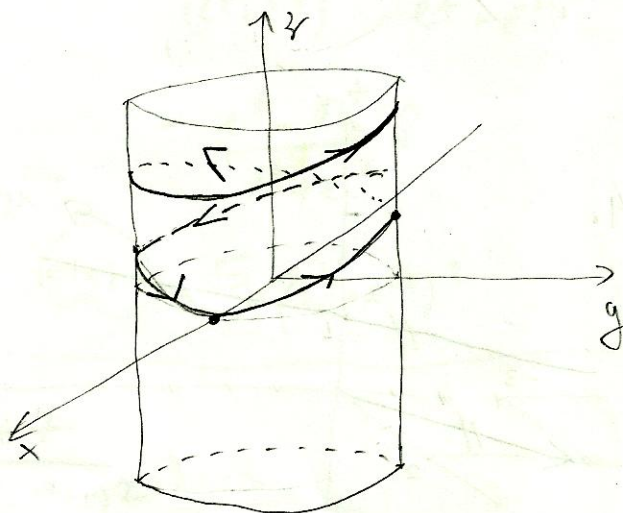
$A(-2,0,-3)$
 $B(0,4,-3)$
 $C(2,0,-5)$

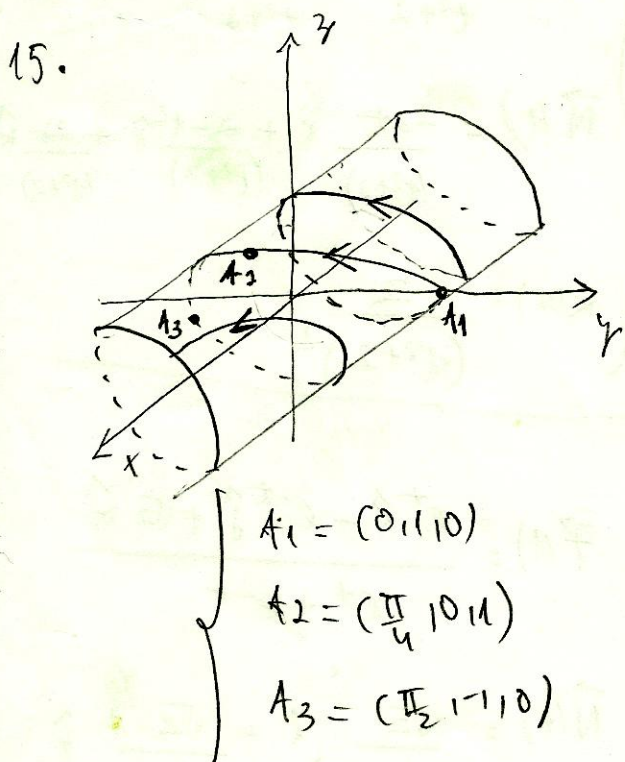
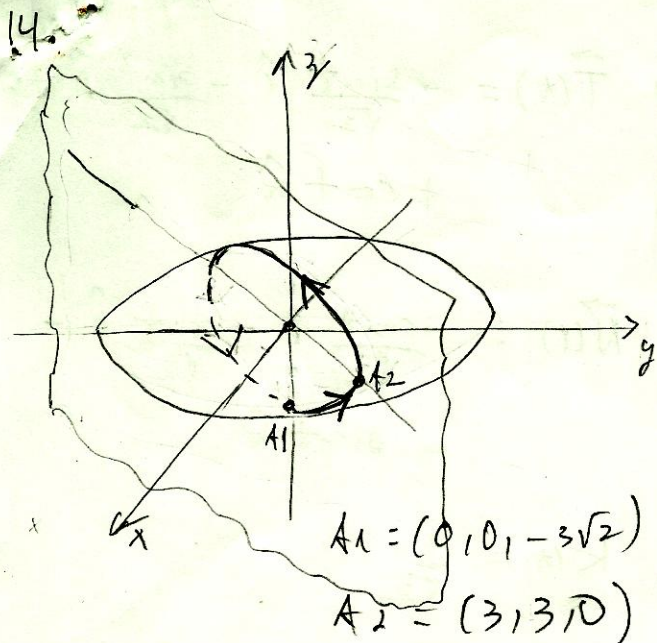
plano $z=-3$

10.



13.





16.

$$\vec{r}(t) = -7t \hat{i} + (3+2t) \hat{j} + t \hat{k}$$

$$\left\{ \begin{array}{l} t > 0 \\ \text{(reta)} \end{array} \right.$$

17.

$$\left\{ \begin{array}{l} \vec{r}(t) = \sqrt{2} \cos t \hat{i} + \sqrt{2} \sin t \hat{j} + 4 \hat{k} \\ 0 \leq t \leq 2\pi \end{array} \right.$$

(círculo no plano $z=4$)

18.

$$\vec{r}(t) = \sqrt{5} \sin t \hat{i} + \sqrt{5} \cos t \hat{j} + 5 \hat{k}$$

$$0 \leq t \leq 2\pi$$

19.

$$\vec{r}(t) = t \hat{i} + t \hat{j} + \sqrt{2}|t| \hat{k}$$

$$t > 0$$

20. Suave

21. Suave por partes
 $(I = (-\infty, 0] \cup [0, +\infty))$

22. Suave

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 $(I = (-\infty, 0] \cup [0, +\infty))$

25.

$$\left\{ \begin{array}{l} \vec{r} = (-3+7t) \hat{i} + (2-2t) \hat{j} + (1+4t) \hat{k} \\ t \in \mathbb{R} \end{array} \right.$$

26.
$$\vec{r}(t) = 6 \cos t \hat{i} + 6 \sin t \hat{j}$$

$$0 \leq t \leq 2\pi$$

27.
$$\vec{r}(t) = \begin{cases} t \hat{i}, & 0 \leq t \leq 3 \\ 2 \hat{i} + (t-3) \hat{j}, & 3 \leq t \leq 6 \\ (9-t) \hat{i} + 3 \hat{j}, & 6 \leq t \leq 9 \\ (12-t) \hat{j}, & 9 \leq t \leq 12 \end{cases}$$

28.
$$\vec{r}(t) = \begin{cases} t \hat{i}, & 0 \leq t \leq 2 \\ (4-t) \hat{i} + (t-2) \hat{j}, & 2 \leq t \leq 4 \\ (6-t) \hat{j}, & 4 \leq t \leq 6 \end{cases}$$

29. 6

30. $3 + \ln 2$

31. $\sqrt{3}$

32. $e - \frac{1}{e}$

33. 52

34.
$$\vec{r}(t) = \frac{t}{\sqrt{t^2+1}} \hat{i} + \frac{\hat{j}}{\sqrt{t^2+1}}$$

$$\vec{N}(t) = \frac{\hat{i} - t \hat{j}}{\sqrt{t^2+1}}, \quad k(t) = \frac{1}{2(t^2+1)^{3/2}}$$

35.
$$\vec{r}(t) = -\frac{\sin t}{\sqrt{2}} \hat{i} - \frac{2 \sin t}{\sqrt{2}} \hat{j} + \cos t \hat{k}$$

$$\vec{N}(t) = \frac{-\cos t}{\sqrt{2}} \hat{i} - \frac{\cos t}{\sqrt{2}} \hat{j} - \sin t \hat{k}$$

$$k(t) = \frac{1}{\sqrt{2}}$$

36.
$$\vec{r}(t) = \frac{2}{t^2+2} \hat{i} + \frac{2t}{t^2+2} \hat{j} + \frac{t^2}{t^2+2} \hat{k}$$

$$\vec{N}(t) = \frac{-2t}{(t^2+2)} \hat{i} + \frac{2-t^2}{(t^2+2)} \hat{j} + \frac{2t}{(t^2+2)} \hat{k}$$

$$k(t) = \frac{2}{(t^2+2)^2}$$

38.
$$\vec{r}(t) = \frac{2t}{2t^2+1} \hat{i} + \frac{2t^2}{2t^2+1} \hat{j} + \frac{1}{2t^2+1} \hat{k}$$

$$\vec{N}(t) = \frac{1-2t^2}{2t^2+1} \hat{i} + \frac{2t}{2t^2+1} \hat{j} - \frac{2t}{2t^2+1} \hat{k}$$

37.
$$\vec{r}(t) = \frac{e^t \hat{i} - e^{-t} \hat{j} + \sqrt{2} \hat{k}}{e^t + e^{-t}}$$

$$\vec{N}(t) = \frac{\sqrt{2}}{(1+e^{-2t})} \hat{i} + \frac{\sqrt{2}}{e^t + e^{-t}} \hat{j} - \frac{e^t - e^{-t}}{e^t + e^{-t}} \hat{k}$$

$$k(t) = \frac{\sqrt{2}}{(e^t + e^{-t})^2}$$

$$k(t) = \frac{2t}{(2t^2+1)^2}$$