Cálculo C - Lista 15

Transformada de Laplace (I)

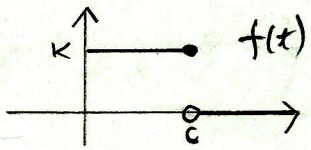
Propriedades:

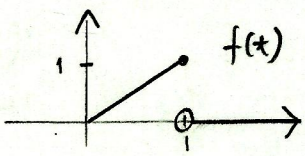
$\mathcal{L}(f') = s\mathcal{L}(f) - f(0)$ (1)

$\mathcal{L}(f^{(n)}) = s^n\mathcal{L}(f) - s^{n-1}f(0) - s^{n-2}f'(0) - \\ - \ldots - f^{(n-1)}(0)$ (2)

$\mathcal{L}\left(\int_0^t f(u)du\right) = \frac{1}{s}\mathcal{L}(f)$ (3)

Encontre a transformada de Laplace das seguintes funções

1. $t + 4$
2. $a + bt + ct^2$
3. 

4. 

5. $$f(t) = \begin{cases} 0, & 0 \leq t < 1 \\ t, & 1 \leq t < 2 \\ 1, & 2 \leq t \end{cases}$$
6. $$f(t) = \begin{cases} 0, & 0 \leq t \leq 5 \\ e^{-t}, & 5 < t \end{cases}$$
7. $(t^2 + \frac{1}{2})^2$
8. $\sin \pi t$
9. $\cos(\omega t + \theta)$
10. $\cos^2 t$
11. $\cos^2 \omega t$
12. $\sinh^2 2t$

Use os teoremas da derivada (1) e (2) e obtenha as transformadas a seguir

13. $\mathcal{L}(t\cos\omega t) = \frac{s^2-\omega^2}{(s^2+\omega^2)^2}$
14. $\mathcal{L}(t\sin\omega t) = \frac{2\omega s}{(s^2+\omega^2)^2}$
15. $\mathcal{L}(t\cosh at) = \frac{s^2+a^2}{(s^2-a^2)^2}$
16. $\mathcal{L}(t\sinh at) = \frac{2as}{(s^2-a^2)^2}$

Use o teorema da integral (3) para calcular $f(t)$ sendo dado $\mathcal{L}(f)$

17. $\frac{1}{s^2+s}$
18. $\frac{10}{s(s^2+9)}$
19. $\frac{1}{s^2(s+1)}$
20. $\frac{1}{s^2}\left(\frac{s-1}{s+1}\right)$
21. $\frac{54}{s^3(s-3)}$
22. $\frac{1}{s^2}\left(\frac{s+1}{s^2+1}\right)$

Lista 15 - Respostas

1. $\frac{1}{s^2} + \frac{4}{s}$

2. $\frac{a}{s} + \frac{b}{s^2} + \frac{2c}{s^3}$

3. $\frac{K(1-e^{-cs})}{s}$

4. $-\frac{e^{-s}}{s} + \frac{(1-e^{-s})}{s^2}$

5. $\frac{(s+1)}{s^2}(e^{-s} - e^{-2s})$

6. $\frac{e^{-5(s+1)}}{s+1}$

7. $24s^{-5} + 2s^{-3} + \frac{1}{4}s^{-1}$

8. $\frac{\pi}{s^2+\pi^2}$

9. $\frac{s\cos\theta - w\sin\theta}{s^2+w^2}$

10. $\frac{1}{2s} + \frac{s}{2s^2+8}$

11. $\frac{1}{2s} + \frac{s}{2s^2+8w^2}$

12. $\frac{1}{2}\left[\frac{s}{s^2-16} - \frac{1}{s}\right]$

17. $1 - e^{-t}$

18. $\frac{10}{9}(1-\cos 3t)$

19. $t + e^{-t} - 1$

20. $2(1-e^{-t}) - t$

21. $2e^{3t} - 9t^2 - 6t - 2$

22. $1 + t - \cos t - \sin t$