Cálculo C - Prova 2

(2.0) 1. Use o teorema de Stokes para calcular $\int_\gamma \vec{F}\cdot d\vec{r}$, onde $\vec{F} = xz\hat{i} + y^2\hat{j} + x^2\hat{k}$ e γ é a curva orientada no sentido anti-horário quando vista de cima e obtida da interseção do cilindro $x^2 + \frac{y^2}{4} = 1$ com o plano $x + y + z = 5$.

(2.0) 2. Use o teorema de Gauss para calcular $\int_\Omega \vec{F}\cdot d\vec{S}$ onde $\vec{F} = -2x\hat{i} + 4y\hat{j} - 7z\hat{k}$ e Ω é a superfície que delimita a região sólida situada dentro da esfera $x^2 + y^2 + z^2 = 4$ e fora do cilindro $x^2 + y^2 = 1$.

(2.0) 3. Resolva $xy' + 6y = 3xy^{4/3}$
[Note que esta equação se reduz a uma equação de Bernoulli]

(1.0) 4. Resolva $x^2y' = 1 - x^2 + y^2 - x^2y^2$ $\quad 1-x^2 + y^2(1-x^2)$

(1.5) 5. Resolva $(y\ln y + ye^x)dx + (x + y\cos y)dy = 0$ $\quad (1-x^2)(1+y^2)$

6. Escolha apenas uma das questões a seguir:

(1.5) (i) Resolva $y'' + 3y' + 2y = x(e^{-x} - e^{-2x})$ usando o método dos coeficientes a determinar.

(ii) Resolva $y'' - 4y' + 4y = \frac{e^{2x}}{x}$ usando o método da variação dos parâmetros.

- $Mdx + Ndy = 0$

Fator integrante dependente de x: $\frac{1}{N(x)}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) \equiv f(x) \therefore F(x) = e^{\int f(x)dx}$.

Fator integrante dependente de y: $\frac{1}{M(x)}\left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) \equiv g(y) \therefore F(y) = e^{\int g(y)dy}$.

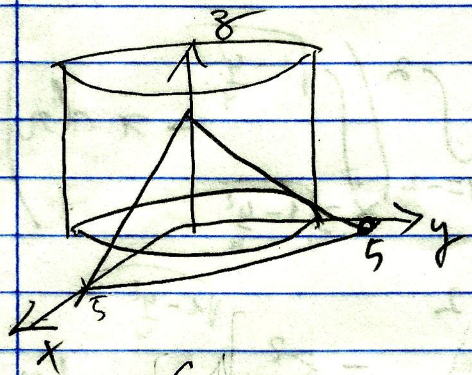
- $y' + p(x)y = g(x)y^a \Rightarrow u(x) = y^{1-a}$

$r(x)$	y_p
$C_nX^n + C_{n-1}x^{n-1} + ... + C_1x + C_0$	$A_nX^n + A_{n-1}x^{n-1} + ... + A_1x + A_0$
Ce^{rx}	Ae^{rx}
$C\cos kx$	$A\cos kx + B\sin kx$
$C\sin kx$	$A\cos kx + B\sin kx$
$(C_nX^n + C_{n-1}x^{n-1} + ... + C_1x + C_0)e^{rx}$	$(A_nX^n + A_{n-1}x^{n-1} + ... + A_1x + A_0)e^{rx}$
$(C_nX^n + C_{n-1}x^{n-1} + ... + C_1x + C_0)\cos kx$	$(A_nX^n + A_{n-1}x^{n-1} + ... + A_1x + A_0)\cos kx +$ $+(B_nX^n + B_{n-1}x^{n-1} + ... + B_1x + B_0)\sin kx$
$(C_nX^n + C_{n-1}x^{n-1} + ... + C_1x + C_0)\sin kx$	$(A_nX^n + A_{n-1}x^{n-1} + ... + A_1x + A_0)\cos kx +$ $+(B_nX^n + B_{n-1}x^{n-1} + ... + B_1x + B_0)\sin kx$
$Ce^{rx}\cos kx$	$Ae^{rx}\cos kx + Be^{rx}\sin kx$
$Ce^{rx}\sin kx$	$Ae^{rx}\cos kx + Be^{rx}\sin kx$
$(C_nX^n + C_{n-1}x^{n-1} + ... + C_1x + C_0)e^{rx}\cos kx$	$(A_nX^n + A_{n-1}x^{n-1} + ... + A_1x + A_0)e^{rx}\cos kx +$ $+(B_nX^n + B_{n-1}x^{n-1} + ... + B_1x + B_0)e^{rx}\sin kx$
$(C_nX^n + C_{n-1}x^{n-1} + ... + C_1x + C_0)e^{rx}\sin kx$	$(A_nX^n + A_{n-1}x^{n-1} + ... + A_1x + A_0)e^{rx}\cos kx +$ $+(B_nX^n + B_{n-1}x^{n-1} + ... + B_1x + B_0)e^{rx}\sin kx$

- $y_h = C_1y_1 + C_2y_2$ $\qquad y_p = u\,y_1 + v\,y_2$

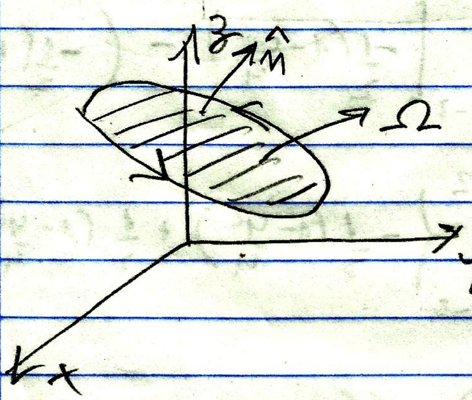
$u = -\int \frac{r(x)y_2}{W(y_1,y_2)}dx \qquad v = \int \frac{r(x)y_1}{W(y_1,y_2)}dx$

1. $\int_\gamma \vec{F}\cdot d\vec{r}$, $\vec{F} = xz\,\vec{i} + y^2\,\vec{j} + x^2\,\vec{k}$



$$\begin{cases} x^2 + \dfrac{y^2}{4} = 1 \\ x + y + z = 5 \end{cases}$$

$$\int_\gamma \vec{F}\cdot d\vec{r} = \int_\Omega (\vec{\nabla}\times\vec{F})\cdot d\vec{s}$$



$$\begin{cases} z = 5 - x - y \\ d\vec{s} = \left(-\dfrac{\partial z}{\partial x}, -\dfrac{\partial z}{\partial y}, 1\right) dx\,dy \\ \quad = (1, 1, 1)\, dx\,dy \end{cases}$$

0.5

$$\vec{\nabla}\times\vec{F} = \det\begin{pmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial_x & \partial_y & \partial_z \\ xz & y^2 & x^2 \end{pmatrix}$$

$$= \hat{i}\,\underbrace{\partial_y x^2}_{0} + \hat{j}\,\partial_z(xz) + \hat{k}\,\underbrace{(\partial_x y^2)}_{0}$$

$$- \hat{k}\,\underbrace{\partial_y(xz)}_{0} - \hat{i}\,\underbrace{\partial_z y^2}_{0} - \hat{j}\,\partial_x x^2$$

$$= \vec{j}\,x - \vec{j}\,2x = -x\,\vec{j}$$

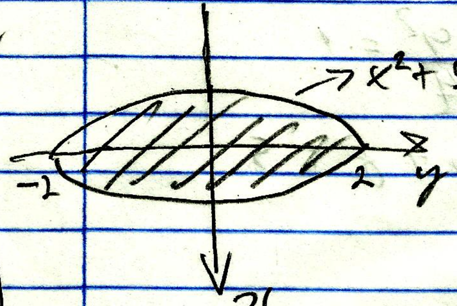
$$= (0, -x, 0)$$

0.5

$\therefore$

$$\int_\gamma \vec{F}\cdot d\vec{r} = \int_{D(x,y)} (0,-x,0)\cdot(1,1,1)\,dx\,dy$$

$$= \int_{D(x,y)} -x\,dx\,dy$$



$$= \int_{y=-2}^{2}\left(\int_{x=\sqrt{1-\frac{y^2}{4}}}^{\sqrt{1-\frac{y^2}{4}}} -x\,dx\right)dy$$

0.5

$$= \int_{y=-2}^{2} \left. -\frac{x^2}{2}\right]_{-\sqrt{1-\frac{y^2}{4}}}^{\sqrt{1-\frac{y^2}{4}}} dy$$

$$= \int_{y=-2}^{2}\left[-\frac{1}{2}\left(1-\frac{y^2}{4}\right) - \left(-\frac{1}{2}\left(1-\frac{y^2}{4}\right)\right)\right]dy$$

$$= \int_{y=-2}^{2}\left\{-\frac{1}{2}\left(1-\frac{y^2}{4}\right) + \frac{1}{2}\left(1-\frac{y^2}{4}\right)\right\}dy$$

$$= 0 \quad //$$

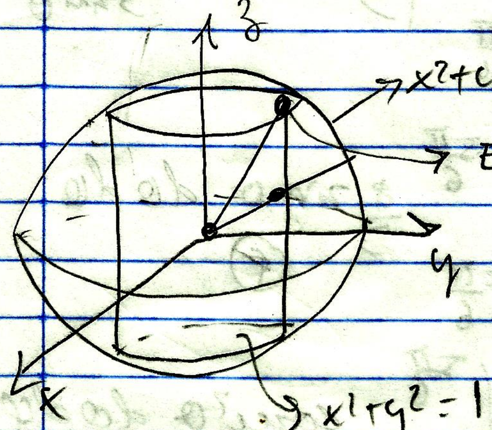
0.5

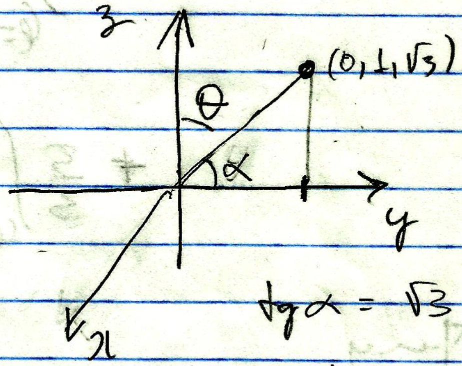
Obs.:

2. $\displaystyle\int_{\Omega} \vec{F}\cdot d\vec{S} = \int_{V} \vec{\nabla}\cdot\vec{F}\, dV$

$\vec{F} = -2x\,\hat{\imath} + 4y\,\hat{\jmath} - 7z\,\hat{k}$

$\vec{\nabla}\cdot\vec{F} = \partial_x(-2x) + \partial_y(4y) + \partial_z(-7z)$
$= -2 + 4 - 7$
$= -5$





$\operatorname{tg}\alpha = \sqrt{3}$

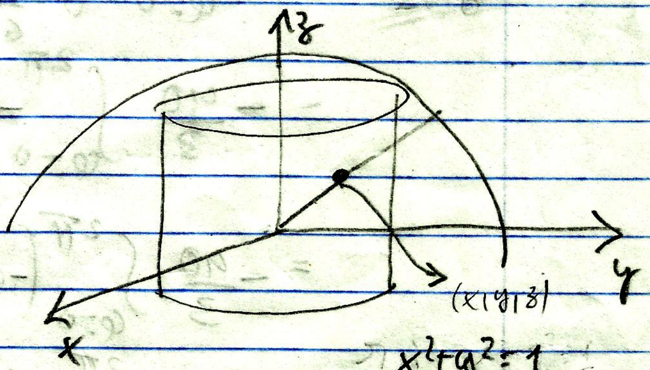
$\alpha = \frac{\pi}{3}$

$\therefore\ \theta = \frac{\pi}{6}$

Coordenadas Esféricas

$$\begin{cases} x = r\sin\theta\cos\varphi \\ y = r\sin\theta\sin\varphi \\ z = r\cos\theta \end{cases}$$

$$\left\{\begin{array}{l} \left.\begin{array}{l} \frac{\pi}{6} \leq \theta \leq \frac{5\pi}{6} \\ 0 \leq \varphi \leq 2\pi \end{array}\right\} \underline{\underline{0.5}} \\ \left.\frac{1}{\sin\theta} \leq r \leq 2\right\} \underline{\underline{0.5}} \end{array}\right.$$



$x^2+y^2=1$

$r^2\sin^2\theta = 1$

$$\int_V -5\,dV = -5\int_{\varphi=0}^{2\pi}\int_{\theta=\frac{\pi}{6}}^{\frac{5\pi}{6}}\int_{\rho=\frac{1}{\sin\theta}}^{2} \rho^2\sin\theta\,d\rho\,d\theta\,d\varphi$$

$$= -5\int_{\varphi=0}^{2\pi}\int_{\theta=\frac{\pi}{6}}^{\frac{5\pi}{6}} \frac{\rho^3}{3}\sin\theta\Big]_{\frac{1}{\sin\theta}}^{2} d\theta\,d\varphi$$

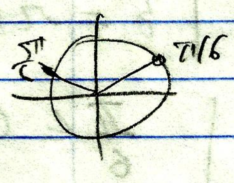
$$= -5\int_{\varphi=0}^{2\pi}\int_{\theta=\frac{\pi}{6}}^{\frac{5\pi}{6}} \left(\frac{8}{3}\sin\theta - \frac{1}{3\sin^3\theta}\sin\theta\right) d\theta\,d\varphi$$

$$= -5\int_{\varphi=0}^{2\pi}\int_{\theta=\frac{\pi}{6}}^{\frac{5\pi}{6}} \frac{8}{3}\sin\theta\,d\theta\,d\varphi \quad (*)$$

$$+\frac{5}{3}\int_{\varphi=0}^{2\pi}\int_{\theta=\frac{\pi}{6}}^{\frac{5\pi}{6}} \csc^2\theta\,d\theta\,d\varphi \quad (**)$$

Then

$$(*) = -5\int_{\varphi=0}^{2\pi}\int_{\theta=\frac{\pi}{6}}^{\frac{5\pi}{6}} \frac{8}{3}\sin\theta\,d\theta\,d\varphi$$



$$= -\frac{40}{3}\int_{\varphi=0}^{2\pi} -\cos\theta\Big]_{\frac{\pi}{6}}^{\frac{5\pi}{6}} d\varphi$$

$$= -\frac{40}{3}\int_{\varphi=0}^{2\pi}\left(-\cos\frac{5\pi}{6} + \cos\frac{\pi}{6}\right) d\varphi$$

$$= -\frac{40}{3}\int_{\varphi=0}^{2\pi}\left(+\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}\right) d\varphi =$$

$$= -\frac{40}{3}\int_{\varphi=0}^{2\pi} +\sqrt{3}\,d\varphi$$

$$= -\frac{40}{3}(+\sqrt{3})\cdot\varphi\Big]_0^{2\pi} = -\frac{40\sqrt{3}}{3}\cdot 2\pi$$

$$= -\frac{80\pi\sqrt{3}}{3}$$

O.K

$$(**) = +\frac{5}{3}\int_{\varphi=0}^{2\pi}\int_{\theta=\frac{\pi}{6}}^{\frac{5\pi}{6}} \operatorname{cosec}^2\theta\, d\theta$$

$$= +\frac{5}{3}\int_{\varphi=0}^{2\pi} -\operatorname{cotg}\theta\, d\theta$$

$$= -\frac{5}{3}\int_{\varphi=0}^{2\pi} \operatorname{cotg}\theta\Big]_{\frac{\pi}{6}}^{\frac{5\pi}{6}} d\theta$$

$$= -\frac{5}{3}\int_{\varphi=0}^{2\pi}\left(\operatorname{cotg}\frac{5\pi}{6} - \operatorname{cotg}\frac{\pi}{6}\right) d\varphi$$

$$\operatorname{cotg}\frac{5\pi}{6} = \frac{\cos\frac{5\pi}{6}}{\operatorname{sen}\frac{5\pi}{6}} = \frac{-\frac{\sqrt{3}}{2}}{\frac{1}{2}} = -\sqrt{3}$$

$$= -\frac{5}{3}\int_{\varphi=0}^{2\pi}(\underbrace{-\sqrt{3}-\sqrt{3}}_{-2\sqrt{3}})\,d\varphi$$

$$= +\frac{10\sqrt{3}}{3}\int_0^{2\pi} d\varphi$$

$$\operatorname{cotg}\frac{\pi}{6} = \frac{\cos\frac{\pi}{6}}{\operatorname{sen}\frac{\pi}{6}} = \sqrt{3}$$

$$= +\frac{10\sqrt{3}}{3}\,\varphi\Big]_0^{2\pi} = +\frac{20\pi\sqrt{3}}{3}$$

Daí

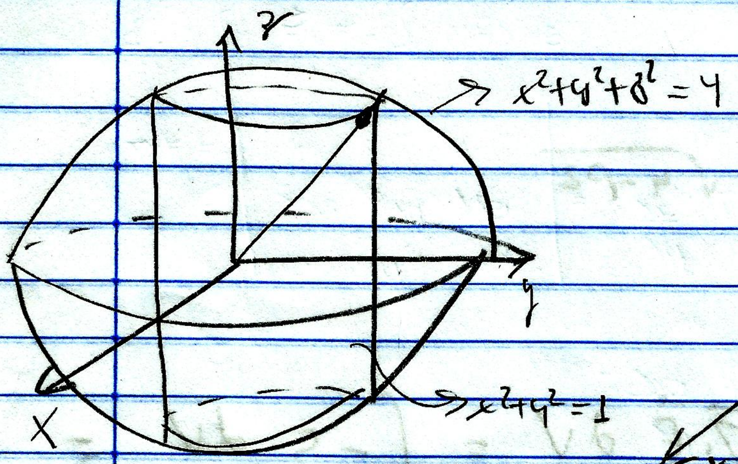
$$\int_V 5\,dV = (*) + (**) = -\frac{80\pi\sqrt{3}}{3} + \frac{20\pi\sqrt{3}}{3} = -\frac{60\pi\sqrt{3}}{3}$$

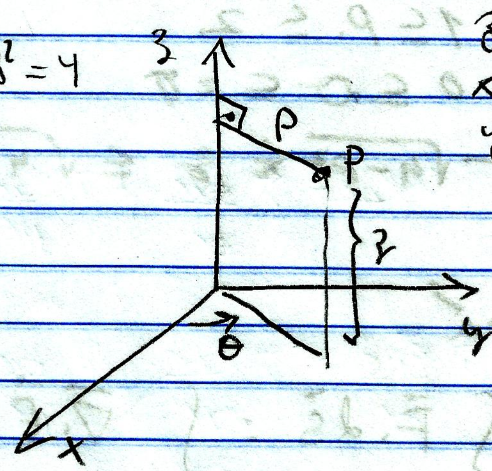
$$= -20\pi\sqrt{3}$$

O.K

Questão 2:
Outra Solução

Coordenadas Cilíndricas





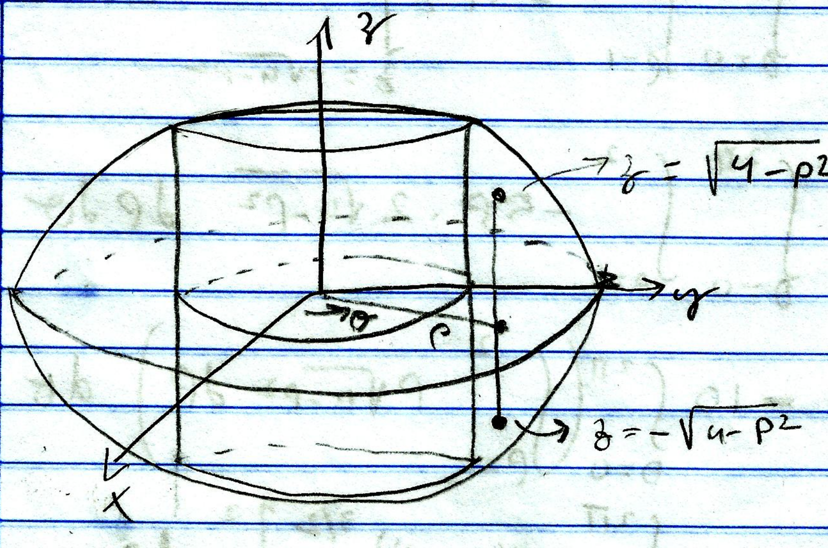
$z = z$
$x = \rho\cos\theta$
$y = \rho\,\mathrm{sen}\,\theta$

$$\begin{cases} x^2+y^2 = 1 \implies \rho^2 = 1 \ \therefore\ \rho = 1 \\ x^2+y^2+z^2 = 4 \implies \rho^2+z^2 = 4 \ \therefore\ z^2 = 4-\rho^2 \end{cases}$$

$$z = \pm\sqrt{4-\rho^2}$$

Para um valor fixo de θ e ρ vemos que a coordenada z satisfaz

$$-\sqrt{4-\rho^2} \leq z \leq \sqrt{4-\rho^2}$$



A parametrização da região se escreve então na forma

$$\begin{cases} 1 \leq \rho \leq 2 \\ 0 \leq \theta \leq 2\pi \\ -\sqrt{4-\rho^2} \leq z \leq \sqrt{4-\rho^2} \end{cases}$$

Daí

$$\int_S \vec{F} \cdot d\vec{S} = \int_V \vec{\nabla} \cdot \vec{F} \, dV = \int_{V(\rho,\theta,z)} -5 \, dV =$$

$$= \int_{V(\rho,\theta,z)} -5 \, \rho \, d\rho \, d\theta \, dz$$

$$= \int_{\theta=0}^{2\pi} \int_{\rho=1}^{2} \left(\int_{z=-\sqrt{4-\rho^2}}^{\sqrt{4-\rho^2}} -5\rho \, dz \right) d\rho \, d\theta$$

$$= \int_{\theta=0}^{2\pi} \int_{\rho=1}^{2} -5\rho z \Big]_{z=-\sqrt{4-\rho^2}}^{\sqrt{4-\rho^2}} d\rho \, d\theta$$

$$= \int_{\theta=0}^{2\pi} \int_{\rho=1}^{2} -5\rho \cdot 2\sqrt{4-\rho^2} \, d\rho \, d\theta$$

$$= -10 \int_{\theta=0}^{2\pi} \left(\int_{\rho=1}^{2} \rho \sqrt{4-\rho^2} \, d\rho \right) d\theta$$

$$= -10 \int_{\theta=0}^{2\pi} -\frac{1}{3}(4-\rho^2)^{3/2} \Big]_{\rho=1}^{2} d\theta =$$

$$= +\frac{10}{3}\int_{\theta=0}^{2\pi}\left[(4-4)^{3/2}-(4-1)^{3/2}\right]d\theta$$

$$= \frac{10}{3}\int_{\theta=0}^{2\pi} -3^{3/2}\,d\theta$$

$$= -\frac{10}{3}\,3^{3/2}\,\theta\Big]_{0}^{2\pi}$$

$$= -10\cdot 3^{1/2}\,2\pi$$

$$= -20\pi\sqrt{3}$$

3. $xy' + 6y = 3xy^{4/3}$ $(*)$

$\therefore$ $y' + \frac{6}{x}y = 3y^{4/3}$; $p(x) = \frac{6}{x}$, $a = \frac{4}{3}$

Seja $u(x) = y^{1-\frac{4}{3}} = y^{-\frac{1}{3}}$

$\therefore$ $y = u(x)^{-3}$

$y' = -3u(x)^{-4}u'(x)$

Substituindo em $(*)$:

$x(-3u^{-4}u') + 6u^{-3} = 3x(u^{-3})^{4/3}$

$-3xu^{-4}u' + 6u^{-3} = 3xu^{-4}$

$(\times u^4)$: $-3xu' + 6u = 3x$

$u' - \frac{2}{x}u = -1$

$u' - \frac{2}{x}u + 1 = 0$

$// du + (-\frac{2}{x}u + 1)dx = 0 //$ (0.5)

$\underbrace{\left(-\frac{2}{x}u + 1\right)}_{M} dx + \underbrace{du}_{N} = 0$ $(**)$

$\frac{1}{N}\left(\frac{\partial M}{\partial u} - \frac{\partial N}{\partial x}\right) = \left(-\frac{2}{x}\right) = f(x) \Longrightarrow$

$$F(x) = e^{\int f(x)\,dx} = e^{\int -\frac{2}{x}\,dx} = e^{-2\ln|x|}$$

$$= e^{\ln|x|^{-2}} = |x|^{-2} = \frac{1}{x^2}$$

$$\therefore \quad F(x) = \frac{1}{x^2}$$

0.5

Daí, voltando a (**)

$$\frac{1}{x^2}\left(-\frac{2u}{x}+1\right)dx + \frac{1}{x^2}\,du = 0$$

$$\left(-\frac{2u}{x^3}+\frac{1}{x^2}\right)dx + \frac{1}{x^2}\,du = 0$$

Seja $\varphi(x,u)$ tq.

$$\frac{\partial \varphi}{\partial u} = \frac{1}{x^2} \Longrightarrow \varphi = \frac{1}{x^2}u + k(x)$$

$$\frac{\partial \varphi}{\partial x} = -\frac{2u}{x^3} + \frac{1}{x^2} \Longrightarrow$$

$$\Longrightarrow \frac{\partial}{\partial x}\left(\frac{u}{x^2} + k(x)\right) = -\frac{2u}{x^3} + \frac{1}{x^2}$$

$$\cancel{-\frac{2u}{x^3}} + k'(x) = \cancel{-\frac{2u}{x^3}} + \frac{1}{x^2}$$

$$k'(x) = \frac{1}{x^2}$$

$$\therefore \quad k(x) = -\frac{1}{x}$$

$$\therefore \quad \varphi(x,u) = \frac{1}{x^2}u - \frac{1}{x}$$

Solução da Eq. (**) :

$\underline{\varphi(x,u)} = c \quad (\text{cte.})$

$$\frac{u}{x^2} - \frac{1}{x} = c$$

$u = y^{-1/3}$

$$\frac{y^{-1/3}}{x^2} - \frac{1}{x} = c$$

$$y^{-1/3} = \left(c + \frac{1}{x}\right) x^2$$

$$\| \; y = \left(cx^2 + x\right)^{-3} \; \|$$

4. $x^2y' = 1 - x^2 + y^2 - x^2y^2$

$x^2y' = 1 - x^2 + y^2(1 - x^2)$

$x^2y' = (1 - x^2)(1 + y^2)$

$\frac{y'}{1+y^2} = \frac{1-x^2}{x^2}$

$\frac{dy}{1+y^2} = \frac{1-x^2}{x^2}\, dx$ $\underline{\underline{0.5}}$

$\int \frac{dy}{1+y^2} = \int \frac{1-x^2}{x^2}\, dx$

$\operatorname{arctg} y = \int (x^{-2} - 1)\, dx$

$= \frac{x^{-1}}{-1} - x + K$

$\therefore \; \boxed{\operatorname{arctg} y = -\frac{1}{x} - x + K}$ $\underline{\underline{1.0}}$

5. $(y\ln y + ye^x)\,dx + (x + y\cos y)\,dy = 0$

$$\begin{cases} M(x,y) = y\ln y + ye^x \\ N(x,y) = x + y\cos y \end{cases}$$

$$\left.\begin{array}{l} \dfrac{\partial M}{\partial y} = \ln y + 1 + e^x \\ \\ \dfrac{\partial N}{\partial x} = 1 \end{array}\right\} \neq \quad \text{Não é exata.}$$

<u>Fator integrante</u>

$$\frac{1}{M}\left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) =$$

$$= \frac{1}{y\ln y + ye^x}\left(\cancel{1} - \ln y - \cancel{1} - e^x\right)$$

$$= \frac{1}{y(\cancel{\ln y + e^x})}(-)(\cancel{\ln y + e^x})$$

$$= -\frac{1}{y} \equiv g(y)$$

$\therefore$

$$F(y) = e^{\int g(y)\,dy} = e^{\int -\frac{1}{y}\,dy}$$

$$= e^{-\ln|y|} = e^{\ln|y|^{-1}} = \frac{1}{|y|} = \frac{1}{y}.$$

0.5

Daí:

$$\frac{1}{y}\left\{ (y\ln y + y e^x)\,dx + (x + y\cos y)\,dy = 0 \right.$$

$$\frac{1}{y}(y\ln y + y e^x)\,dx + \frac{1}{y}(x + y\cos y)\,dy = 0$$

$$(\ln y + e^x)\,dx + \left(\frac{x}{y} + \cos y\right)dy = 0$$

Seja $\varphi(x,y)$ tq.

0.5

$$\begin{cases} \dfrac{\partial\varphi}{\partial x} = \ln y + e^x \Rightarrow \varphi(x,y) = x\ln y + e^x + h(y) \\ \dfrac{\partial\varphi}{\partial y} = \dfrac{x}{y} + \cos y \Rightarrow \end{cases}$$

$$\Rightarrow \frac{\partial}{\partial y}\left(x\ln y + e^x + h(y)\right) = \frac{x}{y} + \cos y$$

$$\cancel{x\frac{1}{y}} + h'(y) = \cancel{\frac{x}{y}} + \cos y$$

$$h'(y) = \cos y$$

$$h(y) = \sin y$$

$\therefore \varphi(x,y) = x\ln y + e^x + \sin y$

Solução da Eq.: $\varphi(x,y) = K$

$$\| x\ln y + e^x + \sin y = K \|$$

1.5

6.

(ii) $y'' + 3y' + 2y = xe^{-x} - xe^{-2x}$

Eq. homogénea:

$y'' + 3y' + 2y = 0$

$\lambda^2 + 3\lambda + 2 = 0$

$\lambda = -2, \lambda = -1$

$y_h = C_1 e^{-x} + C_2 e^{-2x}$

$r = xe^{-x} - xe^{-2x}$

$$y_p = \underbrace{(Ax+B)e^{-x}}_{\text{solução da hom.}} + \underbrace{(Cx+D)e^{-2x}}_{\text{solução da hom.}}$$

$y_p = (Ax^2 + Bx)e^{-x} + (Cx^2 + Dx)e^{-2x}$

0.5

$y'_p = (2Ax + B)e^{-x} - (Ax^2 + Bx)e^{-x} +$

$+ (2Cx + D)e^{-2x} - 2(Cx^2 + Dx)e^{-2x}$

$= (2Ax + B - Ax^2 - Bx)e^{-x} +$

$+ (2Cx + D - 2Cx^2 - 2Dx)e^{-2x}$

$$y'_p = (-Ax^2 + (2A-B)x + B)e^{-x} +$$
$$+ (-2Cx^2 + (2C-2D)x + D)e^{-2x}$$

$$y''_p = (-2Ax + 2A - B)e^{-x} - (-Ax^2 + (2A-B)x + B)e^{-x}$$
$$+ (-4Cx + 2C - 2D)e^{-2x}$$
$$-2(-2Cx^2 + (2C-2D)x + D)e^{-2x}$$

$$= (-2Ax + 2A - B + Ax^2 - (2A-B)x - B)e^{-x}$$
$$+ (-4Cx + 2C - 2D + 4Cx^2 - 4Cx + 4Dx$$
$$- 2D)e^{-2x}$$

$$\therefore y''_p = (2A - 2B + (-4A+B)x + Ax^2)e^{-x}$$
$$+ (2C - 4D + (-8C + 4D)x + 4Cx^2)e^{-2x}$$

Dat:

$$y''_p + 3y'_p + 2y_p = xe^{-x} - xe^{-2x}$$

$$\therefore (2A - 2B + (-4A+B)x + Ax^2)e^{-x}$$
$$+ (2C - 4D + (-8C+4D)x + 4Cx^2)e^{-2x}$$
$$+ (-3Ax^2 + (6A-3B)x + 3B)e^{-x} +$$
$$+ (-6Cx^2 + (6C-6D)x + 3D)e^{-2x}$$
$$+ (2Ax^2 + 2Bx)e^{-x} + (2Cx^2 + 2Dx)e^{-2x} = xe^{-x} - xe^{-2x}$$

$\therefore$

$$x^2 e^{-x}(\cancel{A}-\cancel{3A}+\cancel{2A}) + x e^{-x}(-4A+\cancel{B}+6A-\cancel{3B}+\cancel{2B})$$

$$+ e^{-x}(2A-2B+3B) +$$

$$+ x^2 e^{-2x}(\cancel{4C}-\cancel{6C}+\cancel{2C}) + x e^{-2x}(-8C+\cancel{4D}+6C-\cancel{6D}+\cancel{2D})$$

$$+ e^{-2x}(2C-4D+3D) = xe^{-x} - xe^{-2x}$$

$\therefore$

$$xe^{-x}(2A) + e^{-x}(2A+B) +$$

$$+ xe^{-2x}(-2C) + e^{-2x}(2C-D) = xe^{-x} - xe^{-2x}$$

$\therefore$

$$\underline{10}\left\{\begin{array}{l} 2A = 1 \Rightarrow A = 1/2 \,// \\ 2A+B = 0 \Rightarrow B = -2A = -1 \,// \\ -2C = -1 \Rightarrow C = 1/2 \,// \end{array}\right.$$

$$2C - D = 0 \Rightarrow //D = 2C = 1 \,//$$

$$\therefore \quad y_p = \left(\tfrac{1}{2}x^2 - x\right)e^{-x} + \left(\tfrac{1}{2}x^2 + x\right)e^{-2x}$$

$$// \; y_p = \left(\tfrac{1}{2}x^2 - x\right)e^{-x} + \left(\tfrac{1}{2}x^2 + x\right)e^{-2x}$$

$$y = y_h + y_p$$

$$\boxed{y = C_1 e^{-x} + C_2 e^{-2x} + \left(\tfrac{1}{2}x^2 - x\right)e^{-x} + \left(\tfrac{1}{2}x^2 + x\right)e^{-2x}}$$

6 (ii).

$$y'' - 4y' + 4y = \frac{e^{2x}}{x}$$

Eq. homogénea:

$$y'' - 4y' + 4y = 0$$

$$\lambda^2 - 4\lambda + 4 = 0$$

$$(\lambda - 2)^2 = 0 \Rightarrow \lambda = 2$$

$$y_h = C_1 \underbrace{e^{2x}}_{y_1} + C_2 \underbrace{x e^{2x}}_{y_2}$$

$$\longrightarrow W(y_1, y_2) = \det \begin{pmatrix} e^{2x} & x e^{2x} \\ 2e^{2x} & (2x+1) e^{2x} \end{pmatrix}$$

$$\left\{ \begin{array}{ll} y_1 = e^{2x} & y_2 = x e^{2x} \\ y_1' = 2e^{2x} & y_2' = e^{2x} + 2x e^{2x} \\ & \quad = (2x+1) e^{2x} \end{array} \right.$$

$$= (\cancel{2x}+1) e^{4x} - \cancel{2x} e^{4x}$$

$$= e^{4x}$$

0.5

$$y_p = u\, e^{2x} + v\, x e^{2x} \quad ; \quad r(x) = \frac{e^{2x}}{x}$$

onde

$$u(x) = -\int \frac{\frac{\cancel{e^{2x}}}{\cancel{x}} \cdot \cancel{x} \cancel{e^{2x}}}{\cancel{e^{4x}}} \, dx$$

$$= -\int dx = -x$$

$$\therefore \; // u(x) = -x //$$

0.5

$$v(x) = \int \frac{\cancel{\frac{e^{2x}}{x}} \cdot \cancel{e^{2x}}}{\cancel{e^{4x}}} dx$$

$$= \int \frac{1}{x} dx = \ln|x|$$

$$\therefore \quad v(x) = \ln|x|$$

$$\therefore$$

$$y_p = u e^{2x} + v x e^{2x}$$

$$// \; y_p = -x e^{2x} + x \ln|x| \, e^{2x} \; //$$ 0.5

$$y = y_h + y_p$$

$$\boxed{y = c_1 e^{2x} + c_2 x e^{2x} + x e^{2x} (\ln|x| - 1)}$$

$$= c_1 e^{2x} + \tilde{c}_2 x e^{2x} + x e^{2x} \ln|x|$$