8.5/10

Cálculo C - Prova 3

2.0 1. Resolva usando o método dos coeficientes a determinar

$$y''' - 4y'' + 5y' - 2y = xe^x$$

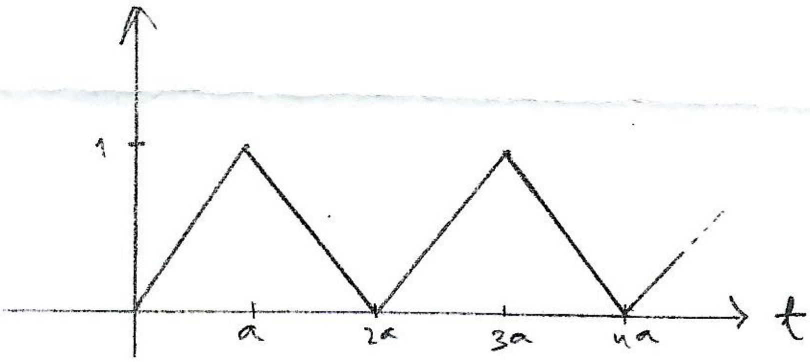
1.5 2. Resolva

$$x^2y'' + 2xy' - 2y = \frac{6}{x^2} + 3x$$

1.0 3. Use transformada de Laplace para resolver o problema de valor inicial

$$y'' + 3y = t^3 \qquad y(0) = 0, \quad y'(0) = 0$$

1.0 4. Seja $f(t)$ a função cujo gráfico é dado a seguir



(i) Escreva $f(t)$ em termos de funções degraus

(ii) Calcule a partir de (i) a transformada de Laplace de $f(t)$

1.0 5. Calcule $\mathcal{L}^{-1}\{\ln\left(\frac{s+2}{s-5}\right)\}$.

1. $y''' - 4y'' + 5y' - 2y = xe^x$

Eq. homogénea: $\lambda^3 - 4\lambda^2 + 5\lambda - 2 = 0$

$\lambda = 1$ é raiz

$$
\begin{array}{l|l}
\lambda^3 - 4\lambda^2 + 5\lambda - 2 & \lambda - 1 \\
\cline{2-2}
-\lambda^3 + \lambda^2 & \lambda^2 - 3\lambda + 2 \\
\cline{1-1}
\quad -3\lambda^2 + 5\lambda - 2 & \\
\quad +3\lambda^2 - 3\lambda & \\
\cline{1-1}
\qquad 2\lambda - 2 & \\
\qquad -2\lambda + 2 & \\
\cline{1-1}
\qquad\ 0 &
\end{array}
$$

$\lambda^2 - 3\lambda + 2 = 0$

$\lambda = \dfrac{3 \pm \sqrt{9-8}}{2} = \dfrac{3 \pm 1}{2} = \begin{cases} 2 \\ 1 \end{cases}$

$\begin{cases} \lambda = 1 : \text{raiz mult. } 2 \\ \lambda = 2 : \text{raiz mult. } 1 \end{cases}$

$y_h = C_1 e^x + C_2 x e^x + C_3 e^{2x}$ 0.5

Eq. não homogénea:

$y_p = (Ax + B)e^x$ → presente na homogénea

$y_p = (Ax^2 + Bx)e^x$ → presente na hom.

$y_p = (Ax^3 + Bx^2)e^x$ 0.5

$$y'_p = (3Ax^2 + 2Bx)e^x + (Ax^3 + Bx^2)e^x$$

$$y''_p = (6Ax + 2B)e^x + (3Ax^2 + 2Bx)e^x +$$

$$+ (3Ax^2 + 2Bx)e^x + (Ax^3 + Bx^2)e^x$$

$$= (2B + 6Ax + 4Bx + 6Ax^2 + Bx^2 + Ax^3)e^x$$

$$y'''_p = (6A + 4B + 12Ax + 2Bx + 3Ax^2)e^x +$$

$$+ (2B + 6Ax + 4Bx + 6Ax^2 + Bx^2 + Ax^3)e^x$$

$$= (6A + 6B + 18Ax + 6Bx + 9Ax^2 + Bx^2 + Ax^3)e^x$$

Dari

$$y''' - 4y'' + 5y' - 2y = xe^x$$

$$(6A + 6B + 18Ax + 6Bx + 9Ax^2 + Bx^2 + Ax^3)e^x$$

$$- 4(2B + 6Ax + 4Bx + 6Ax^2 + Bx^2 + Ax^3)e^x +$$

$$+ 5(2Bx + 3Ax^2 + Bx^2 + Ax^3)e^x$$

$$- 2(Ax^3 + Bx^2)e^x = xe^x$$

$$6A + 6B + 18Ax + \cancel{6Bx} + \cancel{9Ax^2} + \cancel{Bx^2} + \cancel{Ax^3}$$
$$-8B - 24Ax - \cancel{16Bx} - \cancel{24Ax^2} - \cancel{4Bx^2} - \cancel{4Ax^3}$$
$$+\cancel{10Bx} + \cancel{15Ax^2} + \cancel{5Bx^2} + \cancel{5Ax^3}$$
$$-\cancel{2Ax^3} - \cancel{2Bx^2} = x$$

$$6A - 2B - 6Ax = x$$

$$\begin{cases} 6A - 2B = 0 \\ -6A = 1 \end{cases} \Rightarrow A = -\frac{1}{6}$$

$$6\left(-\frac{1}{6}\right) - 2B = 0$$
$$-1 - 2B = 0$$
$$B = -\frac{1}{2}$$

$$y_p = (Ax^3 + Bx^2)\, e^x$$
$$= \left(-\frac{1}{6}x^3 - \frac{1}{2}x^2\right) e^x$$

$$\boxed{y = C_1 e^x + C_2 x e^x + C_3 e^{2x} - \left(\frac{1}{6}x^3 + \frac{1}{2}x^2\right) e^x}$$

1-0

2. $x^2y'' + 2xy' - 2y = \frac{6}{x^2} + 3x \quad ; \quad x > 0$

Eq. homogénea

$$x^2y'' + 2xy' - 2y = 0 \quad (*)$$

$$\begin{cases} y_h = x^m \\ y_h' = m x^{m-1} \\ y_h'' = m(m-1)\, x^{m-2} \end{cases}$$

Subst. em $(*)$:

$$m(m-1)\, x^m + 2m\, x^m - 2x^m = 0$$

$$\therefore \quad m(m-1) + 2m - 2 = 0$$

$$m^2 + m - 2 = 0$$

$$m = \frac{-1 \pm \sqrt{1+8}}{2} = \frac{-1 \pm 3}{2} \begin{matrix} \nearrow -2 \\ \searrow 1 \end{matrix}$$

$$y_h = C_1 x + C_2 x^{-2} \qquad \underline{0{,}25}$$

Eq. não-homogénea

$$x^2 y'' + 2xy' - 2y = \frac{6}{x^2} + 3x$$

$$\therefore \quad y'' + \frac{2}{x} y' - \frac{2}{x^2} y = \frac{6}{x^4} + \frac{3}{x}$$

$$y_h = c_1 x + c_2 \frac{1}{x^2}$$

$$\downarrow$$

$$y_p = u x + v \frac{1}{x^2}$$

onde

$$u(x) = -\int \frac{y_2 r(x)}{W(y_1, y_2)} dx$$

$$v(x) = \int \frac{y_1 r(x)}{W(y_1, y_1)} dx$$

$$W(y_1, y_2) = \det \begin{pmatrix} y_1 & y_2 \\ y_1' & y_2' \end{pmatrix} = \det \begin{pmatrix} x & x^{-2} \\ 1 & -2x^{-3} \end{pmatrix}$$

$$= -2x^{-2} - x^{-2} = -3x^{-2}$$

$$u(x) = -\int \frac{x^{-2}\left(\frac{6}{x^4} + \frac{3}{x}\right)}{-3x^{-2}} dx$$

$$= +\frac{1}{3}\int \left(\frac{6}{x^4} + \frac{3}{x}\right) dx = 2\int \frac{1}{x^4} dx + \int \frac{1}{x} dx$$

$$= 2\frac{x^{-3}}{-3} + \ln x = -\frac{2}{3x^3} + \ln x$$

0.5

$$v(x) = \int \frac{x\left(\frac{6}{x^4} + \frac{3}{x}\right)}{-3x^{-2}} \, dx$$

$$= \int \frac{-1}{3} x^3 \left(\frac{6}{x^4} + \frac{3}{x}\right) dx$$

$$= \int -\frac{2}{x} \, dx - \int x^2 \, dx$$

$$= -2 \ln|x| - \frac{x^3}{3} \qquad \underline{\underline{0.5}}$$

$$\therefore \quad y_p = \left(-\frac{2}{3x^3} + \ln x\right) x + \left(-2\ln|x| - \frac{x^3}{3}\right) \frac{1}{x^2}$$

$$= -\frac{2}{3x^2} + x \ln x - \frac{2}{x^2} \ln x - \frac{1}{3} x$$

$$y_p = -\frac{1}{3} x - \frac{2}{3} x^{-2} + x \ln x - \frac{2}{x^2} \ln x$$

$$y = y_h + y_p = c_1 x + c_2 x^{-2} - \frac{1}{3} x - \frac{2}{3} x^{-2} + x \ln x - \frac{2}{x^2} \ln x$$

$$\boxed{y = \tilde{c}_1 x + \tilde{c}_2 x^{-2} + x \ln x - \frac{2}{x^2} \ln x} \qquad \underline{0.25}$$

$(x > 0)$

3. $y'' + 3y = t^3 \quad ; \quad y(0) = 0 \quad , \quad y'(0) = 0$

$$\mathcal{L}(y'') + 3\mathcal{L}(y) = \mathcal{L}(t^3)$$

$$s^2\mathcal{L}(y) - s\,\cancel{y(0)}^{\to 0} - \cancel{y'(0)}^{\to 0} + 3\,\mathcal{L}(y) = \frac{6}{s^4}$$

$$(s^2+3)\,\mathcal{L}(y) = \frac{6}{s^4}$$

$$\mathcal{L}(y) = \frac{6}{s^4}\,\frac{1}{(s^2+3)}$$

$$y = \mathcal{L}^{-1}\left(\frac{6}{s^4}\,\frac{1}{(s^2+3)}\right)$$

$$= \mathcal{L}^{-1}\left(\frac{6}{s^4}\,\frac{1}{(s^2+3)}\right)$$

1.50

Convolução

$$\frac{1}{s^4}\,\frac{1}{(s^2+3)} = \mathcal{L}(f * g)$$

onde
$$\begin{cases} \dfrac{6}{s^4} = \mathcal{L}^{-1}(f) \to f = t^3 \\[2ex] \dfrac{1}{s^2+3} = \mathcal{L}^{-1}(g) \to g = \dfrac{1}{\sqrt{3}}\sin\sqrt{3}\,t \end{cases}$$

$$f * g = \int_0^t (t-x)^3 \frac{1}{\sqrt{3}} \sin\sqrt{3}x \, dx$$

$$= \frac{1}{\sqrt{3}} \int_0^t (t^3 - 3t^2x + 3tx^2 - x^3) \sin\sqrt{3}x \, dx$$

$$= \frac{1}{\sqrt{3}} t^3 \overset{①}{\int_0^t \sin\sqrt{3}x \, dx} - \frac{3}{\sqrt{3}} t^2 \overset{②}{\int_0^t x \sin\sqrt{3}x \, dx} +$$

$$+ \frac{3t}{\sqrt{3}} \overset{③}{\int_0^t x^2 \sin\sqrt{3}x \, dx} - \frac{1}{\sqrt{3}} \overset{④}{\int_0^t x^3 \sin\sqrt{3}x \, dx}$$

$$① = \int_0^t \sin\sqrt{3}x \, dx = -\frac{1}{\sqrt{3}} \cos\sqrt{3}x \Big]_0^t$$

$$= -\frac{1}{\sqrt{3}} \cos\sqrt{3}t + \frac{1}{\sqrt{3}}$$

$$② : \int x \sin\sqrt{3}x \, dx =$$

$$\left(\begin{array}{ll} u = x & \longrightarrow du = dx \\ dv = \sin\sqrt{3}x \, dx & \rightarrow v = -\frac{1}{\sqrt{3}} \cos\sqrt{3}x \end{array} \right)$$

$$= -\frac{x}{\sqrt{3}} \cos\sqrt{3}x + \frac{1}{\sqrt{3}} \int \cos\sqrt{3}x \, dx$$

$$= -\frac{x}{\sqrt{3}} \cos\sqrt{3}x + \frac{1}{3} \sin\sqrt{3}x$$

$$\therefore \; // \int_0^t x \sin\sqrt{3}x \, dx = -\frac{t}{\sqrt{3}} \cos\sqrt{3}t + \frac{1}{3} \sin\sqrt{3}t \; //$$

(3) $= \int x^2 \sin\sqrt{3}\,x \, dx$

$$\begin{pmatrix} u = x^2 & \longrightarrow & du = 2x\,dx \\ dv = \sin\sqrt{3}x\,dx & \longrightarrow & v = -\frac{1}{\sqrt{3}}\cos\sqrt{3}\,x \end{pmatrix}$$

$$= -\frac{1}{\sqrt{3}} x^2 \cos\sqrt{3}\,x + \frac{2}{\sqrt{3}} \underbrace{\int x \cos\sqrt{3}\,x \, dx}_{(*)}$$

Mà

$$(*) = \int x \cos\sqrt{3}\,x \, dx$$

$$\begin{pmatrix} u = x & \longrightarrow & du = dx \\ dv = \cos\sqrt{3}x & \longrightarrow & v = \frac{1}{\sqrt{3}}\sin\sqrt{3}x \end{pmatrix}$$

$$= \frac{x}{\sqrt{3}} \sin\sqrt{3}x - \frac{1}{\sqrt{3}} \int \sin\sqrt{3}x \, dx$$

$$= \frac{x}{\sqrt{3}} \sin\sqrt{3}x + \frac{1}{3} \cos\sqrt{3}\,x$$

$$(3) = -\frac{1}{\sqrt{3}} x^2 \cos\sqrt{3}\,x + \frac{2}{\sqrt{3}} \left(\frac{x}{\sqrt{3}} \sin\sqrt{3}x + \frac{1}{3}\cos\sqrt{3}x \right)$$

$$= -\frac{1}{\sqrt{3}} x^2 \cos\sqrt{3}\,x + \frac{2x}{3} \sin\sqrt{3}x + \frac{2}{3\sqrt{3}} \cos\sqrt{3}\,x$$

Đặt

$$\int_0^t x^2 \sin\sqrt{3}\,x \, dx =$$

$$= -\frac{1}{\sqrt{3}} t^2 \cos\sqrt{3}\,t + \frac{2t}{3} \sin\sqrt{3}\,t + \frac{2}{3\sqrt{3}} \cos\sqrt{3}\,t - \frac{2}{3\sqrt{3}}$$

(4) $= \int x^3 \sin\sqrt{3}\,x \, dx$

$$\left(\begin{array}{l} u = x^3 \rightarrow du = 3x^2\,dx \\ dv = \sin\sqrt{3}\,x \rightarrow v = -\frac{1}{\sqrt{3}} \cos\sqrt{3}\,x \end{array} \right)$$

$$= -\frac{1}{\sqrt{3}} x^3 \cos\sqrt{3}\,x + \frac{3}{\sqrt{3}} \underbrace{\int x^2 \cos\sqrt{3}\,x \, dx}_{(*_1)}$$

$(*_1) = \int x^2 \cos\sqrt{3}\,x \, dx$

$$\left(\begin{array}{l} u = x^2 \rightarrow du = 2x\,dx \\ dv = \cos\sqrt{3}\,x\,dx \rightarrow v = +\frac{1}{\sqrt{3}} \sin\sqrt{3}\,x \end{array} \right)$$

$$= +\frac{1}{\sqrt{3}} x^2 \sin\sqrt{3}\,x - \frac{2}{\sqrt{3}} \int x \sin\sqrt{3}\,x \, dx$$

$$= \frac{1}{\sqrt{3}} x^2 \sin\sqrt{3}\,x - \frac{2}{\sqrt{3}} \left(-\frac{x}{\sqrt{3}} \cos\sqrt{3}\,x + \frac{1}{3} \sin\sqrt{3}\,x \right)$$

$$= \frac{1}{\sqrt{3}} x^2 \sin\sqrt{3}\,x + \frac{2}{3} x \cos\sqrt{3}\,x - \frac{2}{3\sqrt{3}} \sin\sqrt{3}\,x$$

$$\int x^3 \sin\sqrt{3}\, x \, dx =$$

$$= -\frac{1}{\sqrt{3}} x^3 \cos\sqrt{3}x + \frac{3}{\sqrt{3}} \left(\frac{1}{\sqrt{3}} x^2 \sin\sqrt{3}x + \frac{2}{3} x \cos\sqrt{3}x - \frac{2}{3\sqrt{3}} \sin\sqrt{3}x \right)$$

$$= -\frac{1}{\sqrt{3}} x^3 \cos\sqrt{3}x + x^2 \sin\sqrt{3}x + \frac{2}{\sqrt{3}} x \cos\sqrt{3}x - \frac{2}{3} \sin\sqrt{3}x$$

$$\therefore \int_0^t x^3 \sin\sqrt{3}\, x \, dx =$$

$$= -\frac{1}{\sqrt{3}} t^3 \cos\sqrt{3}t + t^2 \sin\sqrt{3}t + \frac{2}{\sqrt{3}} t \cos\sqrt{3}t - \frac{2}{3} \sin\sqrt{3}t$$

Daí

$$f*g = \frac{1}{\sqrt{3}} t^3 \left(-\frac{1}{\sqrt{3}} \cos\sqrt{3}t + \frac{1}{\sqrt{3}}\right) - \frac{3t^2}{\sqrt{3}} \left(-\frac{t}{\sqrt{3}} \cos\sqrt{3}t + \frac{1}{3} \sin\sqrt{3}t\right)$$

$$+ \frac{3t}{\sqrt{3}} \left(-\frac{1}{\sqrt{3}} t^2 \cos\sqrt{3}t + \frac{2}{3} t \sin\sqrt{3}t + \frac{2}{3\sqrt{3}} \cos\sqrt{3}t - \frac{2}{3\sqrt{3}}\right)$$

$$- \frac{1}{\sqrt{3}} \left(-\frac{1}{\sqrt{3}} t^3 \cos\sqrt{3}t + t^2 \sin\sqrt{3}t + \frac{2}{\sqrt{3}} t \cos\sqrt{3}t - \frac{2}{3} \sin\sqrt{3}t\right)$$

$$f*g = \cancel{-\frac{1}{3}t^3\cos\sqrt{3}\,t} + \frac{1}{3}t^3 + \cancel{t^3\cos\sqrt{3}\,t} - \cancel{\frac{1}{\sqrt{3}}t^2\sin\sqrt{3}\,t}$$

$$\cancel{-t^3\cos\sqrt{3}\,t} + \cancel{\frac{2t^2}{\sqrt{3}}\sin\sqrt{3}\,t} + \cancel{\frac{2}{3}t\cos\sqrt{3}\,t} - \frac{2}{3}t$$

$$+ \cancel{\frac{t^3}{3}\cos\sqrt{3}\,t} - \cancel{\frac{t^2}{\sqrt{3}}\sin\sqrt{3}\,t} - \cancel{\frac{2t}{3}\cos\sqrt{3}\,t}$$

$$+ \frac{2}{3\sqrt{3}}\sin\sqrt{3}\,t$$

$$= -\frac{2}{3}t + \frac{1}{3}t^3 + \frac{2}{3\sqrt{3}}\sin\sqrt{3}\,t$$

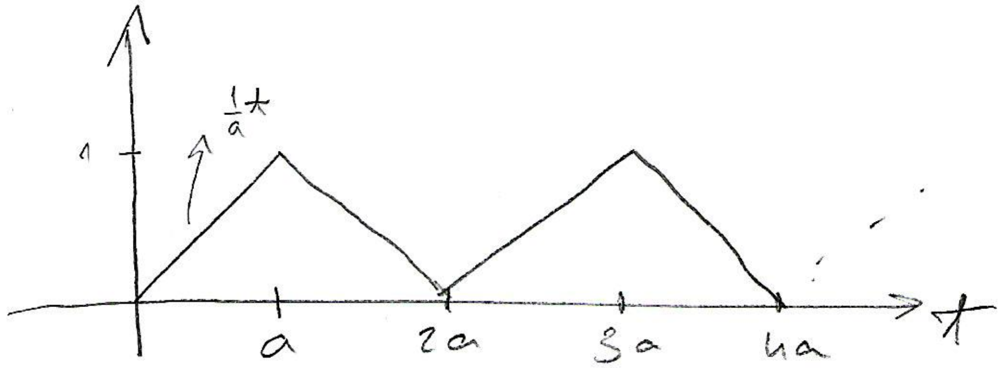
Mo

$$g = \mathcal{L}^{-1}\left(\frac{6}{s^4}\,\frac{1}{(s^2+3)}\right)$$

$$\boxed{y = -\frac{2}{3}t + \frac{1}{3}t^3 + \frac{2}{3\sqrt{3}}\sin\sqrt{3}\,t}$$

4.

i)



$0 < t < a$: $f(t) = \frac{1}{a}t$

$a < t < 2a$: $f(t) = mt + q$

$f(a) = 1$: $1 = ma + q$
$f(2a) = 0$: $0 = 2ma + q$

$1 = -ma$

$m = -\frac{1}{a}$

$q = -2ma = -2\left(-\frac{1}{a}\right)a = 2$

$f(t) = -\frac{1}{a}t + 2$

$2a < t < 3a$: $f(t) = mt + q$

$f(2a) = 0$: $0 = 2ma + q$
$f(3a) = 1$: $1 = 3ma + q$

$q = -2ma = -2$

$ma = 1$

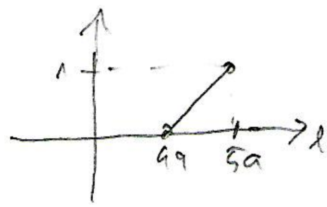
$a = \frac{1}{m}$

$\therefore m = \frac{1}{a}$

$f(t) = \frac{1}{a}t - 2$

$3a < t < 4a$: $f(t) = -\frac{1}{a}t + 4$

$4a < t < 5a$: $f(t) = \frac{1}{a}t - 4$



⋮

Isto é:

$$2na < t < (2n+1)a \; : \; f(t) = \frac{1}{a}t - 2n$$

$$(2n+1)a < t < (2n+2)a \; : \; f(t) = -\frac{1}{a}t + (2n+2)$$

$$(2n+2)a < t < (2n+3)a \; : \; f(t) = \frac{1}{a}t - (2n+2)$$

$$t > 0 \; : \; f(t) = \frac{1}{a}t\left(u(t-0) - u(t-a)\right)$$

$$\left.\begin{aligned} &+ \left(-\frac{1}{a}t + 2\right)\left(u(t-a) - u(t-2a)\right) \\ &+ \left(\frac{1}{a}t - 2\right)\left(u(t-2a) - u(t-3a)\right) \end{aligned}\right] \; 1$$

$$\left.\begin{aligned} &+ \left(-\frac{1}{a}t + 4\right)\left(u(t-3a) - u(t-4a)\right) \\ &+ \left(\frac{1}{a}t - 4\right)\left(u(t-4a) - u(t-5a)\right) \end{aligned}\right] \; 2$$

$$+ \cdots +$$

$$\left.\begin{aligned} &\left(-\frac{1}{a}t + (2n-2)\right)\left(u(t-(2n-3)a) - u(t-(2n-2)a)\right) \\ &+ \left(\frac{1}{a}t - (2n-2)\right)\left(u(t-(2n-2)a) - u(t-(2n-1)a)\right) \end{aligned}\right]$$

$$+ \cdots$$

$\therefore$

$$f(t) = \frac{1}{a} t\, u(t-0) - \frac{1}{a} t\, u(t-a)$$

$$+ \left(-\frac{1}{a} t + 2\right) u(t-a) + \left(\frac{1}{a} t - 2\right) u(t-2a)$$

$$+ \left(\frac{1}{a} t - 2\right) u(t-2a) - \left(\frac{1}{a} t - 2\right) u(t-3a)$$

$$\left(-\frac{1}{a} t + 4\right) u(t-3a) + \left(\frac{1}{a} t - 4\right) u(t-4a)$$

$$+ \left(\frac{1}{a} t - 4\right) u(t-4a) - \left(\frac{1}{a} t - 4\right) u(t-5a) + \cdots$$

$$+ \cdots + \left(-\frac{1}{a} t + (2n-2)\right) u(t-(2n+1)a)$$

$$+ \left(\frac{1}{a} t - (2n-2)\right) u(t-(2n+2)a)$$

$$+ \left(\frac{1}{a} t - (2n-2)\right) u(t-(2n+2)a) + \cdots$$

$$= \frac{1}{a} t\, u(t) + \left(-\frac{2}{a} t + 2\right) u(t-a) +$$

$$+ \left(\frac{2}{a} t - 4\right) u(t-2a) + \left(-\frac{2}{a} t + 6\right) u(t-3a)$$

$$+ \left(\frac{2}{a} t - 8\right) u(t-4a) + \cdots +$$

$$\boxed{f(t) = \frac{1}{a} t\, u(t) + \sum_{n=1}^{\infty} (-1)^n \left[\frac{2}{a} t - 2n\right] u(t-na)}$$

1.0

ii) $\mathcal{L}(f(t)) = \frac{1}{a}\mathcal{L}(u(t)\,t) + \sum_{n=1}^{\infty}(-1)^n \mathcal{L}\left(\left(\frac{2}{a}t - 2n\right)u(t-na)\right)$

$= \frac{1}{a}\mathcal{L}(u(t)\,t) + \sum_{n=1}^{\infty}(-1)^n \mathcal{L}\left(\frac{2}{a}(t-na)\,u(t-na)\right)$

$= \frac{1}{a}\mathcal{L}(u(t)\,t) + \sum_{n=1}^{\infty}(-1)^n \frac{2}{a}\mathcal{L}\left(u(t-na)(t-na)\right)$

$= \frac{1}{a}\frac{1}{s^2} + \sum_{n=1}^{\infty}(-1)^n \frac{2}{a}\,e^{-nas}\,\frac{1}{s^2}$

$$\boxed{\mathcal{L}(f(t)) = \frac{1}{as^2} + \sum_{n=1}^{\infty}(-1)^n \frac{2e^{-nas}}{as^2} \quad ; \; s>0}$$

1.0

$= \frac{1}{as^2}\left[1 + 2\sum_{n=1}^{\infty}(-1)^n e^{-nas}\right]$

$S_n = -e^{-as} + e^{-2as} - e^{-3as} + \dots + (-1)^n e^{-nas}$

$e^{-as}S_n = \quad -e^{-2as} + e^{-3as} - \dots - + (-1)^{n+1}e^{-(n+1)as}$

$S_n(1+e^{-as}) = -e^{-as} + (-1)^{n+1}e^{-(n+1)as}$

$S_n = \frac{-e^{-as} + (-1)^{n+1}e^{-(n+1)as}}{1+e^{-as}}$

$\lim_{n\to\infty} S_n = \frac{-e^{-as}}{1+e^{-as}}$; $\mathcal{L}(f(t)) = \frac{1}{as^2}\left[1 - \frac{2e^{-as}}{1+e^{-as}}\right]$

$= \frac{1}{as^2}\left(\frac{1-e^{-as}}{1+e^{-as}}\right)$

5.

$$\mathcal{L}^{-1}\left(\ln\left(\frac{s+2}{s-5}\right)\right)$$

Seja

$$\mathcal{L}\left(t^n f(t)\right) = (-1)^n \frac{d^n}{ds^n} F(s)$$

$$\Leftrightarrow$$

$$\left| \begin{array}{l} t^n f(t) = (-1)^n \mathcal{L}^{-1}\left(\frac{d^n}{ds^n} F(s)\right) \\ \text{onde } F(s) = \mathcal{L}(f(t)) \end{array}\right.$$

isto é:

$$(*) \quad t^n \mathcal{L}^{-1}(F(s)) = (-1)^n \mathcal{L}^{-1}\left(\frac{d^n}{ds^n} F(s)\right)$$

Seja $F(s) = \ln\left(\frac{s+2}{s-5}\right)$

Então

$$\frac{d}{ds} F(s) = \left(\frac{s-5}{s+2}\right) \cdot \left[\frac{(s+2)'(s-5) - (s+2)(s-5)'}{(s-5)^2}\right]$$

$$= \left(\frac{\cancel{s-5}}{s+2}\right) \cdot \left[\frac{s-5-(s+2)}{(s-5)^{\cancel{2}}}\right]$$

$$= \frac{1}{(s+2)} \frac{(-7)}{(s-5)}$$

$$= -7 \frac{1}{(s+2)(s-5)}$$

1.0

$$= \left[\frac{1}{s+2} - \frac{1}{s-5}\right]$$

Daí, de (*) tem-se:

$$t\,\mathcal{L}^{-1}\left(\ln\frac{(s+2)}{(s-5)}\right) = -\mathcal{L}^{-1}\left(\frac{d}{ds}\ln\frac{(s+2)}{(s-5)}\right)$$

$$= -\mathcal{L}^{-1}\left(\frac{1}{s+2} - \frac{1}{s-5}\right)$$

$$= -\mathcal{L}^{-1}\left(\frac{1}{s+2}\right) + \mathcal{L}^{-1}\left(\frac{1}{s-5}\right)$$

$$t\,\mathcal{L}^{-1}\left(\ln\left(\frac{s+2}{s-5}\right)\right) = -e^{-2t} + e^{5t}$$

$$\therefore \boxed{\mathcal{L}^{-1}\left(\ln\left(\frac{s+2}{s-5}\right)\right) = -\frac{1}{t}e^{-2t} + \frac{1}{t}e^{5t}}$$