$$f(s) = \frac{s+1}{s^2(s^2+4s+5)} = \frac{A}{s} + \frac{B}{s^2} + \frac{Cs+D}{s^2+4s+5}$$

$\therefore$

$$\frac{s+1}{s^2(s^2+4s+5)} = \frac{As(s^2+4s+5)+B(s^2+4s+5)+(Cs+D)s^2}{s^2(s^2+4s+5)}$$

$$\frac{s+1}{s^2(s^2+4s+5)} = \frac{(As^3+4As^2+5As+Bs^2+4Bs+5B+ +Cs^3+Ds^2)}{s^2(s^2+4s+5)}$$

$$\frac{s+1}{s^2(s^2+4s+5)} = \frac{\{(A+C)s^3+(4A+B+D)s^2+(5A+4B)s +5B\}}{s^2(s^2+4s+5)}$$

$$\Rightarrow \begin{cases} A+C=0 \\ 4A+B+D=0 \\ 5A+4B=1 \\ 5B=1 \Rightarrow \| B=\frac{1}{5} \| \end{cases}$$

$$5A+4B=1 \Rightarrow \| A = \frac{1-4B}{5} = \frac{1-\frac{4}{5}}{5} = \frac{1}{25} \|$$

$$A+C=0 \Rightarrow \| C = -A = -\frac{1}{25} \|$$

$$4A + B + D = 0$$

$$\therefore \; D = -4A - B$$

$$= -\frac{4}{25} - \frac{1}{5} = \frac{-9}{25}$$

$\therefore$

$$\frac{s+1}{s^2(s^2+4s+5)} = \frac{1}{25}\frac{1}{s} + \frac{1}{5}\frac{1}{s^2} + \frac{-\frac{1}{25}s - \frac{9}{25}}{s^2+4s+5}$$

$$(*) \qquad (**) \qquad (***)$$

$$(*) = \frac{1}{25}\frac{1}{s}$$

$$\therefore \; \mathcal{L}^{-1}\left(\frac{1}{25}\frac{1}{s}\right) = \frac{1}{25}$$

$$(**) = \frac{1}{5}\frac{1}{s^2}$$

$$\therefore \; \mathcal{L}^{-1}\left(\frac{1}{5}\frac{1}{s^2}\right) = \frac{1}{5}t$$

$$(***) = \frac{-\frac{1}{25}s - \frac{9}{25}}{s^2+4s+5} = \frac{-\frac{1}{25}(s+2) + \frac{2}{25} - \frac{9}{25}}{(s+2)^2+1}$$

$$= -\frac{1}{25}\frac{s+2}{(s+2)^2+1} - \frac{7}{25}\frac{1}{(s+2)^2+1}$$

$$\mathcal{L}^{-1}\left(-\frac{1}{25}\,\frac{s+2}{[(s+2)^2+1]}\right) = -\frac{1}{25}\,\mathcal{L}^{-1}\left(\frac{s+2}{(s+2)^2+1}\right)$$

Mas $\quad \mathcal{L}(\cos t) = \dfrac{s}{s^2+1}$

$$\mathcal{L}(e^{-2t}\cos t) = \frac{s+2}{(s+2)^2+1}$$

$$\therefore \quad \mathcal{L}^{-1}\left(-\frac{1}{25}\,\frac{s+2}{(s+2)^2+1}\right) = -\frac{1}{25}\,e^{-2t}\cos t$$

$$\mathcal{L}^{-1}\left(-\frac{7}{25}\,\frac{1}{[(s+2)^2+1]}\right) = -\frac{7}{25}\,\mathcal{L}^{-1}\left(\frac{1}{(s+2)^2+1}\right)$$

Mas $\quad \mathcal{L}(\sin t) = \dfrac{1}{s^2+1}$

$$\mathcal{L}(e^{-2t}\sin t) = \frac{1}{(s+2)^2+1}$$

$$\therefore \quad \mathcal{L}^{-1}\left(-\frac{7}{25}\,\frac{1}{[(s+2)^2+1]}\right) = -\frac{7}{25}\,e^{-2t}\sin t$$

Dec :

$$f(t) = \mathcal{L}^{-1}\left(\frac{s+1}{s^2(s^2+4s+5)}\right)$$

$$= \mathcal{L}^{-1}\left(\frac{1}{25}\frac{1}{s}\right) + \mathcal{L}^{-1}\left(\frac{1}{5}\frac{1}{s^2}\right) +$$

$$+ \mathcal{L}^{-1}\left(-\frac{1}{25}\frac{(s+2)}{[(s+2)^2+1]}\right) +$$

$$+ \mathcal{L}^{-1}\left(-\frac{7}{25}\frac{1}{[(s+2)^2+1]}\right)$$

$$= \boxed{\frac{1}{25} + \frac{1}{5}t - \frac{1}{25}e^{-2t}\cos t - \frac{7}{25}e^{-2t}\sin t}$$