Cálculo C – Lista 1

1. $\vec{F}(t) = t\,\hat{i} + t^2\,\hat{j} + t^3\,\hat{k}$

$f_1(t) = t$, $f_2(t) = t^2$, $f_3(t) = t^3$

$$\left.\begin{array}{l} \text{Dom } f_1 = \mathbb{R} \\ \text{Dom } f_2 = \mathbb{R} \\ \text{Dom } f_3 = \mathbb{R} \end{array}\right\} \Rightarrow \text{Dom } F = \text{Dom } f_1 \cap \text{Dom } f_2 \cap \text{Dom } f_3 = \mathbb{R}$$

$$\therefore \boxed{\begin{array}{l} \text{Dom } \vec{F} = \mathbb{R} \\ f_1 = t \;,\; f_2 = t^2 \;,\; f_3 = t^3 \end{array}}$$

2. $\vec{F}(t) = \sqrt{t+1}\,\hat{i} + \sqrt{t-1}\,\hat{j} + \hat{k}$

$$\left|\begin{array}{l} f_1(t) = \sqrt{t+1} \;,\; f_2(t) = \sqrt{t-1} \\ f_3(t) = 1 \end{array}\right.$$

$\underline{\text{Dom } f_1}$: $t+1 \geq 0 \Rightarrow t \geq -1$

$\text{Dom } f_1 = [-1, +\infty)$

$\underline{\text{Dom } f_2}$: $t-1 \geq 0 \Rightarrow t \geq 1$

$\text{Dom } f_2 = [1, +\infty)$

$\underline{\text{Dom } f_3}$: $\mathbb{R}$

$\text{Dom } f = \text{Dom } f_1 \cap \text{Dom } f_2 \cap \text{Dom } f_3$

$= [-1, +\infty) \cap [1, +\infty) \cap \mathbb{R}$ ...

$$\therefore \boxed{\begin{array}{l} \text{Dom } \vec{F} = [1, +\infty) \\ f_1(t) = \sqrt{t+1} \;,\; f_2(t) = \sqrt{t-1} \;,\; f_3(t) = 1 \end{array}}$$

3. $\vec{F}(t) = \text{tgh}\, t\,\hat{i} - \dfrac{1}{t^2-4}\,\hat{k}$

$$\left\{\begin{array}{l} f_1(t) = \text{tgh}\, t = \dfrac{e^t - e^{-t}}{e^t + e^{-t}} \\ \text{Dom } f_1 = \mathbb{R} \end{array}\right.$$

$$\left\{ f_2(t) = 0 \;,\; \text{Dom } f_2 = \mathbb{R} \right.$$

$$\left\{\begin{array}{l} f_3(t) = \dfrac{-1}{t^2-4} \\ \underline{\text{Dom } f_3} : \; t^2 - 4 \neq 0 \;\therefore\; t \neq \pm 2 \\ \text{Dom } f_3 = \mathbb{R} - \{\pm 2\} \end{array}\right.$$

$\therefore$ $\text{Dom } f = \text{Dom } f_1 \cap \text{Dom } f_2 \cap \text{Dom } f_3$

$= \mathbb{R} \cap \mathbb{R} \cap (\mathbb{R} - \{\pm 2\})$

$= \mathbb{R} - \{\pm 2\}$ //

$$\therefore \boxed{\begin{array}{l} \text{Dom } \vec{F} = \mathbb{R} - \{\pm 2\} \\ f_1(t) = \text{tgh}\, t \\ f_2(t) = 0 \\ f_3(t) = -\dfrac{1}{t^2-4} \end{array}}$$

4. $\vec{F}(t) = \left[(t^2-1)\hat{i} + \ln t\,\hat{j} + \cot t\,\hat{k}\right] \times$

$\times \left[(4-t^2)\hat{i} + e^{-5t}\hat{j} + \frac{1}{t}\hat{k}\right]$

$= (t^2-1)e^{-5t}\,\hat{i}\times\hat{j} +$

$(t^2-1)\frac{1}{t}\,\hat{i}\times\hat{k} +$

$+ \ln t\,(4-t^2)\,\hat{j}\times\hat{i}$

$+ \ln t\,\frac{1}{t}\,\hat{j}\times\hat{k}$

$+ \cot t\,(4-t^2)\,\hat{k}\times\hat{i}$

$+ \cot t\, e^{-5t}\,\hat{k}\times\hat{j}$

$= (t^2-1)e^{-5t}\,\hat{k} + \frac{t^2-1}{t}(-\hat{j})$

$+ (4-t^2)\ln t\,(-\hat{k}) +$

$+ \frac{\ln t}{t}\,\hat{i} +$

$+ \cot t\,(4-t^2)\,\hat{j}$

$+ \cot t\, e^{-5t}(-\hat{i})$

$= \left(\frac{\ln t}{t} - \cot t\, e^{-5t}\right)\hat{i}$

$+ \left((4-t^2)\cot t - \frac{t^2-1}{t}\right)\hat{j}$

$+ \left((t^2-1)e^{-5t} - (4-t^2)\ln t\right)\hat{k}$

• $f_1(t) = \frac{\ln t}{t} - \cot t\, e^{-5t}$

<u>Dom f_1</u> :

$\ln t \Rightarrow t > 0$

$\frac{1}{t} \Rightarrow t \neq 0$

$\cot t \Rightarrow t \neq n\pi,\ n\in\mathbb{Z}$

$\text{Dom } f_1 = (0,+\infty) - \{n\pi,\ n\in\mathbb{Z}\}$

• $f_2(t) = (4-t^2)\cot t - \frac{t^2-1}{t}$

$\cot t \Rightarrow t \neq n\pi,\ n\in\mathbb{Z}$

$\frac{1}{t} \Rightarrow t \neq 0$

$\text{Dom } f_2 = \mathbb{R} - \{n\pi \mid n\in\mathbb{Z}\}$

• $f_3(t) = (t^2-1)e^{-5t} - (4-t^2)\ln t$

$\ln t \Rightarrow t > 0.$

$\text{Dom } f_3 = (0,+\infty)$

$\therefore \text{Dom } \vec{F} = \text{Dom } f_1 \cap \text{Dom } f_2 \cap \text{Dom } f_3$

$= (0,+\infty) - \{n\pi \mid n\in\mathbb{Z}\}$

$= (0,\pi)\cup(\pi,2\pi)\cup\ldots$

$\text{Dom } \vec{F} = (0,\pi)\cup(\pi,2\pi)\cup(2\pi,3\pi)\cup\ldots$

$f_1 = \frac{\ln t}{t} - (\cot t)\,e^{-5t}$

$f_2 = (4-t^2)\cot t - \frac{t^2-1}{t}$; $f_3 = (t^2-1)e^{-5t} - (4-t^2)\ln t$

5. $\vec{F}(t) = (t\,\hat{i} + \hat{j}) \times (\hat{i} - t^2\,\hat{j} + 2\sqrt{t}\,\hat{k})$

$= -t^3\,\hat{i}\times\hat{j} + 2t\sqrt{t}\;\hat{i}\times\hat{k} + \hat{j}\times\hat{i} + 2\sqrt{t}\;\hat{j}\times\hat{k}$

$= -t^3\,\hat{k} + 2t\sqrt{t}\,(-\hat{j}) - \hat{k} + 2\sqrt{t}\,\hat{i}$

$\therefore$

$\vec{F}(t) = 2\sqrt{t}\,\hat{i} - 2t\sqrt{t}\,\hat{j} - (t^3+1)\,\hat{k}$

$\begin{cases} f_1(t) = 2\sqrt{t} \\ \text{Dom } f_1 : \; t \geq 0 \end{cases}$

$\begin{cases} f_2(t) = -2t\sqrt{t} \\ \text{Dom } f_2 : \; t \geq 0 \end{cases}$

$\begin{cases} f_3(t) = t^3+1 \\ \text{Dom } f_3 = \mathbb{R} \end{cases}$

$\text{Dom } \vec{F} = \text{Dom } f_1 \cap \text{Dom } f_2 \cap \text{Dom } f_3$

$= [0,+\infty) \cap [0,+\infty) \cap \mathbb{R}$

$= [0,+\infty)$

$\text{Dom } \vec{F} = [0,+\infty)$

$f_1 = 2\sqrt{t}\;, \quad f_2 = -2t\sqrt{t}\;, \quad f_3 = -(t^3+1)$

6. $\vec{F} - \vec{G}$

$\begin{cases} \vec{F} = 2t\,\hat{i} + t^2\,\hat{j} - \ln t\,\hat{k} \\ \vec{G} = e^{t}\,\hat{i} + e^{-t}\,\hat{j} + 2t\,\hat{k} \end{cases}$

$\vec{F} - \vec{G} = (2t - e^{t})\,\hat{i} + (e^2 - e^{-t})\,\hat{j} + (-\ln t - 2t)\,\hat{k}$

- $f_1(t) = 2t - e^{-t}$

  $\text{Dom } f_1 : \; \mathbb{R}$

- $f_2(t) = e^2 - e^{-t}$

  $\text{Dom } f_2 = \mathbb{R}$

- $f_3(t) = -\ln t - 2t$

  $\text{Dom } f_3 : \; t > 0$

$\text{Dom } (\vec{F} - \vec{G}) = \text{Dom } f_1 \cap \text{Dom } f_2 \cap \text{Dom } f_3$

$= \mathbb{R} \cap \mathbb{R} \cap (0,+\infty)$

$= (0,+\infty)$

$\therefore$

$\boxed{\begin{array}{l} \text{Dom } (\vec{F} - \vec{G}) = (0,+\infty) \\ f_1(t) = 2t - e^{t} \\ f_2(t) = e^2 - e^{-t} \\ f_3(t) = -\ln t - 2t \end{array}}$

7. $\underline{2\vec{F} - 3\vec{G}}$

$\vec{F}(t) = t\,\hat{i} + t^2\,\hat{j} + t^3\,\hat{k}$

$\vec{G}(t) = \cos t\,\hat{i} + \sin t\,\hat{j} + \hat{k}$

$2\vec{F} - 3\vec{G} = 2t\,\hat{i} + 2t^2\,\hat{j} + 2t^3\hat{k} - 3\cos t\,\hat{i} - 3\sin t\,\hat{j} - 3\hat{k}$

$$2\vec{F} - 3\vec{G} = (2t - 3\cos t)\,\hat{i} + (2t^2 - 3\sin t)\,\hat{j} + (2t^3 - 3)\,k$$

- $f_1(t) = 2t - 3\cos t$

  $\underline{\text{Dom } f_1 : \mathbb{R}}$

- $f_2(t) = 2t^2 - 3\sin t$

  $\underline{\text{Dom } f_2 = \mathbb{R}}$

- $f_3(t) = 2t^3 - 3$

  $\underline{\text{Dom } f_3 = \mathbb{R}}$

$\text{Dom } (2\vec{F} - 3\vec{G}) = \text{Dom } f_1 \cap \text{Dom } f_2 \cap \text{Dom } f_3 = \mathbb{R}$

$\therefore$

$$\boxed{\begin{aligned} &\text{Dom } (2\vec{F} - 3\vec{G}) = \mathbb{R} \\ &f_1 = 2t - 3\cos t \\ &f_2 = 2t^2 - 3\sin t \\ &f_3 = 2t^3 - 3 \end{aligned}}$$

8. $\vec{F} \times \vec{G}$

$\vec{F}(t) = t\,\hat{i} + t^2\,\hat{j} + t^3\,\hat{k}$

$\vec{G}(t) = \cos t\,\hat{i} + \sin t\,\hat{j} + \hat{k}$

$$\vec{F} \times \vec{G} = (t\,\hat{i} + t^2\hat{j} + t^3\hat{k}) \times (\cos t\,\hat{i} + \sin t\,\hat{j} + \hat{k})$$

$$= t\sin t\,\hat{i}\times\hat{j} + t\,\hat{i}\times\hat{k} + t^2\cos t\,\hat{j}\times\hat{i} + t^2\,\hat{j}\times\hat{k} + t^3\cos t\,\hat{k}\times\hat{i} + t^3\sin t\,\hat{k}\times\hat{j}$$

$$= t\sin t\,\hat{k} + t(-\hat{j}) + t^2\cos t\,(-\hat{k}) + t^2\,\hat{i} + t^3\cos t\,\hat{j} + t^3\sin t\,(-\hat{i})$$

$$\vec{F} \times \vec{G} = (t^2 - t^3\sin t)\,\hat{i} + (t^3\cos t - t)\,\hat{j} + (t\sin t - t^2\cos t)\,\hat{k}$$

$\therefore$

$$\boxed{\begin{aligned} &\text{Dom } \vec{F}\times\vec{G} = \mathbb{R} \\ &f_1(t) = t^2 - t^3\sin t \\ &f_2(t) = t^3\cos t - t \\ &f_3(t) = t\sin t - t^2\cos t \end{aligned}}$$

9. $\vec{F} \times \vec{G}$

$$\begin{cases} \vec{F}(t) = t\,\hat{j} - \frac{1}{\sqrt{t}}\,\hat{k} \\ \vec{G}(t) = (t - \sin t)\,\hat{i} + (1-\cos t)\,\hat{j} \end{cases}$$

$$\vec{F} \times \vec{G} = t(t-\sin t)\,\underbrace{\hat{j}\times\hat{i}}_{-\hat{k}} + $$
$$+ \frac{-1}{\sqrt{t}}(t-\sin t)\,\underbrace{\hat{k}\times\hat{i}}_{\hat{j}}$$
$$+ \frac{-1}{\sqrt{t}}(1-\cos t)\,\underbrace{\hat{k}\times\hat{j}}_{-\hat{i}}$$

$$= -t(t-\sin t)\,\hat{k}$$
$$- \frac{1}{\sqrt{t}}(t-\sin t)\,\hat{j}$$
$$\frac{1}{\sqrt{t}}(1-\cos t)\,\hat{i}$$

- $f_1(t) = \frac{1}{\sqrt{t}}(1-\cos t)$

Dom f_1 : $(0,+\infty)$

- $f_2(t) = -\frac{1}{\sqrt{t}}(t-\sin t)$

Dom f_2 : $(0,+\infty)$

- $f_3(t) = -t(t-\sin t)$

Dom $f_3 = \mathbb{R}$

$\therefore$

Dom $\vec{F}\times\vec{G}$ = Dom $f_1 \cap$ Dom $f_2 \cap$ Dom f_3
$= (0,+\infty)$

$\therefore$

$$\boxed{\begin{array}{l} \text{Dom } \vec{F}\times\vec{G} = (0,+\infty) \\ f_1(t) = \frac{1}{\sqrt{t}}(1-\cos t) \\ f_2(t) = -\frac{1}{\sqrt{t}}(t-\sin t) \\ f_3(t) = -t(t-\sin t) \end{array}}$$

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10. $f\vec{F}$

$$\begin{cases} \vec{F}(t) = \ln t\,\hat{i} - 4e^{2t}\,\hat{j} + \frac{\sqrt{t-1}}{t}\,\hat{k} \\ f(t) = \sqrt{t} \end{cases}$$

$$f\vec{F} = \sqrt{t}\ln t\,\hat{i} - 4\sqrt{t}\,e^{2t}\,\hat{j} + \frac{\sqrt{t-1}}{\sqrt{t}}\,\hat{k}$$

- $f_1(t) = \sqrt{t}\ln t$

$$\left.\begin{array}{l} \sqrt{t} \to t \geq 0 \\ \ln t \to t > 0 \end{array}\right\} \Rightarrow t > 0$$

$\underline{\text{Dom } f_1 = (0,+\infty)}$

- $f_2(t) = -4\sqrt{t}\,e^{2t}$

$\sqrt{t} \to t \geq 0$; $\underline{\text{Dom } f_2 = [0,+\infty)}$

- $f_3(t) = \frac{\sqrt{t-1}}{\sqrt{t}}$

$$\left.\begin{array}{l} \sqrt{t-1} \to t \geq 1 \\ 1/\sqrt{t} \to t > 0 \end{array}\right\} \Rightarrow [1,+\infty)$$

Dom $f_3 = [1,+\infty)$

0. Cont.

$$\text{Dom}(f\vec{F}) = \text{Dom } f_1 \cap \text{Dom } f_2 \cap \text{Dom } f_3$$
$$= [1, +\infty)$$

$\therefore$

$$\boxed{\begin{aligned} &\text{Dom}(f\vec{F}) = [1, +\infty) \\ &f_1(t) = \sqrt{t}\ \ln t \\ &f_2(t) = -4\sqrt{t}\ e^{2t} \\ &f_3(t) = \frac{\sqrt{t-1}}{\sqrt{t}} \end{aligned}}$$

11. $\vec{F} \circ g$

$$\begin{cases} \vec{F}(t) = \cos t\ \hat{\imath} + \sin t\ \hat{\jmath} + \sqrt{t+2}\ \hat{k} \\ g(t) = t^{1/3} \end{cases}$$

$$(\vec{F} \circ g)(t) = \left[(\cos t) \circ t^{1/3}\right] \hat{\imath} +$$
$$+ \left[(\sin t) \circ t^{1/3}\right] \hat{\jmath}$$
$$+ \left[\left(\sqrt{t+2}\right) \circ t^{1/3}\right] \hat{k}$$

$$= \cos t^{1/3}\ \hat{\imath} + \sin t^{1/3}\ \hat{\jmath}$$
$$+ \sqrt{t^{1/3}+2}\ \hat{k}$$

$f_1(t) = \cos t^{1/3}$

$\text{Dom } f_1 = \mathbb{R}$

$\cdot\ f_2(t) = \sin t^{1/3}$

$\text{Dom } f_2 = \mathbb{R}$

$\cdot\ f_3(t) = \sqrt{t^{1/3}+2}$

$\text{Dom } f_3$: $\quad t^{1/3}+2 \geq 0$

$t^{1/3} \geq -2$

$t \geq -8$

$\text{Dom } f_3 = [-8, +\infty)$

$\therefore$

$$\text{Dom } F = \text{Dom } f_1 \cap \text{Dom } f_2 \cap \text{Dom } f_3$$
$$= \mathbb{R} \cap \mathbb{R} \cap [-8, +\infty)$$
$$= [-8, +\infty)$$

$\therefore$

$$\boxed{\begin{aligned} &\text{Dom } \vec{F} \circ g = [-8, +\infty) \\ &f_1(t) = \cos t^{1/3} \\ &f_2(t) = \sin t^{1/3} \\ &f_3(t) = \sqrt{t^{1/3}+2} \end{aligned}}$$

12. $\vec{F} \circ g$

$$\begin{cases} \vec{F}(t) = e^{-2t}\,\hat{i} + e^{t^2}\,\hat{j} + t^3\,\hat{k} \\ g(t) = \ln t \end{cases}$$

$$(\vec{F} \circ g) = (e^{-2t}) \circ \ln t\;\hat{i} + (e^{t^2}) \circ \ln t\;\hat{j} + (t^3) \circ \ln t\;\hat{k}$$

$$= \underbrace{e^{-2\ln t}}\,\hat{i} + \underbrace{e^{(\ln t)^2}}\,\hat{j} + (\ln t)^3\,\hat{k}$$

$$\left|\; \vec{F} \circ g = \underbrace{t^{-2}}\,\hat{i} + \underbrace{t^{\ln t}}\,\hat{j} + (\ln t)^3\,\hat{k} \right.$$

Obs.:

$$\begin{cases} e^{-2\ln t} = e^{\ln t^{-2}} = t^{-2} \\ e^{(\ln t)^2} = e^{\ln t \,\ln t} = (e^{\ln t})^{\ln t} = t^{\ln t} \end{cases}$$

$\text{Dom}\, \vec{F} \circ g \subset \text{Dom}\, g = (0, +\infty)$

• $f_1(t) = t^{-2}$

$\text{Dom}\, f_1 = \mathbb{R} - \{0\}$

• $f_2(t) = t^{\ln t}$

$\text{Dom}\, f_2$: $\ln t \to t > 0$

$t^{(\;)} \to t > 0$

$\therefore \text{Dom}\, f_2 = (0, +\infty)$

• $f_3(t) = (\ln t)^3$

$\text{Dom}\, f_3$: $\ln t \to t > 0$

$\text{Dom}\, f_3 = (0, +\infty)$

$\text{Dom}\, \vec{F} \circ g = \text{Dom}\, f_1 \cap \text{Dom}\, f_2 \cap \text{Dom}\, f_3$

$= (\mathbb{R} - \{0\}) \cap (0, +\infty) \cap (0, +\infty)$

$= (0, +\infty)$

$\therefore$

$$\boxed{\begin{array}{l} \text{Dom}\, \vec{F} \circ g = (0, +\infty) \\ f_1(t) = \dfrac{1}{t^2} \\ f_2(t) = t^{\ln t} \\ f_3(t) = (\ln t)^3 \end{array}}$$

13. $\lim\limits_{t\to 4} (\hat{\imath} - \hat{\jmath} + \hat{k}) = \hat{\imath} - \hat{\jmath} + \hat{k}$

14. $\lim\limits_{t\to -1} (3\hat{\imath} + t\hat{\jmath} + t^5\hat{k}) =$

$= \lim\limits_{t\to -1} 3\,\hat{\imath} + \lim\limits_{t\to -1} t\,\hat{\jmath} + \lim\limits_{t\to -1} t^5\,\hat{k}$

$= 3\hat{\imath} + (-1)\hat{\jmath} + (-1)\hat{k}$

$= 3\hat{\imath} - \hat{\jmath} - \hat{k}$

15. $\lim\limits_{t\to \pi} (\operatorname{tg} t\,\hat{\imath} + 3t\,\hat{\jmath} - 4\hat{k})$

$= \lim\limits_{t\to\pi} \operatorname{tg} t\,\hat{\imath} + \lim\limits_{t\to\pi} 3t\,\hat{\jmath} -$

$- \lim\limits_{t\to\pi} 4\,\hat{k}$

$= 0\,\hat{\imath} + 3\pi\,\hat{\jmath} - 4\hat{k}$

$= 3\pi\,\hat{\jmath} - 4\hat{k}$

16. $\lim\limits_{t\to 0} \left(\frac{\sin t}{t}\,\hat{\imath} + e^t\hat{\jmath} + (t+\sqrt{2})\,\hat{k}\right)$

$= \lim\limits_{t\to 0} \frac{\sin t}{t}\,\hat{\imath} + \lim\limits_{t\to 0} e^t\,\hat{\jmath} +$

$+ \lim\limits_{t\to 0} t+\sqrt{2}\;\hat{k}$

$= \hat{\imath} + \hat{\jmath} + \sqrt{2}\,\hat{k}$

17. $\vec{F}(t) = \begin{cases} 5\,\hat{\imath} - \sqrt{2t^2+2t+4}\;\hat{\jmath} + e^{-(t-2)}\hat{k}, & t<2 \\ (t^2+1)\,\hat{\imath} + (4-t^3)\,\hat{\jmath} + \hat{k}, & t>2 \end{cases}$

$\lim\limits_{t\to 2^-} \vec{F}(t) = \lim\limits_{t\to 2^-} \left(5\hat{\imath} - \sqrt{2t^2+2t+4}\,\hat{\jmath} + e^{-(t-2)}\hat{k}\right)$

$= 5\,\hat{\imath} - 4\,\hat{\jmath} + \hat{k}$

$\lim\limits_{t\to 2^+} \vec{F}(t) = \lim\limits_{t\to 2^+} \left((t^2+1)\,\hat{\imath} + (4-t^3)\,\hat{\jmath} + \hat{k}\right)$

$= 5\,\hat{\imath} - 4\,\hat{\jmath} + \hat{k}$

$\therefore$

$\lim\limits_{t\to 2^-} \vec{F}(t) = \lim\limits_{t\to 2^+} F(t) = 5\hat{\imath} - 4\hat{\jmath} + \hat{k}$

$\therefore$

$$\boxed{\lim_{t\to 2} \vec{F}(t) = 5\,\hat{\imath} - 4\,\hat{\jmath} + \hat{k}}$$

8. $\vec{F}(t) = \begin{cases} t\,\hat{i} + e^{-1/t^2}\hat{j} + t^2\hat{k} & ; t \neq 0 \\ \hat{j} & , t = 0 \end{cases}$

$$\lim_{t\to 0} \vec{F}(t) = \lim_{t\to 0} (t\,\hat{i} + e^{-1/t^2}\hat{j} + t^2\hat{k})$$

$$= \lim_{t\to 0} \overset{0}{t}\,\hat{i} + \lim_{t\to 0} \overset{0}{e^{-1/t^2}}\,\hat{j}$$

$$+ \lim_{t\to 0} \overset{0}{t^2}\,\hat{k}$$

$$= 0 \;//$$

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19. $\begin{cases} \vec{F}(t) = e^{-1/t^2}\,\hat{i} + \cos t\,\hat{j} + t^3\hat{k} \\ \vec{G}(t) = -\pi\,\hat{i} + \dfrac{1+\cos t}{t}\,\hat{j} \end{cases}$

$$\lim_{t\to 0} (\vec{F}-\vec{G})(t) =$$

$$= \lim_{t\to 0} \left[\left(e^{-1/t^2} + \pi\right)\hat{i} + \left(\cos t - \frac{1+\cos t}{t}\right)\hat{j} + t^3\hat{k} \right]$$

$$= \lim_{t\to 0} \left(e^{-1/t^2} + \pi\right)\hat{i} +$$

$$+ \lim_{t\to 0} \left(\cos t - \frac{1+\cos t}{t}\right)\hat{j}$$

$$+ \lim_{t\to 0} t^3\,\hat{k}$$

Mas

$$\lim_{t\to 0} \left(\underbrace{e^{-1/t^2}}_{0} + \pi\right) = \pi$$

$$\lim_{t\to 0} \left(\underbrace{\cos t}_{1} - \frac{1+\cos t}{t}\right) = \pm\infty$$

$$\lim_{t\to 0} (\vec{F}-\vec{G})(t) \text{ não existe}$$

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20. $\begin{cases} \vec{F}(t) = \dfrac{\sin(t-1)}{t-1}\,\hat{i} + \dfrac{t+3}{t-2}\,\hat{j} + \cos\pi t\,\hat{k} \\ \vec{G}(t) = (t^2+1)\,\hat{i} - \dfrac{t-2}{t+3}\,\hat{j} - \sqrt{t^2+1}\,\hat{k} \end{cases}$

$$\vec{F}\cdot\vec{G}(t) = \frac{\sin(t-1)}{t-1}(t^2+1) + \cancel{-\frac{t+3}{t-2}\,\frac{t-2}{t+3}} - \cos\pi t\,\sqrt{t^2+1}$$

$$\vec{F}\cdot\vec{G}(t) = \frac{\sin(t-1)}{t-1}(t^2+1) - 1 - \cos\pi t\,\sqrt{t^2+1}$$

$$\therefore \lim_{t\to 1} \vec{F}\cdot\vec{G} = \lim_{t\to 1} \left(\overbrace{\frac{\sin(t-1)}{t-1}}^{1}\,\overbrace{(t^2+1)}^{2} - 1 - \underbrace{\cos\pi t}_{-1}\,\sqrt{t^2+1} \right)$$

$$= 1\cdot 2 - 1 - (-1)\sqrt{2}$$

$$= 2 - 1 + \sqrt{2}$$

$$= 1 + \sqrt{2} \;//$$

21. $$\lim_{t\to 3}\left(\frac{t^2-5t+6}{t-3}\,\hat{i} + \frac{t^2-2t-3}{t-3}\,\hat{j} + \frac{t^2+4t-21}{t-3}\,\hat{k}\right)$$

$$= \lim_{t\to 3}\frac{t^2-5t+6}{t-3}\,\hat{i} + \lim_{t\to 3}\frac{t^2-2t-3}{t-3}\,\hat{j} + \lim_{t\to 3}\frac{t^2+4t-21}{t-3}\,\hat{k}$$

Mas

$$\lim_{t\to 3}\frac{t^2-5t+6}{t-3} = \lim_{t\to 3}\frac{\cancel{(t-3)}(t-2)}{\cancel{(t-3)}} = \lim_{t\to 3} t-2 = 1$$

$$\lim_{t\to 3}\frac{t^2-2t-3}{t-3} = \lim_{t\to 3}\frac{\cancel{(t-3)}(t+1)}{\cancel{(t-3)}} = \lim_{t\to 3}(t+1) = 4$$

$$\lim_{t\to 3}\frac{t^2+4t-21}{t-3} = \lim_{t\to 3}\frac{\cancel{(t-3)}(t+7)}{\cancel{t-3}} = \lim_{t\to 3}(t+7)$$

$\therefore$

$$\lim_{t\to 3}\left(\frac{t^2-5t+6}{t-3}\,\hat{i} + \frac{t^2-2t-3}{t-3}\,\hat{j} + \frac{t^2+4t-21}{t-3}\,\hat{k}\right) = \hat{i} + 4\hat{j} + 10\hat{k}$$

22. $$\lim_{t\to 1}\left(\frac{t^2+1}{t-1}\,\hat{i} + \frac{t^2-1}{t+1}\,\hat{j} + \frac{t^2+7t-8}{t-1}\,\hat{k}\right)$$

$$= \lim_{t\to 1}\frac{t^2+1}{t-1}\,\hat{i} + \lim_{t\to 1}\frac{t^2-1}{t+1}\,\hat{j} + \lim_{t\to 1}\frac{t^2+7t-8}{t-1}\,\hat{k}$$

Mas

$$\lim_{t\to 1}\frac{t^2+1}{t-1} = \pm\infty$$

Logo

$$\boxed{\lim_{t\to 1}\left(\frac{t^2+1}{t-1}\,\hat{i} + \frac{t^2-1}{t+1}\,\hat{j} + \frac{t^2+7t-8}{t-1}\,\hat{k}\right)}$$

não existe

23.

$$\lim_{t\to 1}\left(\frac{\ln t}{t-1}\,\hat{\imath} - \frac{e^t - e}{t-1}\,\hat{\jmath} + \frac{t+1}{t^2}\,\hat{k}\right)$$ [Silverman]

$$= \lim_{t\to 1}\frac{\ln t}{t-1}\,\hat{\imath} - \lim_{t\to 1}\frac{e^t-e}{t-1}\,\hat{\jmath} + \lim_{t\to 1}\frac{t+1}{t^2}\,\hat{k}$$

Mas

$$\lim_{t\to 1}\frac{\ln t}{t-1} \overset{\text{L'Hop.}}{=} \lim_{t\to 1}\frac{\frac{1}{t}}{1} = \lim_{t\to 1}\frac{1}{t} = 1$$

$$\lim_{t\to 1}\frac{e^t-e}{t-1} \overset{\text{L'Hop.}}{=} \lim_{t\to 1}\frac{e^t}{1} = e$$

$$\lim_{t\to 1}\frac{t+1}{t^2} = 2$$

$\therefore$

$$\lim_{t\to 1}\left(\frac{\ln t}{t-1}\,\hat{\imath} - \frac{e^t-e}{t-1}\,\hat{\jmath} + \frac{t+1}{t^2}\,\hat{k}\right) = \hat{\imath} - e\,\hat{\jmath} + 2\,\hat{k}$$

24.

$$\lim_{t\to 0}\left(\frac{\sqrt{1+t}-1}{t}\,\hat{\imath} + \frac{t-1}{t+1}\,\hat{\jmath} - \frac{\sinh t}{t}\,\hat{k}\right)$$

$$= \lim_{t\to 0}\frac{\sqrt{1+t}-1}{t}\,\hat{\imath} + \lim_{t\to 0}\frac{t-1}{t+1}\,\hat{\jmath} - \lim_{t\to 0}\frac{\sinh t}{t}\,\hat{k}$$

Mas

$$\lim_{t\to 0}\frac{\sqrt{1+t}-1}{t} \overset{\text{L'Hop.}}{=} \lim_{t\to 0}\frac{\frac{1}{2\sqrt{1+t}}}{1} = \frac{1}{2}$$

$$\lim_{t\to 0}\frac{t-1}{t+1} = -1$$

$$\lim_{t\to 0}\frac{\sinh t}{t} = \lim_{t\to 0}\frac{e^t-e^{-t}}{2t} \overset{\text{L'Hop.}}{=} \lim_{t\to 0}\frac{e^t+e^{-t}}{2} = 1$$

$\therefore$

$$\lim_{t\to 0}\left(\frac{\sqrt{1+t}-1}{t}\,\hat{\imath} + \frac{t-1}{t+1}\,\hat{\jmath} - \frac{\sinh t}{t}\,\hat{k}\right) = \frac{1}{2}\,\hat{\imath} - \hat{\jmath} - \hat{k}$$

25.

$$\lim_{t\to 0}\left(\frac{\sin t}{t}\hat{\imath}+\frac{1-\cos t}{t}\hat{\jmath}+e^{2t}\hat{k}\right)$$

$$=\lim_{t\to 0}\frac{\sin t}{t}\hat{\imath}+\lim_{t\to 0}\cancel{\frac{1-\cos t}{t}}^{\,0}\hat{\jmath}+$$

$$+\lim_{t\to 0}e^{2t}\hat{k}$$

$$=1\hat{\imath}+\hat{k}$$

26.

$$\lim_{t\to 0}\left(\frac{\sin 3t}{t}\hat{\imath}+\frac{\operatorname{tg}2t}{3t}\hat{\jmath}+\right.$$
$$\left.+\ln(1+t)\hat{k}\right)$$

$$=\lim_{t\to 0}\frac{\sin 3t}{t}\hat{\imath}+\lim_{t\to 0}\frac{\operatorname{tg}2t}{3t}\hat{\jmath}+$$

$$+\lim_{t\to 0}\ln(1+t)\hat{k}$$

Mas

$$\lim_{t\to 0}\frac{\sin 3t}{t}\overset{\text{L'Hop.}}{=}\lim_{t\to 0}\frac{3\cos 3t}{1}$$

$$=3$$

$$\lim_{t\to 0}\frac{\operatorname{tg}2t}{3t}\overset{\text{L'Hop.}}{=}\lim_{t\to 0}\frac{2\sec^2 2t}{3}$$

$$=\frac{2}{3}$$

$$\lim_{t\to 0}\ln(1+t)=0$$

$\therefore$

$$\lim_{t\to 0}\left(\frac{\sin 3t}{t}\hat{\imath}+\frac{\operatorname{tg}2t}{3t}\hat{\jmath}+\ln(1+t)\hat{k}\right)=$$

$$=3\hat{\imath}+\frac{2}{3}\hat{\jmath}$$

27.

$$\lim_{t\to 3}\left\{\left(\frac{9-t^2}{3-t}\right)\hat{\imath}+\left(\frac{t^2+t-12}{t-3}\right)\hat{\jmath}+\right.$$
$$\left.+\frac{(t^3-13t-12)}{t-3}\hat{k}\right\}=(\star)$$

$$\lim_{t\to 3}\frac{9-t^2}{3-t}=\lim_{t\to 3}3+t=6$$

$$\lim_{t\to 3}\frac{t^2+t-12}{t-3}=\lim_{t\to 3}\frac{\cancel{(t-3)}(t+4)}{\cancel{(t-3)}}$$

$$=7$$

$$\lim_{t\to 3}\frac{t^3-13t+12}{t-3}\overset{\text{L'Hop.}}{=}\lim_{t\to 3}\frac{3t^2-13}{1}$$

$$=27-13=\underline{14}$$

$\therefore$

$$(\star)=6\hat{\imath}+7\hat{\jmath}+14\hat{k}$$

28.

$$\lim_{t\to\infty}\left(\frac{1}{t}\,\hat{\imath}+\frac{t-1}{t+1}\,\hat{\jmath}+\frac{\sin t^3}{t^2}\,\hat{k}\right) = (\star)$$

$$=\overset{0}{\cancel{\lim_{t\to\infty}\frac{1}{t}}}\,\hat{\imath}+\lim_{t\to\infty}\frac{t-1}{t+1}\,\hat{\jmath}+$$

$$+\lim_{t\to\infty}\frac{\sin t^3}{t^2}\,\hat{k}$$

Mas

$$\begin{cases}\lim\limits_{t\to\infty}\dfrac{t-1}{t+1} = 1\\[2ex] \lim\limits_{t\to\infty}\dfrac{\sin t^3}{t^2} = 0\end{cases}$$

$\therefore$

$$\boxed{\lim_{t\to\infty}\left(\frac{1}{t}\,\hat{\imath}+\frac{t-1}{t+1}\,\hat{\jmath}+\frac{\sin t^3}{t^2}\,\hat{k}\right) = \hat{\jmath}}$$

29.

$$\lim_{t\to 0^+}\left(t\,\hat{\imath}+2t^{1/4}\,\hat{\jmath}-\frac{\ln t}{t}\,\hat{k}\right)$$

$$=\lim_{t\to 0^+} t\,\hat{\imath}+2\lim_{t\to 0^+} t^{1/4}\,\hat{\jmath}$$

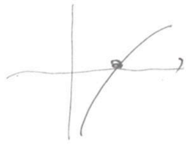
$$-\lim_{t\to 0^+}\frac{\ln t}{t}\,\hat{k}$$

Mas

$$\lim_{t\to 0^+} t = 0$$

$$\lim_{t\to 0^+} t^{1/4} = 0$$

$$\lim_{t\to 0^+}\frac{\ln t}{t} = \frac{-\infty}{0^+} = -\infty$$



O limite não existe

30.

$$\lim_{t\to 1^+}\left(e^{\frac{1}{1-t}}\,\hat{\imath}+\sqrt{t-1}\,\hat{\jmath}+\ln t\,\hat{k}\right)$$

$$\lim_{t\to 1^+} e^{\frac{1}{1-t}} = e^{\frac{1}{0^-}} = e^{-\infty} = 0$$

$$\lim_{t\to 1^+}\sqrt{t-1} = 0$$

$$\lim_{t\to 1^+}\ln t = 0$$

$\therefore$

$$\lim_{t\to 1^+}\left(e^{\frac{1}{1-t}}\,\hat{\imath}+\sqrt{t-1}\,\hat{\jmath}+\ln t\,\hat{k}\right) = 0$$

31.

$$\lim_{t\to 1^-}\left(\sqrt{1-t}\,\hat{\imath} - (1-t)\ln(1-t)\,\hat{\jmath}\right) =$$

Mas

$$\lim_{t\to 1^-}\sqrt{1-t} = 0$$

$$\lim_{t\to 1^-}\underset{\downarrow\\0}{(1-t)}\,\underbrace{\ln(1-t)} = 0$$

$\therefore$

$$\lim_{t\to 1^-}\left(\sqrt{1-t}\,\hat{\imath} - (1-t)\ln(1-t)\,\hat{\jmath}\right)$$

$$= 0$$

32.

$$\vec{F}(t) = 4t\,\hat{\jmath} - \sqrt{5}\,\hat{k}$$

$$\lim_{t\to t_0}\vec{F}(t) = 4t_0\,\hat{\jmath} - \sqrt{5}\,\hat{k} = \vec{F}(t_0)$$

$\forall\, t_0 \in \mathbb{R}$.

$\therefore$ $\boxed{\vec{F}(t) \text{ é contínua } \forall t \in \mathbb{R}}$

34.

$$\vec{F}(t) = \begin{cases} (2t+1)\,\hat{\imath} + (2t-1)\,\hat{\jmath} + 4t\,\hat{k} & (t < -3) \\ (t-2)\,\hat{\imath} - (t+10)\,\hat{\jmath} + (2t-9)\,\hat{k} & (t \geq -3) \end{cases}$$

Se $t < -3$ tem-se

$$\vec{F}(t) = f_1(t)\,\hat{\imath} + f_2(t)\,\hat{\jmath} + f_3(t)\,\hat{k}$$

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$$\left.\begin{aligned} f_1(t) &= 2t+1 \\ f_2(t) &= 2t-1 \\ f_3(t) &= 4t \end{aligned}\right\} \text{ são contínuas para } t < -3$$

Logo

$\vec{F}(t)$ é contínua $\forall\, t < -3$.

Analogamente,

Se $t > -3$:

$$\vec{F}(t) = (t-2)\,\hat{\imath} - (t+10)\,\hat{\jmath} + (2t-9)\,\hat{k}$$

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$$\left.\begin{aligned} f_1(t) &= (t-2) \\ f_2(t) &= -(t+10) \\ f_3(t) &= 2t-9 \end{aligned}\right\} \text{ são contínuas para } t > -3$$

Logo

$\vec{F}(t)$ é contínua $\forall\, t > -3$

34. (Cont.)

$t = -3$

$$\lim_{t\to -3^-} \vec{F}(t) = \lim_{t\to -3^-} \left[(2t+1)\hat{i} + (2t-1)\hat{j} + 4t\,\hat{k}\right]$$

$$= -5\hat{i} - 7\hat{j} - 12\hat{k}$$

$$\lim_{t\to -3^+} \vec{F}(t) = \lim_{t\to -3^+} \left[(t-2)\hat{i} - (t+10)\hat{j} + (2t-9)\hat{k}\right]$$

$$= -5\hat{i} - 7\hat{j} - 15\hat{k}$$

Vemos que

$$\lim_{t\to -3^-} \vec{F}(t) = -5\hat{i} - 7\hat{j} - 12\hat{k} \neq$$
$$\neq -5\hat{i} - 7\hat{j} - 15\hat{k} = \lim_{t\to -3^+} \vec{F}(t)$$

Logo

$$\boxed{\nexists \lim_{t\to -3} \vec{F}(t)}$$

$\therefore$ $\boxed{\vec{F}(t) \text{ é contínua em } \mathbb{R} - \{-3\}}$

33.

$$\vec{F}(t) = \begin{cases} \dfrac{\sin t}{t}\hat{i} + \hat{j} + \hat{k}, & t < 0 \\ (t^2+1)\vec{i} + \ln(t+3)\hat{j} + \hat{k}, & t \geq 0 \end{cases}$$

Se $t < 0$:

$\vec{F}(t) = \dfrac{\sin t}{t}\hat{i} + \hat{j} + \hat{k}$ é contínua

Se $t > 0$

$\vec{F}(t) = (t^2+1)\vec{i} + \ln(t+3)\hat{j} + \hat{k}$ é contínua

Se $t = 0$:

$$\lim_{t\to 0^-} \vec{F}(t) = \lim_{t\to 0^-}\left(\frac{\sin t}{t}\hat{i} + \hat{j} + \hat{k}\right)$$
$$= \hat{i} + \hat{j} + \hat{k}$$

$$\lim_{t\to 0^+} \vec{F}(t) = \lim_{t\to 0^+}\left((t^2+1)\hat{i} + \ln(t+3)\hat{j} + \hat{k}\right)$$
$$= \hat{i} + \ln 3\,\hat{j} + \hat{k}$$

$$\therefore \lim_{t\to 0^-} \vec{F}(t) \neq \lim_{t\to 0^+} \vec{F}(t)$$

$\therefore$ $\vec{F}(t)$ não é contínua em $t = 0$

$\therefore$ $\boxed{\vec{F}(t) \text{ é contínua em } \mathbb{R} - \{0\}}$

35.

$$\vec{F}(t) = \begin{cases} t\,\hat{i} + (3t-2)\,\hat{j} + (3-t)\,\hat{k} & (-\infty < t < 2) \\ 2\,\hat{i} + (2+t)\,\hat{j} + t\,\hat{k}; & 2 \leq t < \infty \end{cases}$$

$\vec{F}(t)$ é contínua em $\mathbb{R} - \{2\}$

$\underline{t=2}$

$$\lim_{t\to 2^-} \vec{F}(t) = \lim_{t\to 2^-} \left(t\,\hat{i} + (3t-2)\,\hat{j} + (3-t)\,\hat{k} \right) = 2\hat{i} + 4\hat{j} + \hat{k}$$

$$\lim_{t\to 2^+} \vec{F}(t) = \lim_{t\to 2^+} \left(2\,\hat{i} + (2+t)\,\hat{j} + t\,\hat{k} \right) = 2\hat{i} + 4\hat{j} + 2\hat{k}$$

$$\lim_{t\to 2} \vec{F}(t) \not\exists$$

$\therefore$

$\boxed{\vec{F}(t) \text{ é contínua em } \mathbb{R} - \{2\}}$

36.

$$\vec{F}(t) = \begin{cases} t^2\,\hat{i} + (3-t)\,\hat{j} + t\,\hat{k}, & -\infty < t < 1 \\ t\,\hat{i} + 2t\,\hat{j} + (2-t^2)\,\hat{k}; & 1 \leq t < \infty \end{cases}$$

Por inspeção vemos que

$\vec{F}(t)$ é contínua em $\mathbb{R} - \{1\}$.

$\underline{t=1}$

$$\lim_{t\to 1^-} \vec{F}(t) = \lim_{t\to 1^-} \left(t^2\hat{i} + (3-t)\hat{j} + t\hat{k} \right) = \hat{i} + 2\hat{j} + 1\hat{k}$$

$$\lim_{t\to 1^+} \vec{F}(t) = \lim_{t\to 1^+} \left(t\,\hat{i} + 2t\,\hat{j} + (2-t^2)\,\hat{k} \right) = \hat{i} + 2\hat{j} + \hat{k}$$

$\therefore$

$$\lim_{t\to 1^-} \vec{F}(t) = \lim_{t\to 1^+} \vec{F}(t)$$

$\therefore$

$$\lim_{t\to 1} \vec{F}(t) \text{ existe} = \vec{F}(1)$$

$\therefore$

$\boxed{\vec{F}(t) \text{ é contínua } \forall\, t \in \mathbb{R}}$

37.

$$\frac{d\vec{F}}{dt} = \frac{d}{dt}t\,\hat{i} + \frac{d}{dt}\sqrt{t}\,\hat{j}$$

$$= \hat{i} + \frac{1}{2\sqrt{t}}\hat{j}$$

38.

$$\vec{F}(t) = \sqrt{t}\,\hat{i} + t^{-3/2}\hat{j} + \ln(2t-1)\,\hat{k}$$

$$\vec{F}'(t) = \frac{1}{2\sqrt{t}}\hat{i} - \frac{3}{2}t^{-5/2}\hat{j} + \frac{2}{2t-1}\hat{k}$$

39.

$$\vec{F}(t) = \arcsin t\,\hat{i} + \sqrt{1+t^2}\,\hat{j} + e^{-t^3}\hat{k}$$

$$\vec{F}'(t) = \frac{1}{\sqrt{1-t^2}}\hat{i} + \frac{t}{\sqrt{1+t^2}}\hat{j} + -3t^2 e^{-t^3}\hat{k}$$

10.

$$\vec{F}(t) = e^{\sqrt{t}}\hat{i} + 3\hat{j} - \arccos 2t\,\hat{k}$$

$$\vec{F}'(t) = \frac{1}{2\sqrt{t}}e^{\sqrt{t}}\hat{i} - \left(-\frac{2}{\sqrt{1-4t^2}}\hat{k}\right)$$

$$\vec{F}'(t) = \frac{e^{\sqrt{t}}}{\ldots}\hat{i} + \frac{2}{\ldots}\hat{k}$$

41.

$$\vec{F}(t) = \operatorname{tg} t\,\hat{i} + \hat{j} + \sec t\,\hat{k}$$

$$\vec{F}'(t) = \sec^2 t\,\hat{i} + \sec t\operatorname{tg} t\,\hat{k}$$

42.

$$\vec{F} = e^t\cos t\,\hat{i} - e^t\sin t\,\hat{k}$$

$$\vec{F}'(t) = (e^t\cos t - e^t\sin t)\,\hat{i} + (-e^t\sin t - e^t\cos t)\,\hat{k}$$

43.

$$\vec{F}(t) = \cosh t\,\hat{i} + \sinh t\,\hat{j} - \sqrt{t}\,\hat{k}$$

$$\begin{cases} \cosh t = \dfrac{e^t + e^{-t}}{2} \\ \dfrac{d}{dt}\cosh t = \dfrac{e^t - e^{-t}}{2} = \sinh t \end{cases}$$

$$\begin{cases} \sinh t = \dfrac{e^t - e^{-t}}{2} \\ \dfrac{d}{dt}\sinh t = \dfrac{e^t + e^{-t}}{2} = \cosh t \end{cases}$$

$$\vec{F}'(t) = \sinh t\,\hat{i} + \cosh t\,\hat{j} - \frac{1}{2\sqrt{t}}\hat{k}$$

44.

$$\vec{u}(t) = t\hat{i} - t^2\hat{j} + 2t\hat{k}$$

$$\frac{d\vec{u}}{dt} = \hat{i} - 2t\hat{j} + 2\hat{k}$$

---

45. $\frac{d}{dt}\left(f(t)\,\vec{v}(t)\right) =$

$$= \frac{d}{dt}\left((t^2+3)(\hat{i} - 2t\hat{j} + 3t^2\hat{k})\right)$$

$$= \frac{d}{dt}\left((t^2+3)\hat{i} - 2t(t^2+3)\hat{j} + 3t^2(t^2+3)\hat{k}\right)$$

$$= 2t\hat{i} + \left(-2(t^2+3) - 4t^2\right)\hat{j} + \left(6t(t^2+3) + 6t^3\right)\hat{k}$$

$$= 2t\hat{i} + (-6t^2-6)\hat{j} + (12t^3+18t)\hat{k}$$

---

46. $\frac{d}{dt}\left(g(t)\,\vec{u}(t)\right) =$

$$= \frac{d}{dt}\left((2t^3-3t)(t\hat{i} - t^2\hat{j} + 2t\hat{k})\right)$$

$$= \frac{d}{dt}\left((2t^4-3t^2)\hat{i} + (-2t^5+3t^3)\hat{j} + (4t^4-6t^2)\hat{k}\right)$$

$$= (8t^3-6t)\hat{i} + (-10t^4+9t^2)\hat{j} + (16t^3-12t)\hat{k}$$

---

47. $\frac{d}{dt}(\vec{u}\times\vec{v})$

$$\vec{u}\times\vec{v} = (t\hat{i} - t^2\hat{j} + 2t\hat{k}) \times (\hat{i} - 2t\hat{j} + 3t^2\hat{k})$$

$$= t(-2t)\,\hat{i}\times\hat{j} + t(3t^2)\,\hat{i}\times\hat{k} - t^2\,\hat{j}\times\hat{i} - t^2 3t^2\,\hat{j}\times\hat{k} + 2t\,\hat{k}\times\hat{i} - 4t^2\,\hat{k}\times\hat{j}$$

$$= -2t^2\hat{k} + 3t^3(-\hat{j}) - t^2(-\hat{k}) - 3t^4\hat{i} + 2t\hat{j} - 4t^2(-\hat{i})$$

$$= (4t^2-3t^4)\hat{i} + (2t-3t^3)\hat{j} + (t^2-2t^2)\hat{k}$$

$$\vec{u}\times\vec{v} = (4t^2-3t^4)\hat{i} + (2t-3t^3)\hat{j} - t^2\hat{k}$$

47. Cont.

$$\frac{d}{dt}\vec{u}\times\vec{v} = (8t-12t^3)\,\hat{i} + \\ +(2-9t^2)\,\hat{j} \\ -2t\,\hat{k}$$

48.

$$\frac{d}{dt}(\vec{u}\times t\vec{v})$$

Aqui,

$$\vec{u}\times(t\vec{v}) =$$

$$= t\,\vec{u}\times\vec{v}$$

$$= t(t\hat{i}-t^2\hat{j}+2t\hat{k})\times \\ \times(\hat{i}-2t\hat{j}+3t^2\hat{k})$$

$$= t\Big(-2t^2\hat{k} + 3t^3(-\hat{j}) \\ -t^2(-\hat{k}) - 3t^4\hat{i} \\ + 2t\hat{j} - 4t^2(-\hat{i})\Big)$$

$$= t\Big((4t^2-3t^4)\hat{i} + (2t-3t^3)\hat{j} \\ + (\underbrace{t^2-2t^2}_{-t^2})\hat{k}\Big)$$

$$= (4t^3-3t^5)\hat{i} + (2t^2-3t^4)\hat{j} - t^3\hat{k}$$

$\therefore$

$$\frac{d}{dt}(t\,\vec{u}\times\vec{v}) =$$

$$= \frac{d}{dt}\Big((4t^3-3t^5)\hat{i} + \\ +(2t^2-3t^4)\hat{j} \\ -t^3\hat{k}\Big)$$

$$= (12t^2-15t^4)\hat{i} + \\ +(4t-12t^3)\hat{j} \\ -3t^2\hat{k}$$

Uma vez que

$$\vec{u}\times(t\vec{v}) = (t\,\vec{u})\times\vec{v} = t(\vec{u}\times\vec{v})$$

temos que...

$$\begin{cases} \frac{d}{dt}(t\vec{u}\times\vec{v}) = \frac{d}{dt}\big(t(\vec{u}\times\vec{v})\big) \\ = (12t^2-15t^4)\hat{i} + (4t-12t^3)\hat{j} \\ \quad -3t^2\hat{k} \end{cases}$$

49. $\frac{d}{dt}(2\vec{u}\cdot\vec{v})$

$$2\vec{u}\cdot\vec{v} = 2(t, -t^2, 2t)\cdot(1, -2t, 3t^2)$$

$$= 2(t + 2t^3 + 6t^3)$$

$$= 2(t + 8t^3)$$

$$= 2t + 16t^3$$

$$\therefore \frac{d}{dt}(2\vec{u}\cdot\vec{v}) = 2 + 48t^2 \;//$$

50. $\frac{d}{dt}(3\vec{u} + 4\vec{v})$

$$3\vec{u} + 4\vec{v} = 3(t\hat{i} - t^2\hat{j} + 2t\hat{k}) + 4(\hat{i} - 2t\hat{j} + 3t^2\hat{k})$$

$$= (3t+4)\hat{i} + (-3t^2 - 8t)\hat{j} + (6t + 12t^2)\hat{k}$$

$$\therefore \frac{d}{dt}(3\vec{u} + 4\vec{v}) =$$

$$= 3\hat{i} + (-6t - 8)\hat{j} + (6 + 24t)\hat{k} \;//$$

51. $\frac{d}{dt}\left(f(t)\vec{u} + g(t)\vec{v}\right)$

$$f(t)\vec{u} + g(t)\vec{v} =$$

$$= (t^2+3)(t\hat{i} - t^2\hat{j} + 2t\hat{k}) + (2t^3 - 3t)(\hat{i} - 2t\hat{j} + 3t^2\hat{k})$$

$$= t(t^2+3)\hat{i} - t^2(t^2+3)\hat{j} + 2t(t^2+3)\hat{k} + (2t^3 - 3t)\hat{i} - 2t(2t^3 - 3t)\hat{j} + 3t^2(2t^3 - 3t)\hat{k}$$

$$= (t^3 + \cancel{3t} + 2t^3 - \cancel{3t})\hat{i} + (-t^4 - 3t^2 - 4t^4 + 6t^2)\hat{j} + (2t^3 + 6t + 6t^5 - 9t^3)\hat{k}$$

$$= 3t^3\hat{i} + (-5t^4 + 3t^2)\hat{j} + (6t - 7t^3 + 6t^5)\hat{k}$$

$$\therefore \boxed{\frac{d}{dt}\left(f(t)\vec{u} + g(t)\vec{v}\right) = 9t^2\hat{i} + (-20t^3 + 6t)\hat{j} + (6 - 21t^2 + 30t^4)\hat{k}}$$

52.

$$\vec{u} \times \frac{d\vec{v}}{dt} - f(t)\, \vec{u} \cdot \frac{d\vec{v}}{dt}\, \vec{v}$$

$$\frac{d\vec{v}}{dt} = \frac{d}{dt}\left(\hat{i} - 2t\,\hat{j} + 3t^2\,\hat{k}\right)$$

$$= -2\,\hat{j} + 6t\,\hat{k}$$

$$\vec{u} \times \frac{d\vec{v}}{dt} = \left(t\hat{i} - t^2\hat{j} + 2t\hat{k}\right) \times$$
$$\times \left(-2\hat{j} + 6t\,\hat{k}\right)$$

$$= -2t\,\hat{k} + 6t^2(-\hat{j})$$
$$-6t^3\,\hat{i} - 4t(-\hat{i})$$

$$= (-6t^3 + 4t)\,\hat{i} - 6t^2\,\hat{j} - 2t\,\hat{k}$$

$$\vec{u} \cdot \frac{d\vec{v}}{dt} = \left(t\hat{i} - t^2\hat{j} + 2t\hat{k}\right) \cdot$$
$$\cdot \left(-2\hat{j} + 6t\,\hat{k}\right)$$

$$= 2t^2 + 12t^2$$

$$= 14t^2$$

$$f(t)\, \vec{u} \cdot \frac{d\vec{v}}{dt}\, \vec{v} =$$

$$= (t^2+3)\; 14t^2\left(\hat{i} - 2t\,\hat{j} + 3t^2\,\hat{k}\right)$$

$$= \left(14t^4 + 42t^2\right)\hat{i} - \left(14t^4 + 42t^2\right)2t\,\hat{j}$$

$\therefore$

$$\vec{u} \times \frac{d\vec{v}}{dt} - f(t)\, \vec{u} \cdot \frac{d\vec{v}}{dt}\, \vec{v} =$$

$$= (-6t^3 + 4t)\,\hat{i} - 6t^2\,\hat{j} - 2t\,\hat{k}$$
$$- (14t^4 + 42t^2)\,\hat{i} + (14t^4 + 42t^2)2t\,\hat{j}$$
$$- (14t^4 + 42t^2)\,3t^2\,\hat{k}$$

$$= \left(-6t^3 + 4t - 14t^4 - 42t^2\right)\hat{i}$$
$$+ \left(-6t^2 + 28t^5 + 84t^3\right)\hat{j}$$
$$\left(-2t - 42t^6 - 126t^4\right)\hat{k}$$

54. $\begin{cases} \vec{u}(t) : \text{diferenciável} \\ |\vec{u}(t)| = cte \end{cases}$

Seja $\dfrac{d\vec{u}}{dt} \neq 0$.

Seja $\vec{u} = (u_1, u_2, u_3)$

então

$$|\vec{u}| = \sqrt{u_1^2 + u_2^2 + u_3^2} = cte$$

$$\therefore \quad u_1^2(t) + u_2^2(t) + u_3^2(t) = cte^2$$

Derivando essa expressão obtém-se

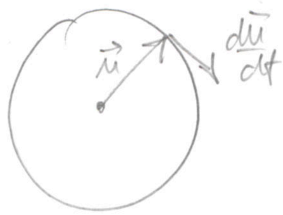
$$2u_1(t)\frac{du_1(t)}{dt} + 2u_2(t)\frac{du_2(t)}{dt} + 2u_3(t)\frac{du_3(t)}{dt} = 0$$

$$2\left(u_1(t), u_2(t), u_3(t)\right) \cdot \left(\frac{du_1(t)}{dt}, \frac{du_2(t)}{dt}, \frac{du_3(t)}{dt}\right) = 0$$

$$\therefore \quad \boxed{\vec{u} \cdot \frac{d\vec{u}}{dt} = 0}$$

Um movimento com $|\vec{u}(t)| = cte$ é um movimento circular no caso de $\vec{u}(t)$ ser a posição de uma partícula. Daí

$$\vec{u} \cdot \frac{d\vec{u}}{dt} = 0$$



nos diz que num movimento circular a velocidade é perpendicular ao raio vetor.

55. $\left.\begin{array}{l}\vec{v} = \vec{v}(s) \\ s = s(t)\end{array}\right\}$ diferenciável

Seja

$$\vec{v} = v_1 \hat{i} + v_2 \hat{j} + v_3 \hat{k}$$

$$\begin{array}{l} v = v(s) \\ s = s(t) \end{array} \Rightarrow v = v(s(t))$$

$$\frac{d\vec{v}}{dt} = \frac{dv_1}{dt}(s(t))\,\hat{i} + \frac{dv_2}{dt}(s(t))\,\hat{j} + \frac{dv_3}{dt}(s(t))\,\hat{k}$$

$$= \frac{dv_1}{ds}\frac{ds}{dt}\hat{i} + \frac{dv_2}{ds}\frac{ds}{dt}\hat{j} + \frac{dv_3}{ds}\frac{ds}{dt}\hat{k}$$

$$= \underline{\left(\frac{dv_1}{ds}\hat{i} + \frac{dv_2}{ds}\hat{j} + \frac{dv_3}{ds}\hat{k}\right)} \cdot \frac{ds}{dt}$$

$$\boxed{\frac{d\vec{v}(s(t))}{dt} = \frac{d\vec{v}(s)}{ds}\,\frac{ds(t)}{dt}}$$

56.

Seja

$$\vec{F}(t) = F_1(t)\,\hat{i} + F_2(t)\,\hat{j} + F_3(t)\,\hat{k}$$

As condições

$\vec{F}$ contínua em $[a,b]$
diferenciável em (a,b)

Valem também para as funções componentes F_1, F_2, F_3.

Daí temos do teorema do valor médio aplicado a cada função componente que

$\exists\, \underline{c_1}$, $a < c_1 < b$ tq.

$$F_1'(c_1) = \frac{F_1(b) - F_1(a)}{b-a}$$

$\exists\, \underline{c_2}$, $a < c_2 < b$

$$F_2'(c_2) = \frac{F_2(b) - F_2(a)}{b-a}$$

$\exists\, \underline{c_3}$, $a < c_3 < b$

$$F_3'(c_3) = \frac{F_3(b) - F_3(a)}{\ldots}$$

No entanto não há nada que garante que $c_1 = c_2 = c_3$, que seria a condição que garantiria que

$$\vec{F}'(c) = \frac{\vec{F}(b) - \vec{F}(a)}{b-a}.$$

$\therefore$

<u>O resultado não vale</u>

58.

$$\vec{F}(t) = \sin t \, \hat{i} - \cos t \, \hat{j}$$

$$\vec{F}'(t) = \cos t \, \hat{i} + \sin t \, \hat{j}$$

$$\vec{F}''(t) = -\sin t \, \hat{i} + \cos t \, \hat{j}$$

$$= -\vec{F}(t)$$

$$\therefore \boxed{\vec{F}''(t) \parallel \vec{F}(t), \forall t}$$

59.

$$\frac{d}{dt}(\vec{F} \times \vec{F}') =$$

$$= \cancel{\vec{F}' \times \vec{F}'}^{\,0} + \vec{F} \times \vec{F}''$$

$$= \underline{\vec{F} \times \vec{F}''}$$

---

60. $\vec{F}(t) \parallel \vec{F}''(t), \forall t$

Seja $\vec{F}(t) \times \vec{F}'(t)$

Então

$$\frac{d}{dt}(\vec{F}(t) \times \vec{F}'(t)) =$$

$$= \cancel{\vec{F}'(t) \times \vec{F}'(t)}^{\,0} + \vec{F}(t) \times \vec{F}''(t)$$

$$= \underline{\vec{F}(t) \times \vec{F}''(t)}$$

$$= 0 \text{ (suposição)}$$

$$\therefore \boxed{\vec{F}(t) \times \vec{F}'(t) = \text{cte} \quad \forall t}$$

61.

$$\int dt\,\left(t^2\hat{i} - (3t-1)\hat{j} - \frac{1}{t^3}\hat{k}\right) =$$

$$= \left(\int dt\, t^2\right)\hat{i} - \left(\int dt\,(3t-1)\right)\hat{j} - \left(\int dt\,\frac{1}{t^3}\right)\hat{k}$$

$$= \frac{t^3}{3}\hat{i} - \left(\frac{3t^2}{2} - t\right)\hat{j} - \frac{t^{-2}}{-2}\hat{k}$$

$$= \frac{t^3}{3}\hat{i} - \left(\frac{3t^2}{2} - t\right)\hat{j} + \frac{1}{2t^2}\hat{k}$$

62.

$$\int dt\,\left(t\cos t\,\hat{i} + t\sin t\,\vec{j} + 3t^4\hat{k}\right)$$

$$= \overset{①}{\int dt\, t\cos t}\,\hat{i} + \overset{②}{\int dt\, t\sin t}\,\hat{j} + \overset{③}{\int dt\, 3t^4}\,\hat{k}$$

$$① = \int dt\, t\cos t = +t\sin t - \int \sin t\, dt$$

$u = t \to du = dt$

$dv = \cos t\, dt \to v = +\sin t$

$$= +t\sin t - \int \sin t\, dt$$

$$= +t\sin t + \cos t$$

$$② = \int dt\, t\sin t =$$

$u = t \to du = dt$

$dv = \sin t\, dt \to v = -\cos t$

$$= -t\cos t + \int \cos t\, dt$$

$$= -t\cos t + \sin t$$

$$③ = \int dt\, 3t^4 = \frac{3t^5}{5}$$

$\therefore$

$$\int dt\,\left(t\cos t\,\hat{i} + t\sin t\,\hat{j} + 3t^4\hat{k}\right)$$

$$= (t\sin t + \cos t)\,\hat{i} + (-t\cos t + \sin t)\,\hat{j} + \frac{3t^5}{5}\hat{k}$$

63.

$$\int_a^b dt\,\left(t\hat{i} + t^2\hat{j} - t^3\hat{k}\right) =$$

$$= \left.\frac{t^2}{2}\right]_a^b \hat{i} + \left.\frac{t^3}{3}\right]_a^b \hat{j} - \left.\frac{t^4}{4}\right]_a^b \hat{k}$$

$$= \left(\frac{b^2}{2} - \frac{a^2}{2}\right)\hat{i} + \left(\frac{b^3}{3} - \frac{a^3}{3}\right)\hat{j} + \left(-\frac{b^4}{4} + \frac{a^4}{4}\right)\hat{k}$$

64.

$$\int_0^1 dt\,\left(t^2\,\hat{i} - 2t\,\hat{j} + \sqrt{t}\,\hat{k}\right)$$

$$= \frac{t^3}{3}\Big]_0^1 \hat{i} - \frac{2t^2}{2}\Big]_0^1 \hat{j} + \frac{t^{3/2}}{\frac{3}{2}}\Big]_0^1 \hat{k}$$

$$= \frac{1}{3}\,\hat{i} - \hat{j} + \frac{2}{3}\,\hat{k}$$

65.

$$\int_0^{\frac{\pi}{4}} dt\left(\cos t\,\hat{i} + \sin 2t\,\hat{j} + \cos^2 t\,\hat{k}\right)$$

$$= \underbrace{\int_0^{\frac{\pi}{4}} dt\,\cos t}_{1}\;\hat{i} + \underbrace{\int_0^{\frac{\pi}{4}} dt\,\sin 2t}_{2}\;\hat{j} + \underbrace{\int_0^{\frac{\pi}{4}} dt\,\cos^2 t}_{3}\;\hat{k}$$

$$① = \int_0^{\frac{\pi}{4}} dt\,\cos t = \sin t\Big]_0^{\pi/4} = \frac{\sqrt{2}}{2}$$

$$② = \int_0^{\frac{\pi}{4}} dt\,\sin 2t = \int_0^{\frac{\pi}{2}} \frac{du}{2}\,\sin u$$

$(u = 2t)$

$$= \frac{1}{2}(-)\cos u\Big]_0^{\pi/2}$$

$$= \frac{1}{2}$$

$$③ = \int_0^{\frac{\pi}{4}} dt\,\cos^2 t$$

$$\int dt\,\cos^2 t = \int dt\,\frac{1+\cos 2t}{2}$$

$$= \frac{1}{2}t + \frac{\sin 2t}{4}$$

$\therefore$

$$\int_0^{\frac{\pi}{4}} dt\,\cos^2 t = \frac{1}{2}t + \frac{\sin 2t}{4}\Big]_0^{\frac{\pi}{4}}$$

$$= \frac{\pi}{8} + \frac{1}{4}$$

$\therefore$

$$\int_0^{\frac{\pi}{4}} dt\left(\cos t\,\hat{i} + \sin 2t\,\hat{j} + \cos^2 t\,\hat{k}\right)$$

$$= \frac{\sqrt{2}}{2}\,\hat{i} + \frac{1}{2}\,\hat{j} + \left(\frac{\pi}{8} + \frac{1}{4}\right)\hat{k}$$

66.

$$\int_1^4 dt\,\left(e^t\hat{\imath} + t^2\hat{\jmath} + \ln 2t\,\hat{k}\right) = (\star)$$

$$= e^t\Big]_1^4\hat{\imath} + \frac{t^3}{3}\Big]_1^4\hat{\jmath} +$$

$$+ \int_1^4 dt\,\ln 2t\,\hat{k}$$

Mas

$$\int dt\,\ln 2t = \frac{1}{2}\int dx\,\ln x$$

$x = 2t$

$u = \ln x \rightarrow du = \frac{1}{x}dx$

$dv = dx \rightarrow v = x$

$\therefore$

$$\int dx \ln x = x\ln x - \int \frac{x}{x}dx$$

$$= x\ln x - x$$

$\therefore$

$$\int dt\,\ln 2t = \frac{1}{2}\left(x\ln x - x\right)$$

$$= \frac{1}{2}\left(2t\ln 2t - 2t\right)$$

$$= t\ln 2t - t$$

$$\int_1^4 dt\,\ln 2t = \left(t\ln 2t - t\right)\Big]_1^4$$

$$= 4\ln 8 - 4$$

$$- \ln 2 + 1$$

$$= 4\cdot 3\ln 2 - \ln 2 - 3$$

$$= 11\ln 2 - 3$$

$\therefore$

$$(\star) = (e^4 - e)\,\hat{\imath} + \left(\frac{64}{3} - \frac{1}{3}\right)\hat{\jmath} +$$

$$+ (11\ln 2 - 3)\,\hat{k}$$

$$= (e^4 - e)\,\hat{\imath} + 21\,\hat{\jmath} +$$

$$+ (11\ln 2 - 3)\,\hat{k}$$

67.

$$\vec{F}(t) = \int_0^t ds\,\left(s\tan s^3\,\hat{\imath} + \right.$$

$$\left. + \cos e^s\,\hat{\jmath} + e^{s^2}\hat{k}\right)$$

$$\frac{d\vec{F}}{dt} = t\tan t^3\,\hat{\imath} + \cos e^t\,\hat{\jmath}$$

$$+ e^{t^2}\hat{k}$$

68.

$$\vec{F}(t) = \int_0^{t^2} ds \left(\cos s \,\hat{i} + e^{-s^2} \hat{j} + \tan s \,\hat{k} \right)$$

seja $z \equiv t^2$

$$\frac{d\vec{F}}{dt} \equiv \underbrace{\frac{d\vec{F}}{dz}} \frac{dz}{dt}$$

$$\equiv \left(\cos t^2 \hat{i} + e^{-t^4} \hat{j} + tg\, t^2 \hat{k} \right) 2t$$

$$\equiv 2t \cos t^2 \hat{i} + 2t e^{-t^4} \hat{j} + 2t\, tg\, t^2 \hat{k}$$

Cálculo C - Respostas Lista 1

1. $\text{Dom } \vec{F} = \mathbb{R}$

   $F_1(t) = t \ , \ F_2(t) = t^2, \ F_3(t) = t^3$

2. $\text{Dom } \vec{F} = [1, +\infty)$

   $F_1(t) = \sqrt{t+1} \ , \ F_2(t) = \sqrt{t-1}$

   $F_3(t) = 1$

3. $\text{Dom } \vec{F} = \mathbb{R} - \{\pm 2\}$

   $F_1(t) = \text{tgh } t$

   $F_2(t) = 0 \ , \ F_3(t) = \dfrac{-1}{t^2-4}$

4. $\text{Dom } \vec{F} = (0,\pi) \cup (\pi, 2\pi) \cup (2\pi, 3\pi) \cup \dots$

   $= (0,\pi) - \{n\pi \mid n \in \mathbb{N}\}$

   $F_1(t) = \dfrac{\ln t}{t} - e^{-5t} \cot g\, t$

   $F_2(t) = (4-t^2) \cot g\, t - \dfrac{t^2-1}{t}$

   $F_3(t) = e^{-5t}(t^2-1) - (4-t^2) \ln t$

5. $\text{Dom } \vec{F} = [0, +\infty)$

   $F_1(t) = 2\sqrt{t}$

   $F_2(t) = -2t\sqrt{t}$

   $F_3(t) = -(t^3+1)$

6. $\text{Dom } (\vec{F} - \vec{G}) = (0, +\infty)$

   $(\vec{F} - \vec{G})_1 = 2t - e^t$

   $(\vec{F} - \vec{G})_2 = e^2 - e^{-t}$

   $(\vec{F} - \vec{G})_3 = -\ln t - 2t$

7. $\text{Dom } (2\vec{F} - 3\vec{G}) = \mathbb{R}$

   $(2\vec{F} - 3\vec{G})_1 = 2t - 3\cos t$

   $(2\vec{F} - 3\vec{G})_2 = 2t^2 - 3\sin t$

   $(2\vec{F} - 3\vec{G})_2 = 2t^3 - 3$

8. $\text{Dom } \vec{F} \times \vec{G} = \mathbb{R}$

   $(\vec{F} \times \vec{G})_1 = t^2 - t^3 \sin t$

   $(\vec{F} \times \vec{G})_2 = t^3 \cos t - t$

   $(\vec{F} \times \vec{G})_3 = t \sin t - t^2 \cos t$

9.

$\mathrm{Dom}\ \vec{F}\times\vec{G} = (0,+\infty)$

$(\vec{F}\times\vec{G})_1(t) = \frac{1}{\sqrt{t}}(1-\cos t)$

$(\vec{F}\times\vec{G})_2(t) = \frac{-1}{\sqrt{t}}(t-\sin t)$

$(\vec{F}\times\vec{G})_3(t) = -t(t-\sin t)$

10.

$\mathrm{Dom}\ f\vec{F} = [1,+\infty)$

$(f\vec{F})_1 = \sqrt{t}\ln t$

$(f\vec{F})_2 = -4\sqrt{t}\,e^{2t}$

$(f\vec{F})_3 = \frac{\sqrt{t-1}}{\sqrt{t}}$

11. $\mathrm{Dom}\ \vec{F}\circ g = [-8,+\infty)$

$(\vec{F}\circ g)_1 = \cos t^{1/3}$

$(\vec{F}\circ g)_2 = \sin t^{1/3}$

$(\vec{F}\circ g)_3 = \sqrt{t^{1/3}+2}$

12.

$\mathrm{Dom}\ \vec{F}\circ g = (0,+\infty)$

$(\vec{F}\circ g)_1 = \frac{1}{t^2}$

$(\vec{F}\circ g)_2 = t^{\ln t}$

$(\vec{F}\circ g)_3 = (\ln t)^3$

13. $\hat{i}-\hat{j}+\hat{k}$

14. $3\hat{i}-\hat{j}-\hat{k}$

15. $3\pi\,\hat{j}-4\hat{k}$

16. $\hat{i}+\hat{j}+\sqrt{2}\,\hat{k}$

17. $5\hat{i}-4\hat{j}+\hat{k}$

18. $\vec{0}$

19. O limite não existe

20. $1+\sqrt{2}$

21. $\hat{i}+4\hat{j}+10\hat{k}$

22. O limite não existe

23. $\hat{i}-e\hat{j}+2\hat{k}$

24. $\frac{1}{2}\hat{i}-\hat{j}-\hat{k}$

25. $\hat{i}+\hat{k}$

26. $3\hat{i}+\frac{2}{3}\hat{j}$

27. $6\hat{i}+7\hat{j}+14\hat{k}$

28. $\hat{j}$

29. O limite não existe

30. $\vec{0}$

31. $\vec{0}$

32. $(-\infty, +\infty) = \mathbb{R}$

33. $\mathbb{R} - \{0\}$

34. $\mathbb{R} - \{-3\}$

35. $\mathbb{R} - \{2\}$

36. $(-\infty, +\infty)$

37. $\hat{i} + \frac{1}{2\sqrt{t}}\hat{j}$

38. $\frac{1}{2\sqrt{t}}\hat{i} - \frac{3}{2}t^{-5/2}\hat{j} + \frac{2}{2t-1}\hat{k}$

39. $\frac{1}{\sqrt{1-t^2}}\hat{i} + \frac{t}{\sqrt{1+t^2}}\hat{j} -$
$- 3t^2 e^{-t^3}\hat{k}$

40. $\frac{e^{\sqrt{t}}}{2\sqrt{t}}\hat{i} + \frac{2}{\sqrt{1-4t^2}}\hat{k}$

41. $\sec^2 t\ \hat{i} + \sec t\ tg\, t\ \hat{k}$

42. $(e^t \cos t - e^t \sin t)\hat{i} +$
$+ (-e^t \sin t - e^t \cos t)\hat{k}$

43.
$\sinh t\ \hat{i} + \cosh t\ \hat{j} - \frac{1}{2\sqrt{t}}\hat{k}$

44. $\hat{i} - 2t\hat{j} + 2\hat{k}$

45. $2t\,\hat{i} + (-6t^2 - 6)\hat{j} +$
$+ (12t^3 + 18t)\hat{k}$

46.
$(8t^3 - 6t)\hat{i} + (-10t^4 + 9t^2)\hat{j} +$
$+ (16t^3 - 12t)\hat{k}$

47.
$(8t - 12t^3)\hat{i} + (2 - 9t^2)\hat{j} - 2t\hat{k}$

48. $(\cancel{4t}\ 12t^2 - 15t^4)\hat{i} + (4t - 12t^3)\hat{j} -$
$- 3t^2\hat{k}$

49. $2t + 16t^3$

50. $3\hat{i} + (-6t - 8)\hat{j} + (6 + 24t)\hat{k}$

51. $9t^2\hat{i} + (-20t^3 + 6t)\hat{j} +$
$+ (6 - 21t^2 + 30t^4)\hat{k}$

52. $(-14t^4 - 6t^3 - 42t^2 + 4t)\hat{i}$
$+ (28t^5 + 84t^3 - 6t^2)\hat{j}$
$+ (-42t^6 - 126t^4 - 2t)\hat{k}$

61. $\frac{t^3}{3}\hat{i} - \left(\frac{3t^2}{2} - t\right)\hat{j} + \frac{1}{2t^2}\hat{k}$

62. $(t\sin t + \cos t)\hat{i} +$

$+ (-t\cos t + \sin t)\hat{j} + \frac{3t^5}{5}\hat{k}$

63. $\left(\frac{b^2}{2} - \frac{a^2}{2}\right)\hat{i} + \left(\frac{b^3}{3} - \frac{a^3}{3}\right)\hat{j} +$

$+ \left(-\frac{b^4}{4} + \frac{a^4}{4}\right)\hat{k}$

64. $\frac{1}{3}\hat{i} - \hat{j} + \frac{2}{3}\hat{k}$

65. $\frac{\sqrt{2}}{2}\hat{i} + \frac{1}{2}\hat{j} + \left(\frac{\pi}{8} + \frac{1}{4}\right)\hat{k}$

66. $(e^4 - e)\hat{i} + 21\hat{j} + (11\ln 2 - 3)\hat{k}$

67. $t\tan t^3\,\hat{i} + \cos e^t\,\hat{j} + e^{t^2}\hat{k}$

68. $2t\cos t^2\,\hat{i} + 2t\,e^{-t^4}\hat{j} + 2t\tan t^2\,\hat{k}$