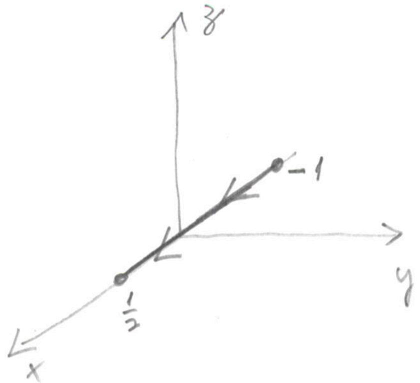
Cálculo C - Respostas Lista 2

1.



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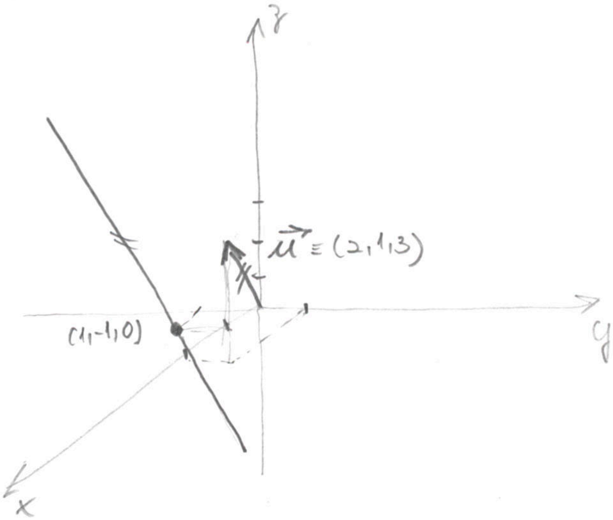

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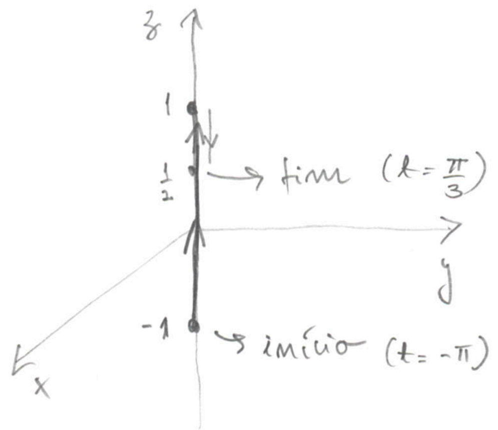

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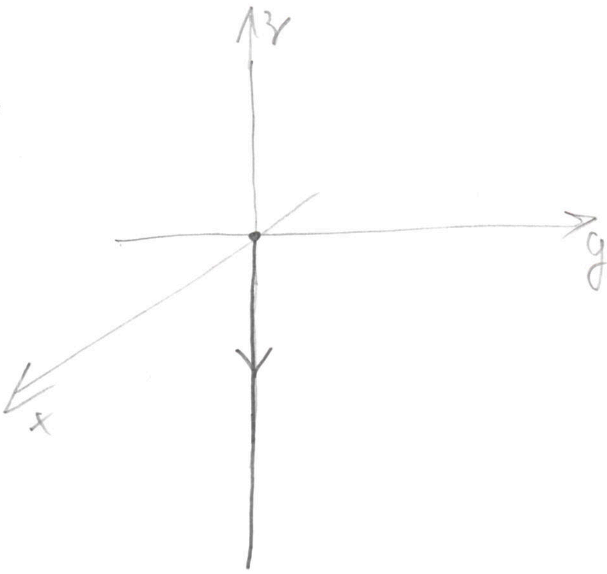

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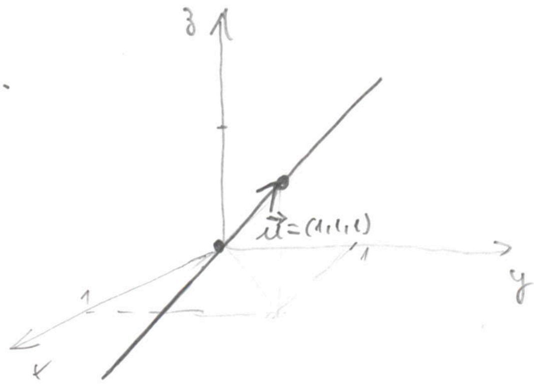

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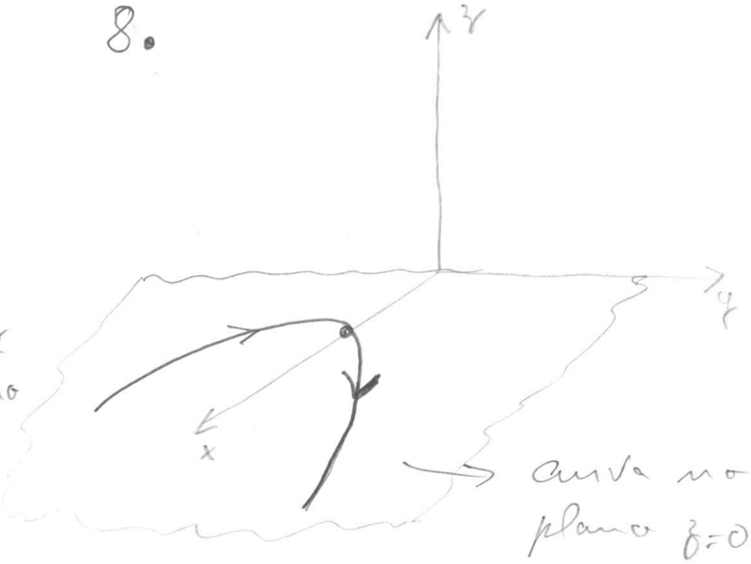

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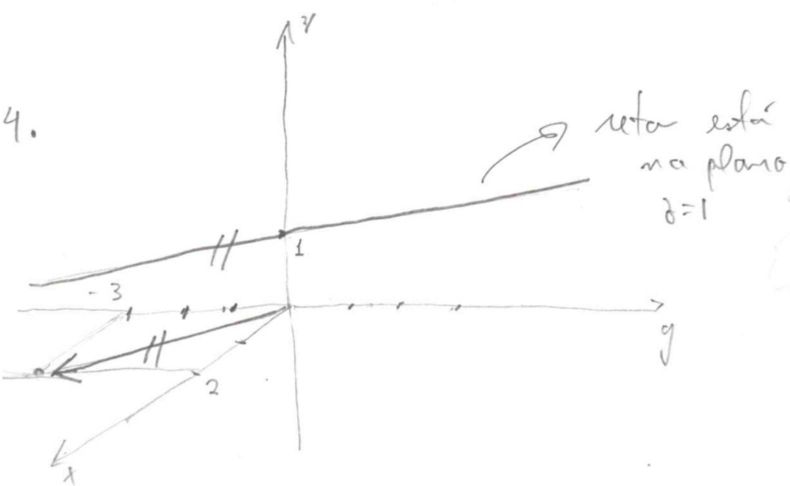
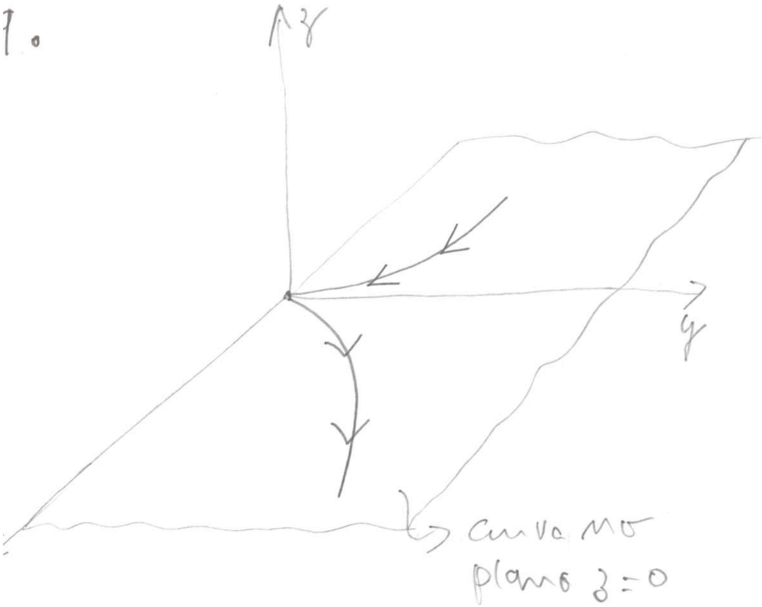
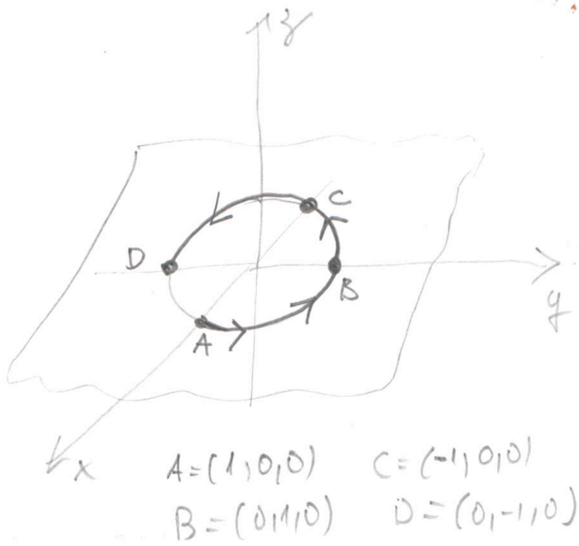


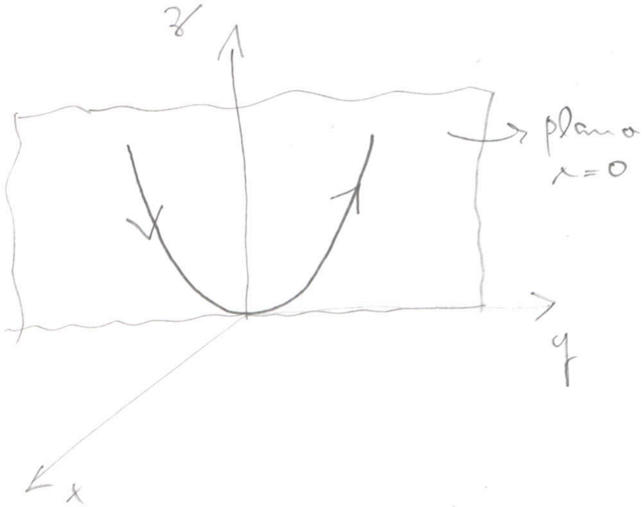
1.


z
y
curva no plano z=0

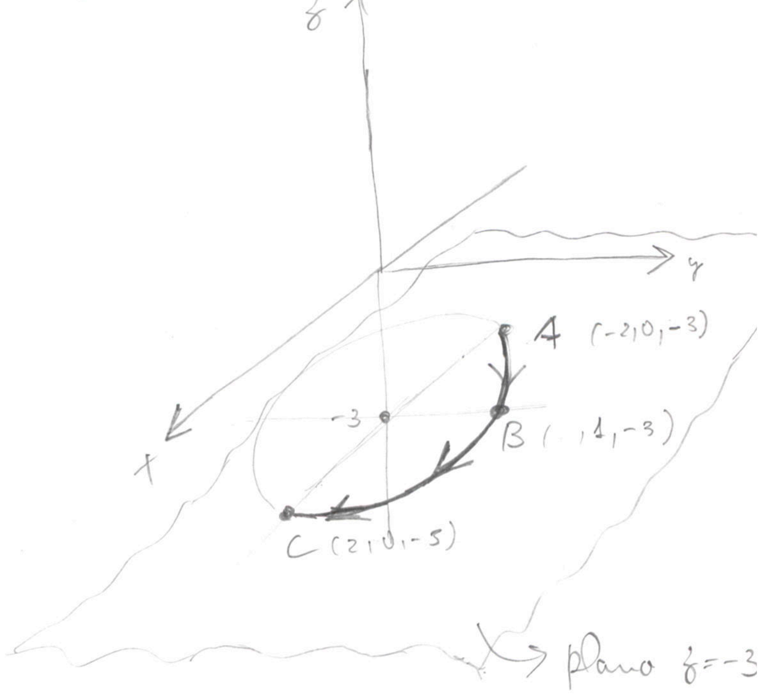
11.


z
C
D
B
A
y
x
A=(1,0,0) C=(-1,0,0)
B=(0,1,0) D=(0,-1,0)

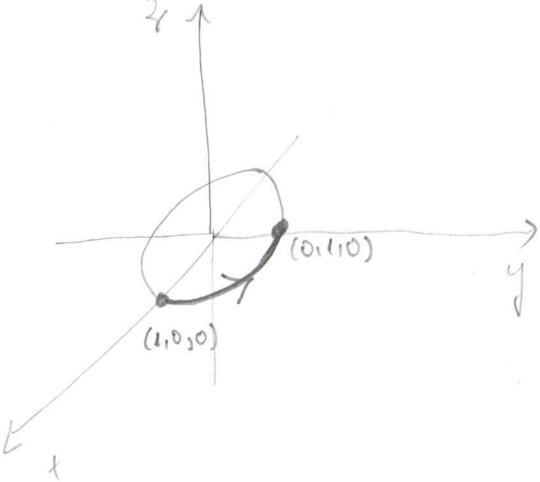
7.


z
plano x=0
y
x

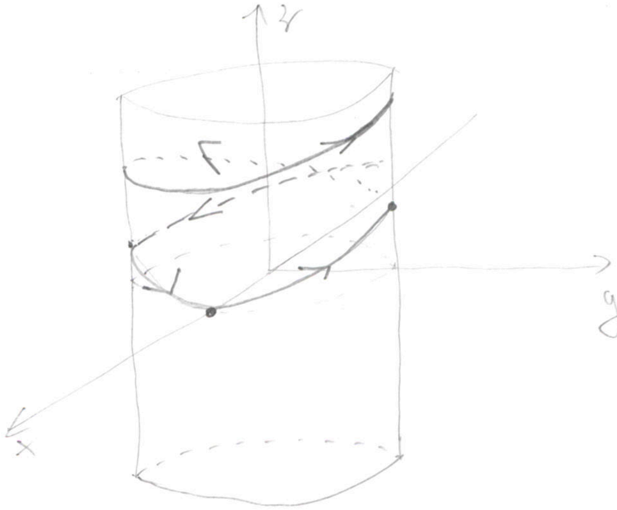
12.


z
y
A (-2,0,-3)
-3
B (,1,-3)
x
C (2,0,-5)
plano z=-3

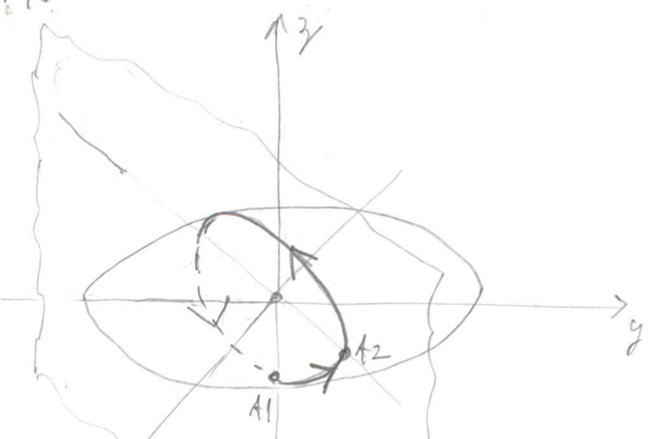
10.


z
(0,1,0)
y
(1,0,0)
x

13.


z
y
x

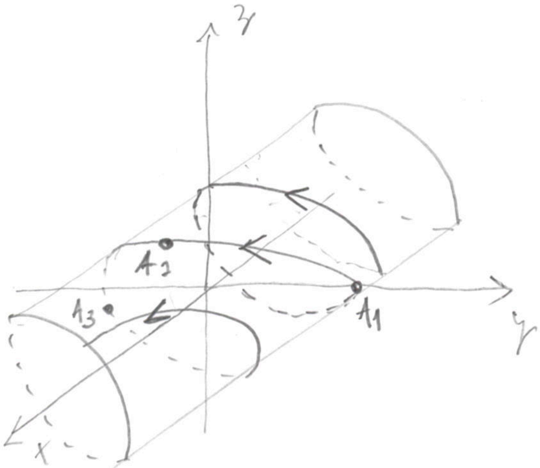
14.



$A_1 = (0, 0, -3\sqrt{2})$

$A_2 = (3, 3, 0)$

15.



$$\begin{cases} A_1 = (0, 1, 0) \\ A_2 = (\frac{\pi}{4}, 0, 1) \\ A_3 = (\frac{\pi}{2}, -1, 0) \end{cases}$$

6. $\begin{cases} \vec{r}(t) = -7t\,\hat{i} + (3+2t)\,\hat{j} + t\,\hat{k} \\ t \geq 0 \\ \text{(reta)} \end{cases}$

17.

$$\begin{cases} \vec{r}(t) = \sqrt{2}\cos t\,\hat{i} + \sqrt{2}\sin t\,\hat{j} + 4\hat{k} \\ 0 \leq t \leq 2\pi \end{cases}$$

(círculo no plano $z = 4$)

18.

$\vec{r}(t) = \sqrt{5}\sin t\,\hat{i} + \sqrt{5}\cos t\,\hat{j} + 5\hat{k}$

$0 \leq t \leq 2\pi$

19.

$\vec{r}(t) = t\,\hat{i} + t\,\hat{j} + \sqrt{2}\,|t|\,\hat{k}$

$t \geq 0$

20. Suave

21. Suave por partes
($I = (-\infty, 0] \cup [0, +\infty)$)

22. Suave

23. Suave por partes
($I = (-\infty, 0] \cup [0, +\infty)$)

24. Suave por partes
($I = (-\infty, 0] \cup [0, +\infty)$)

25. $\begin{cases} \vec{r} = (-3+7t)\,\hat{i} + (2-2t)\,\hat{j} + (1+4t)\,\hat{k} \\ t \in \mathbb{R} \end{cases}$

26.

$$\begin{cases} \vec{r}(t) = 6\cos t\,\hat{i} + 6\sin t\,\hat{j} \\ 0 \le t \le 2\pi \end{cases}$$

27.

$$\vec{r}(t) = \begin{cases} t\,\hat{i}, & 0 \le t \le 3 \\ 3\hat{i} + (t-3)\,\hat{j}, & 3 \le t \le 6 \\ (9-t)\,\hat{i} + 3\hat{j}, & 6 \le t \le 9 \\ (12-t)\,\hat{j}, & 9 \le t \le 12 \end{cases}$$

28.

$$\vec{r}(t) = \begin{cases} t\,\hat{i}, & 0 \le t \le 2 \\ (4-t)\,\hat{i} + (t-2)\,\hat{j}, & 2 \le t \le 4 \\ (6-t)\,\hat{j}; & 4 \le t \le 6 \end{cases}$$

29. 6

30. $3+\ln 2$

31. $\sqrt{3}$

32. $e - \frac{1}{e}$

33. 52

34.

$$\begin{cases} \vec{T}(t) = \frac{t}{\sqrt{t^2+1}}\,\hat{i} + \frac{\hat{j}}{\sqrt{t^2+1}} \\ \vec{N}(t) = \frac{\hat{i} - t\,\hat{j}}{\sqrt{t^2+1}}, \quad \kappa(t) = \frac{1}{2(t^2+1)^{3/2}} \end{cases}$$

35.

$$\begin{cases} \vec{T}(t) = -\frac{\sin t}{\sqrt{2}}\,\hat{i} - \frac{\sin t}{\sqrt{2}}\,\hat{j} + \cos t\,\hat{k} \\ \vec{N}(t) = -\frac{\cos t}{\sqrt{2}}\,\hat{i} - \frac{\cos t}{\sqrt{2}}\,\hat{j} - \sin t\,\hat{k} \\ \kappa(t) = \frac{1}{\sqrt{2}} \end{cases}$$

36.

$$\begin{cases} \vec{T}(t) = \frac{2}{t^2+2}\,\hat{i} + \frac{2t}{t^2+2}\,\hat{j} + \frac{t^2}{t^2+2}\,\hat{k} \\ \vec{N}(t) = \frac{-2t}{(t^2+2)}\,\hat{i} + \frac{2-t^2}{(t^2+2)}\,\hat{j} + \frac{2t}{(t^2+2)}\,\hat{k} \\ \kappa(t) = \frac{2}{(t^2+2)^2} \end{cases}$$

37.

$$\begin{cases} \vec{T}(t) = \frac{e^t\,\hat{i} - e^{-t}\,\hat{j} + \sqrt{2}\,\hat{k}}{e^t + e^{-t}} \\ \vec{N}(t) = \frac{\sqrt{2}}{(e^t+e^{-t})}\,\hat{i} + \frac{\sqrt{2}}{e^t+e^{-t}}\,\hat{j} - \frac{e^t - e^{-t}}{e^t+e^{-t}}\,\hat{k} \\ \kappa(t) = \sqrt{2}/(e^t+e^{-t})^2 \end{cases}$$

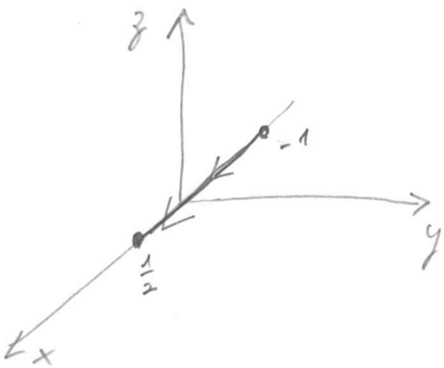
38.

$$\vec{T}(t) = \frac{2t}{2t^2+1}\,\hat{i} + \frac{2t^2}{2t^2+1}\,\hat{j} + \frac{1}{2t^2+1}\,\hat{k}$$

$$\vec{N}(t) = \frac{1-2t^2}{2t^2+1}\,\hat{i} + \frac{2t}{2t^2+1}\,\hat{j} - \frac{2t}{2t^2+1}\,\hat{k}$$

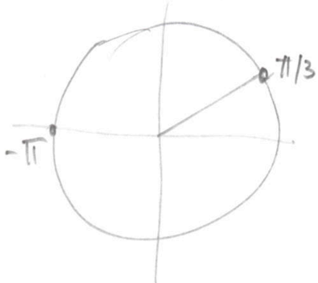
$$\kappa(t) = \frac{2t}{(2t^2+1)^2}$$

1. $\vec{r}(t) = t\,\hat{i}$; $-1 \le t \le \frac{1}{2}$

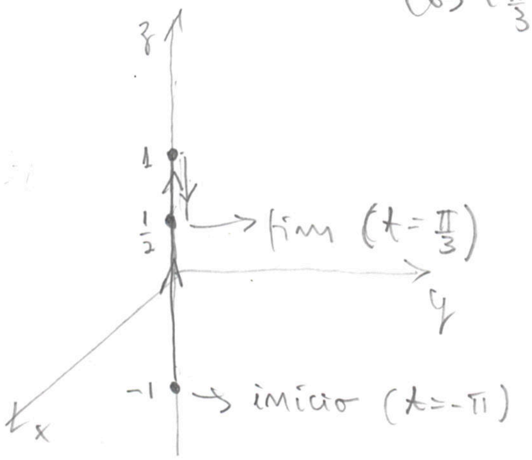


2. $\vec{r}(t) = \cos \pi t\,\hat{k}$; $-1 \le t \le \frac{1}{3}$

$$-1 \le t \le \frac{1}{3} \Rightarrow -\pi \le \pi t \le \frac{\pi}{3}$$



$\cos(-\pi) = -1$
⋮ cresce
$\cos(0) = 1$
⋮ decresce
$\cos(\frac{\pi}{3}) = \frac{1}{2}$



3. $\vec{r}(t) = t\,\hat{i} + t\,\hat{j} + t\,\hat{k}$

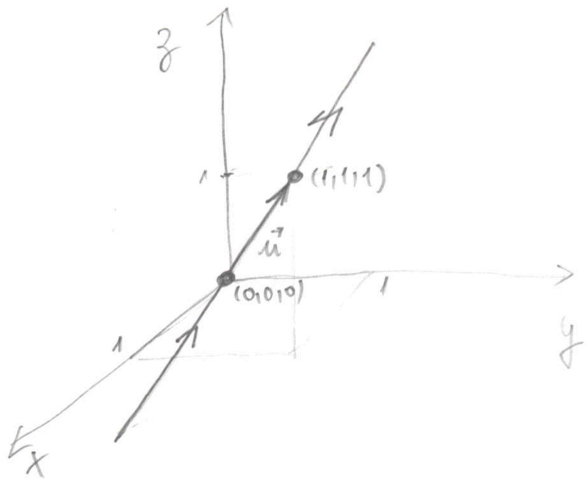
Aqui:

$x(t) = t$, $y(t) = t$, $z(t) = t$

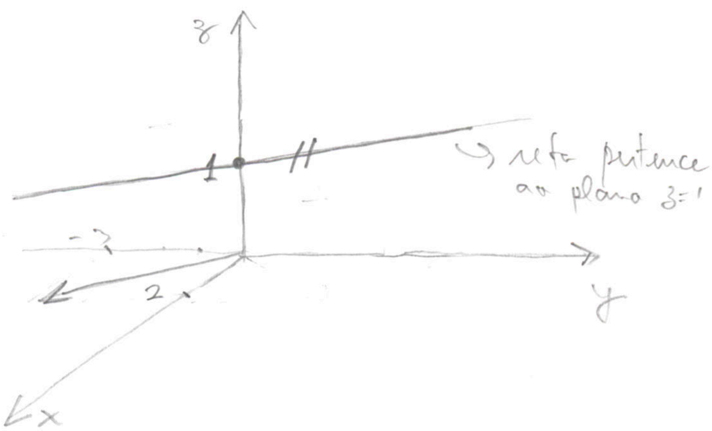
Eqs paramétricas de uma reta paralela ao vetor

$$\vec{u} = (1,1,1)$$

e passando por $(0,0,0)$.



4. $\vec{r}(t) = 2t\,\hat{i} - 3t\,\hat{j} + \hat{k}$



$x(t) = 2t$
$y(t) = -3t$
$z(t) = 1$

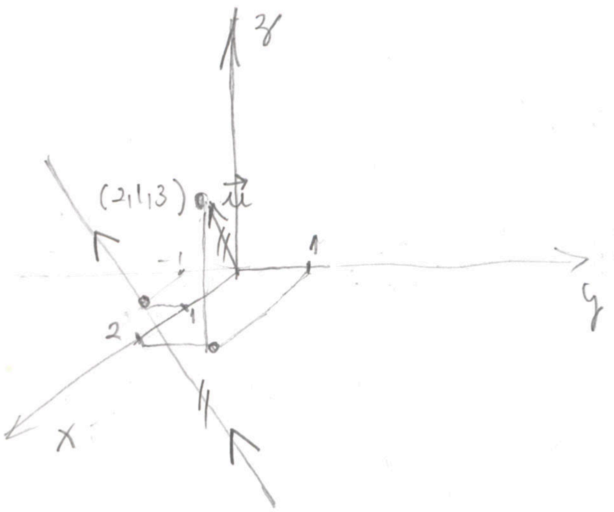
5. $\vec{r}(t) = (2t+1)\,\hat{i} + (t-1)\,\hat{j} + 3t\,\hat{k}$

$x(t) = 2t+1 \quad , \quad y(t) = t-1 \quad , \quad z(t) = 3t$

reta paralela ao vetor

$\vec{u} = (2, 1, 3)$

e passando por $(1, -1, 0)$

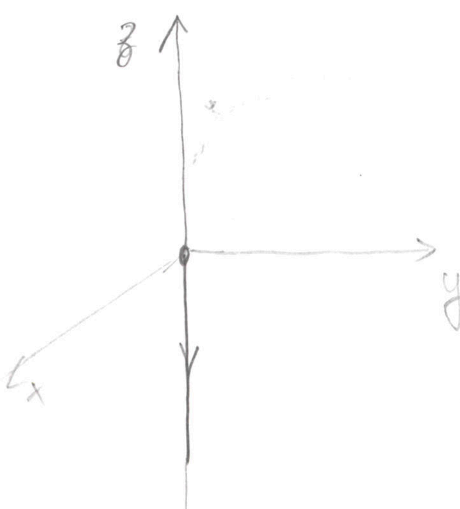


6. $\vec{r}(t) = -16\,t^2\,\hat{k}\ ,\ t \geq 0$

$t=0 \ :\ \vec{r}(0) = 0\,\hat{k}$

$t=1 \ :\ \vec{r}(1) = -16\,\hat{k}$

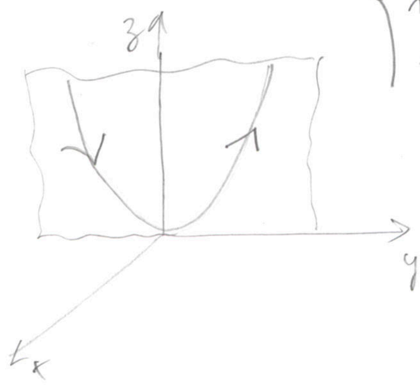
$\vdots$



7. $\vec{r}(t) = t\,\hat{j} + t^2\,\hat{k}$

$y(t) = t \quad , \quad z(t) = t^2$

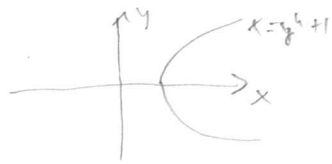
$\therefore \quad z = y^2 \ : \left\{ \begin{array}{l} \text{parábola} \\ \text{no plano} \\ x = 0 \end{array} \right.$



8. $\vec{r}(t) = (t^4+1)\,\hat{i} + t\,\hat{j}$

$x(t) = t^4+1 \quad , \quad y(t) = t$

$\therefore \quad x = y^4 + 1$



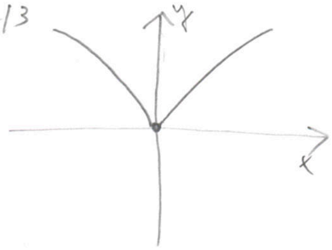
curva no plano $z=0$.

9. $\vec{r}(t) = t^3\hat{i} + t^2\hat{j}$

$x(t) = t^3$, $y(t) = t^2$

$\therefore \quad y(t) = (x^{\frac{1}{3}})^2$

$y = x^{2/3}$

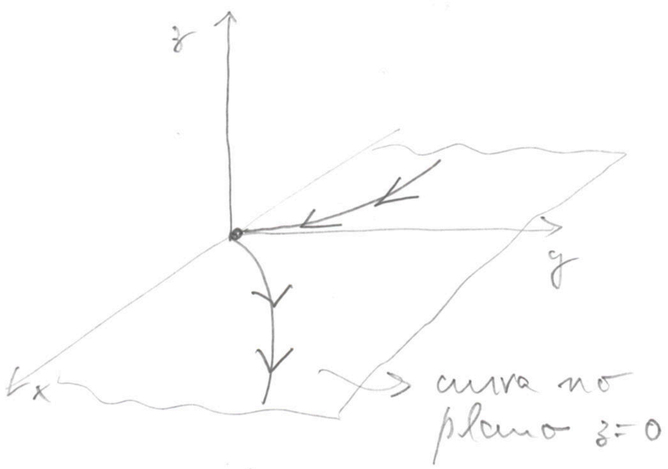


$y' = \frac{2}{3}x^{-1/3}$

$y'' = -\frac{2}{9}x^{-4/3}$

$x > 0 \Rightarrow y'' < 0$ concavidade p/ baixo

$x < 0 \Rightarrow y'' > 0$ concavidade p/ cima



10. $\vec{r}(t) = \cos t\,\hat{i} + \sin t\,\hat{j}$,

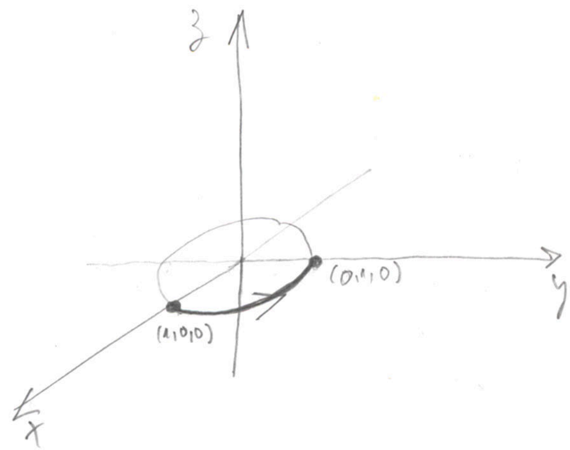
$0 \le t \le \frac{\pi}{2}$

$x(t) = \cos t$, $y(t) = \sin t$, $z(t) = 0$

$t = 0$: $(x(0), y(0), z(0)) = (1,0,0)$

$t = \frac{\pi}{2}$: $(x(\frac{\pi}{2}), y(\frac{\pi}{2}), z(\frac{\pi}{2})) = (0,1,0)$

$x^2 + y^2 = 1$ (círculo)



11. $\vec{r}(t) = \cos 3t\,\hat{i} + \sin 3t\,\hat{j}$

$0 \le t \le \frac{\pi}{2} \Rightarrow 0 \le 3t \le \frac{3\pi}{2}$

$x(t) = \cos 3t$, $y(t) = \sin 3t$, $z(t) = 0$

$t = 0 \Rightarrow (x(t), y(t), z(t)) = (1,0,0) \equiv A$

$t = \frac{\pi}{6} \Rightarrow (x(t), y(t), z(t)) = (\cos\frac{\pi}{2}, \sin\frac{\pi}{2}, 0)$

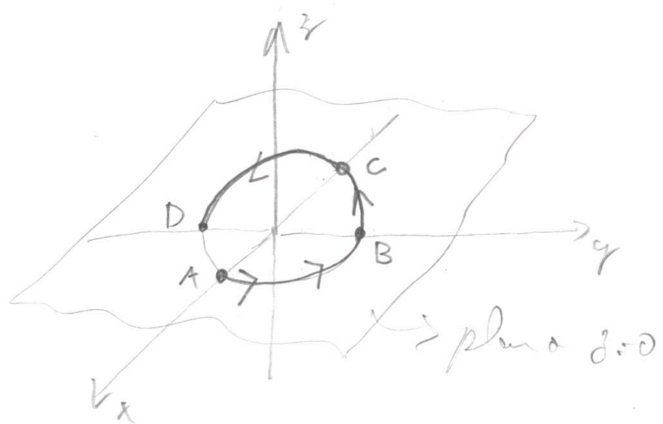
$= (0,1,0) \equiv B$

$t = \frac{\pi}{3} \Rightarrow (x(t), y(t), z(t)) = (\cos\pi, \sin\pi, 0)$

$= (-1,0,0) \equiv$

$t = \frac{\pi}{2} \Rightarrow (x(t), y(t), z(t)) = (\cos\frac{3\pi}{2}, \sin\frac{3\pi}{2}, 0)$

$= (0,-1,0) \equiv D$



2.

$\vec{r}(t) = 2\cos t\,\hat{i} - \sin t\,\hat{j} - 3\hat{k}$

$-\pi \leq t \leq 0$

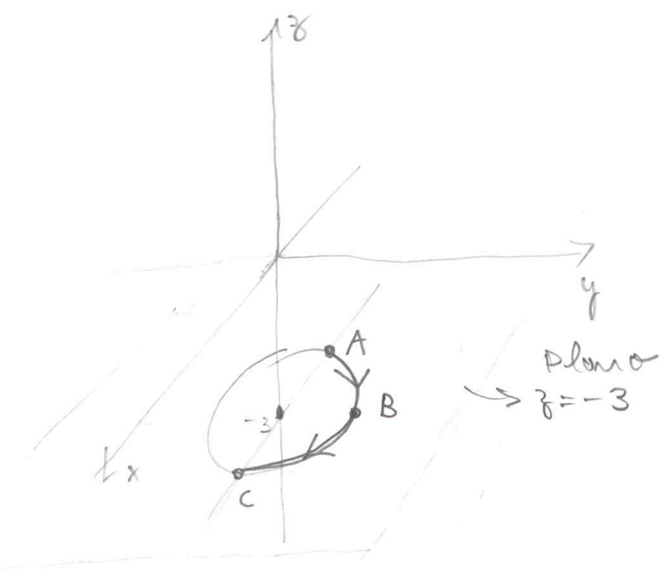
$z(t) = -3 = cte \rightarrow$ curva está no plano $z=-3$.

$\left.\begin{array}{l} x(t) = 2\cos t \\ y(t) = -\sin t \end{array}\right\}$ $\left(\frac{x}{2}\right)^2 + y^2 = 1$ (Elipse)

$t=-\pi$: $(x(t), y(t), z(t)) = (2\cos(-\pi), -\sin(-\pi), -3)$
$= (-2, 0, -3) = A$

$t = -\frac{\pi}{2}$: $(x(t), y(t), z(t)) = (2\cos(-\frac{\pi}{2}), -\sin(-\frac{\pi}{2}), -3)$
$= (0, 1, -3) = B$

$t = 0$: $(x(t), y(t), z(t)) = (2\cos 0, -\sin 0, -3)$
$= (2, 0, -3) = C$



13.

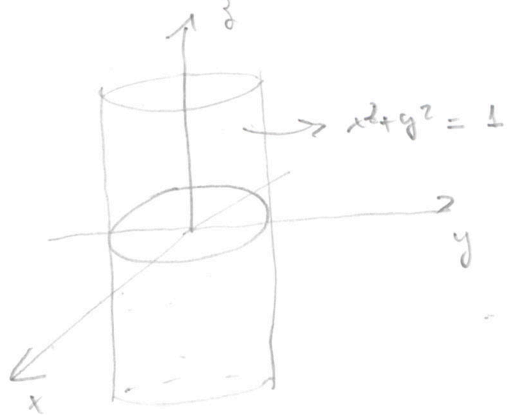
$\vec{r}(t) = \cos t\,\hat{i} + \sin t\,\hat{j} + t^2\hat{k}$

$\begin{cases} x(t) = \cos t \\ y(t) = \sin t \\ z(t) = t^2 \end{cases}$

Para t fixo temos que

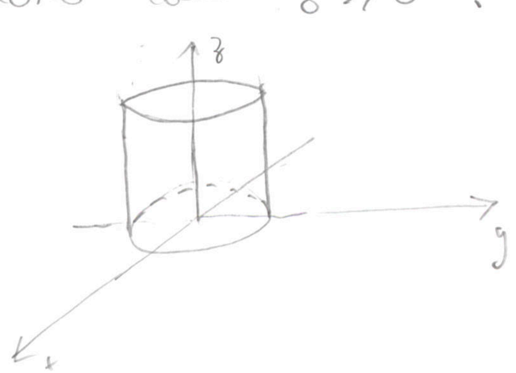
$x^2(t) + y^2(t) = 1$,

assim $(x(t), y(t), z(t))$ estará sempre no cilindro $x^2+y^2 = 1$



Mas $z(t) = t^2 \geq 0$,

logo $(x(t), y(t), z(t))$ estará sempre na parte do cilindro com $z \geq 0$:



13. (Continuação)

O esboço da curva se dá então pela análise de alguns pontos:

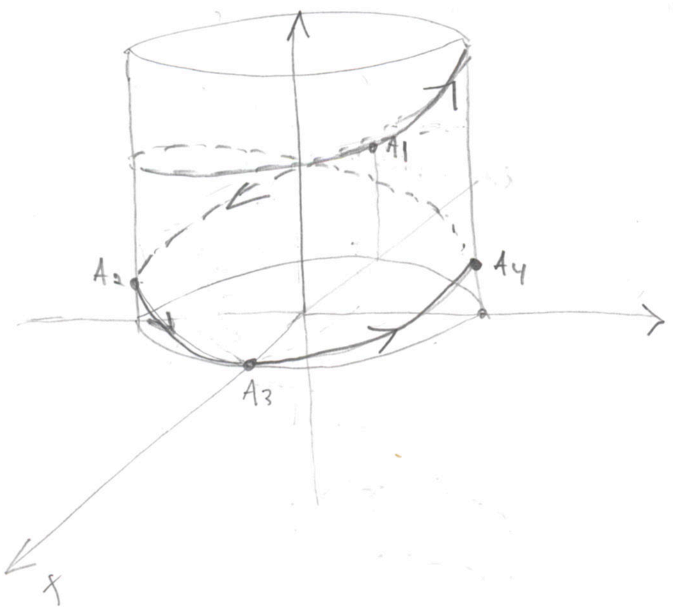
$t=0$: $(x(t),y(t),z(t)) = (\cos 0, \sin 0, 0)$
$= (1,0,0) = \underline{A_3}$

$t=\frac{\pi}{2}$: $(x(t),y(t),z(t)) = \left(\cos\frac{\pi}{2}, \sin\frac{\pi}{2}, \frac{\pi^2}{4}\right)$
$= \left(0, 1, \frac{\pi^2}{4}\right) = \underline{A_4}$

$t=-\frac{\pi}{2}$: $(x(t),y(t),z(t)) = \left(\cos\left(-\frac{\pi}{2}\right), \sin\frac{-\pi}{2}, \frac{\pi^2}{4}\right)$
$= \left(0, -1, \frac{\pi^2}{4}\right) =$
$= \underline{A_2}$

$t=\pi$: $(x(t),y(t),z(t)) = (\cos\pi, \sin\pi, \pi^2)$
$= (-1, 0, \pi^2) = \underline{A_5}$

$t=-\pi$: $(x(t),y(t),z(t)) = (\cos -\pi, \sin -\pi, \pi^2)$
$= (-1, 0, \pi^2) = \underline{A_1}$



14.

$\vec{r}(t) = 3\sin t\,\hat{i} + 3\sin t\,\hat{j} - 3\sqrt{2}\cos t$

$x(t) = 3\sin t$
$y(t) = 3\sin t$
$\Rightarrow x(t) = y(t)\ \forall t$

$z(t) = -3\sqrt{2}\cos t$

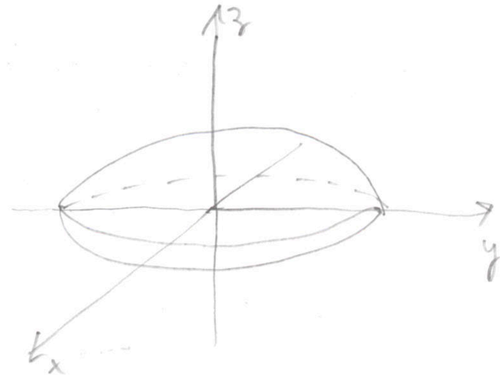
Notamos que

$$\frac{x^2}{18} + \frac{y^2}{18} + \frac{z^2}{18} =$$

$$= \frac{\sin^2 t}{2} + \frac{\sin^2 t}{2} + \cos^2 t$$

$$= 1$$

Isto é, $(x(t),y(t),z(t))$ está na elipse $\frac{x^2}{18} + \frac{y^2}{18} + \frac{z^2}{18} = 1$, $\forall t$

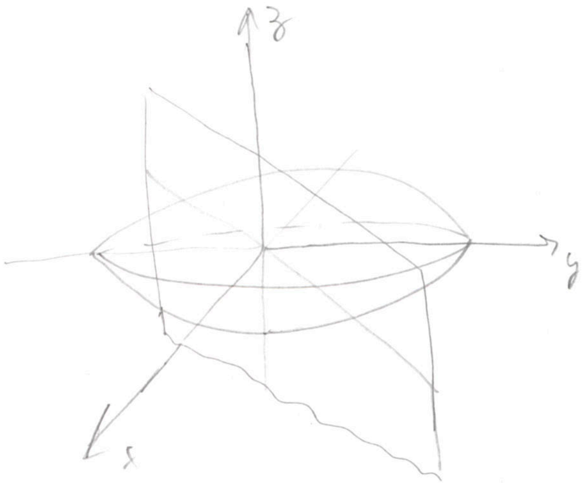


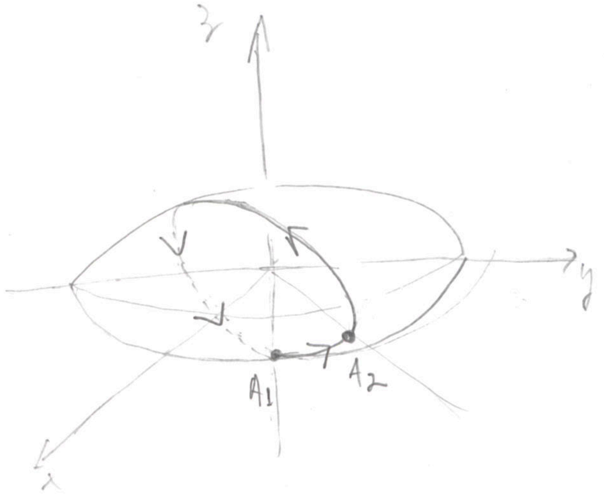
Além disso $x(t) = y(t)$, $\forall t$, implica que $(x(t),y(t),z(t))$ está sobre o plano $x=y$

4. (Continuação)

Isto é $(x(t), y(t), z(t))$ estar na interseção do plano $x=y$ com a elipse

$$\frac{x^2}{18}+\frac{y^2}{18}+\frac{z^2}{18}=1$$





$t=0 : (x(t), y(t), z(t)) = (0, 0, -3\sqrt{2}) = A_1$

$t=\frac{\pi}{2} : (x(t), y(t), z(t)) = (3, 3, 0) = A_2$

15.

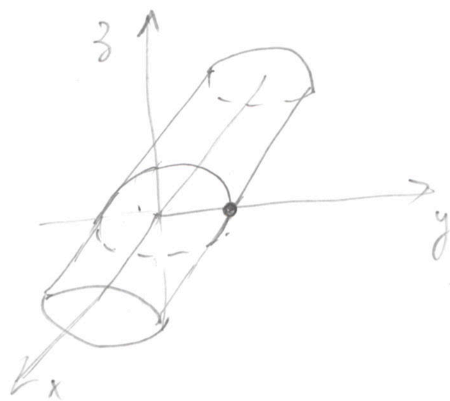
$\vec{r}(t) = t\,\hat{i} + \cos 2t\,\hat{j} + \sin 2t\,\hat{k}$

$$\begin{cases} x(t) = t \\ y(t) = \cos 2t \\ z(t) = \sin 2t \end{cases}$$

Aqui,

$y^2(t) + z^2(t) = 1, \quad \forall t$

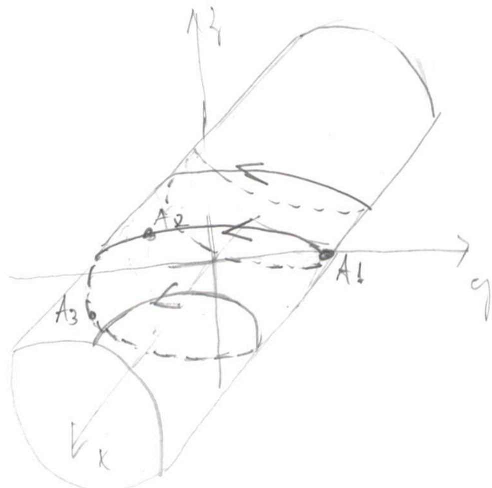
Logo, $(x(t), y(t), z(t))$ deve estar sobre o cilindro $y^2+z^2=1$



$t=0 : (x(t), y(t), z(t)) = (0, 1, 0) = A_1$

$t=\frac{\pi}{4} : (x(t), y(t), z(t)) = (\frac{\pi}{4}, 0, 1) = A_2$

$t=\frac{\pi}{2} : (x(t), y(t), z(t)) = (\frac{\pi}{2}, -1, 0) = A_3$



16.

$$\begin{cases} x+2y+3z=6 \\ y-2z=3 \end{cases}$$

tomemos z como parâmetro

$z=t$, $t\geq 0$ (z cresce)

$\therefore$ $y=3+2z=3+2t$

$x+2y+3z=6$

$x+2(3+2z)+3z=6$

$x+\cancel{6}+4z+3z=\cancel{6}$

$x=-7z$

$\therefore$

$$\boxed{\begin{aligned} x&=-7t \\ y&=3+2t \\ z&=t \quad ; \quad t\geq 0 \end{aligned}}$$

$$\boxed{\vec{r}(t)=-7t\,\hat{i}+(3+2t)\,\hat{j}+t\,\hat{k} \quad t\geq 0}$$

(reta)

17.

$$\begin{cases} x^2+y^2=2 \\ z=4 \end{cases}$$

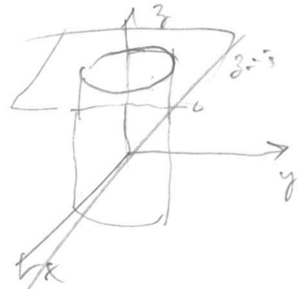
$x^2+y^2=2 \Rightarrow \begin{cases} x=\sqrt{2}\cos t \\ y=\sqrt{2}\sin t \end{cases}$

y aumenta no primeiro octante com a escolha $y=\sqrt{2}\sin t$.

$$\boxed{\vec{r}(t)=\sqrt{2}\cos t\,\hat{i}+\sqrt{2}\sin t\,\hat{j}+4\,\hat{k} \quad ; \quad 0\leq t<2\pi.}$$

18.

$$\begin{cases} x^2+y^2=5 \\ z=x^2+y^2 \end{cases}$$



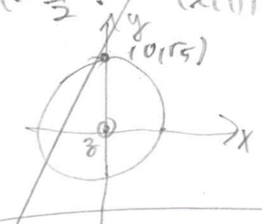
$\therefore$ $z=5$

$x^2+y^2=5 \longrightarrow$ $x=\sqrt{5}\sin t$, $y=\sqrt{5}\cos t$, $0\leq t<2\pi$

x aumenta no primeiro octante

$t=0$: $(x(t),y(t),z(t))=(0,\sqrt{5},5)$

$t=\frac{\pi}{2}$: $(x(t),y(t),z(t))=(\sqrt{5},0,5)$



$$\boxed{\vec{r}(t)=\sqrt{5}\sin t\,\hat{i}+\sqrt{5}\cos t\,\hat{j}+5\,\hat{k} \quad 0\leq t<2\pi}$$

8. $\begin{cases} z = x^2+y^2 \\ x^2+y^2=5 \end{cases}$

$z=5$

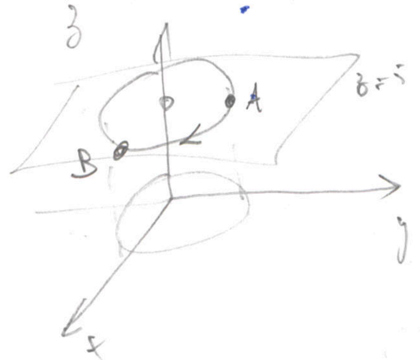
$x^2+y^2=5$ $\qquad x = \sqrt{5}\sin t$

x aumenta no primeiro octante $\qquad y = \sqrt{5}\cos t$

$\qquad 0 \leq t < 2\pi$

$t=0$: $(x(t), y(t), z(t)) = (0, \sqrt{5}, 5) \equiv A$

$t=\frac{\pi}{2}$: $(x(t), y(t), z(t)) = (\sqrt{5}, 0, 5) \equiv B$



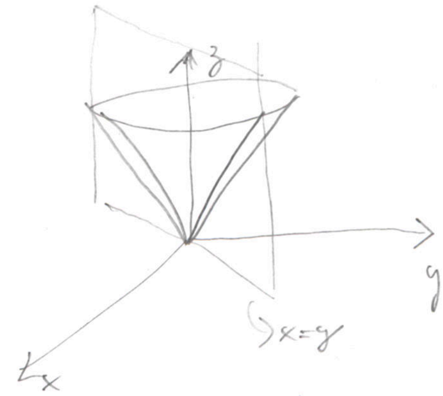
$$\boxed{\begin{gathered}\vec{r}(t) = \sqrt{5}\sin t\,\hat{\imath} + \sqrt{5}\cos t\,\hat{\jmath} + 5\,\hat{k} \\ 0 \leq t < 2\pi\end{gathered}}$$

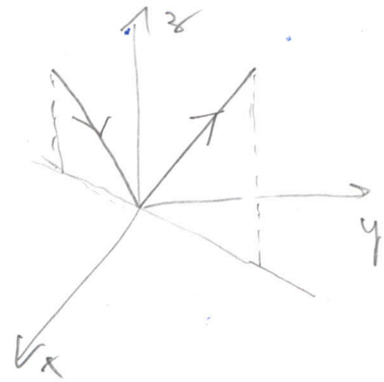
---

19. $\begin{cases} z = \sqrt{x^2+y^2} & \rightarrow \text{cone com } z \geq 0 \\ y = x & \rightarrow \text{plano} \end{cases}$

$z = \sqrt{2x^2} = \sqrt{2}\,|x|$ $\rightarrow$ 2 retas

$\begin{cases} x = t \\ y = t \\ z = \sqrt{2}\,|t| \end{cases}$ $\qquad -\infty < t < +\infty$





$$\boxed{\begin{gathered}\vec{r}(t) = t\,\hat{\imath} + t\,\hat{\jmath} + \sqrt{2}\,|t|\,\hat{k} \\ t \in \mathbb{R}\end{gathered}}$$

20.

$$\begin{cases} \vec{r}(t) = t\,\hat{i} + t^2\,\hat{j} + t^3\,\hat{k} \\ t \in \mathbb{R} \end{cases}$$

$\vec{r}(t)$ suave $\Leftrightarrow$

i) $\vec{r}\,'(t)$ é contínua

ii) $\vec{r}\,'(t_0) \neq 0$

$\vec{r}\,'(t) \neq 0$

$$\begin{cases} \vec{r}\,'(t) = \hat{i} + 2t\,\hat{j} + 3t^2\,\hat{k} \\ t \in \mathbb{R} \end{cases}$$

Temos que

$\vec{r}\,'(t)$ é contínua $\forall t$

(satisfaz (i))

A condição (ii) também se verifica

$\therefore$

$$\begin{cases} \vec{r}(t) = t\,\hat{i} + t^2\,\hat{j} + t^3\,\hat{k} \\ t \in \mathbb{R} \\ \text{é } \underline{\text{parametrização}} \\ \underline{\text{suave}} \end{cases}$$

21.

$\vec{r}(t) = |t|\,\hat{i} + t\,\hat{j} + t\,\hat{k}$

$t \in \mathbb{R}$

$$\vec{r}\,'(t) = \begin{cases} -\hat{i} + \hat{j} + \hat{k}, & t<0 \\ \hat{i} + \hat{j} + \hat{k}, & t>0 \\ \nexists & t=0 \end{cases}$$

$\vec{r}\,'(t)$ é descontínua em $t=0$ logo

$\vec{r}\,'(t)$ é contínua por partes em $I = (-\infty, 0] \cup [0, +\infty)$

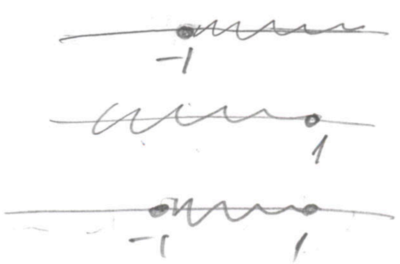
22.

$\vec{r}(t) = (1+t)^{3/2}\,\hat{i} + (1-t)^{3/2}\,\hat{j} + \frac{3t}{2}\,\hat{k}$

Dom $\vec{r}(t)$:

$1+t \geq 0 \rightarrow t \geq -1$

$(1-t) \geq 0 \rightarrow 1 \geq t$



Dom $\vec{r}(t) = [-1, 1]$

$\vec{r}\,'(t) = \frac{3}{2}(1+t)^{1/2}\,\hat{i} - \frac{3}{2}(1-t)^{1/2}\,\hat{j} + \frac{3}{2}\,\hat{k}$

$\vec{r}\,'(t)$ é contínua em $(-1, 1)$

$\vec{r}\,'(t)$ ...

23.

$\vec{r}(t) = \cos^2 t\,\hat{i} + \sin^2 t\,\hat{j} + t^2\,\hat{k}$

$\vec{r}\,'(t) = -2\cos t \sin t\,\hat{i} + 2\sin t \cos t\,\hat{j} + 2t\,\hat{k}$

$\vec{r}\,'(t) = -\sin 2t\,\hat{i} + \sin 2t\,\hat{j} + 2t\,\hat{k}$

$\vec{r}\,'(t)$ é contínua, $\forall t$.

$\vec{r}\,'(t) = \vec{0} \Rightarrow \begin{cases} \sin 2t = 0 \\ 2t = 0 \end{cases}$

$\therefore\ t = 0$

Aqui temos que $\vec{r}(t)$ é suave por partes onde se tem

$I = (-\infty, 0] \cup [0, +\infty)$

---

24. $\begin{cases} \vec{r}(t) = (e^t - t)\,\hat{i} + t^2\,\hat{j} + t^3\,\hat{k} \\ t \in \mathbb{R} \end{cases}$

$\vec{r}\,'(t) = (e^t - 1)\,\hat{i} + 2t\,\hat{j} + 3t^2\,\hat{k}$

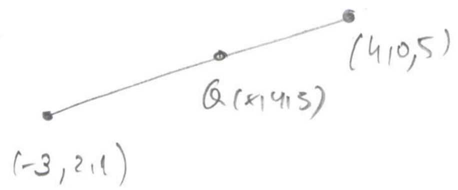
$\vec{r}\,'(t)$ é contínua $\forall t$.

$\vec{r}\,'(t) = 0 \Rightarrow (e^t - 1)\,\hat{i} + 2t\,\hat{j} + 3t^2\,\hat{k} = 0$

$\Rightarrow t = 0$

$\boxed{\vec{r}(t) \text{ é suave por partes } I = (-\infty, 0] \cup [0, +\infty)}$

25.



$\vec{v} = (7, -2, 4)$

$x + 3 = 7t$

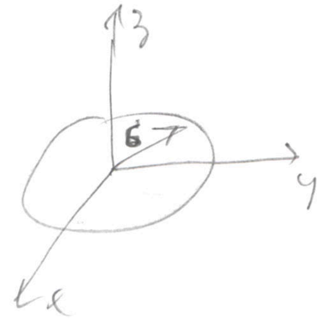
$y - 2 = -2t$

$z - 1 = 4t$

$\therefore$ $\begin{cases} x = -3 + 7t \\ y = 2 - 2t \\ z = 1 + 4t \end{cases}, \ t \in \mathbb{R}$

$\vec{r}(t) = (-3+7t)\,\hat{i} + (2-2t)\,\hat{j} + (1+4t)\,k$

26.



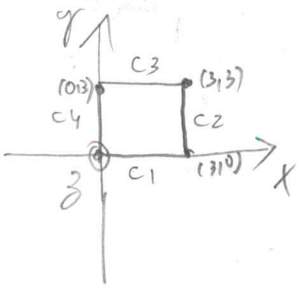
$x = 6\cos t$

$y = 6\sin t$

$\vec{r}(t) = 6\cos t\,\hat{i} + 6\sin t\,\hat{j}$

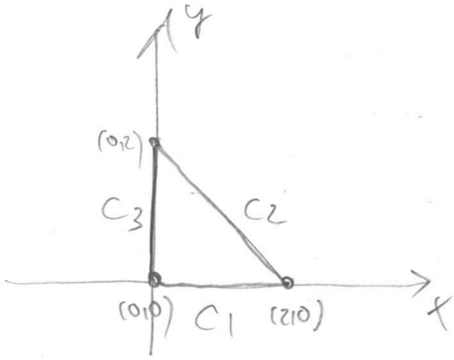
$0 \le t \le 2\pi$

27.



$$\vec{r}(t) = \begin{cases} t\hat{i} \, ; & 0 \le t \le 3 \\ 3\hat{i} + (t-3)\hat{j}, & 3 \le t \le 6 \\ (9-t)\hat{i} + 3\hat{j}, & 6 \le t \le 9 \\ (12-t)\hat{j} & 9 \le t \le 12 \end{cases}$$

28.



C_1: $t\hat{i}$; $0 \le t \le 2$

C_2: $y = 2 - x$

$\underbrace{(4-t)}_{x}\hat{i} + (t-2)\hat{j}$; $2 \le t \le 4$

$2 - x = 2 - (4-t)$
$= -2 + t$

C_3: $(6-t)\hat{j}$ $\quad 4 \le t \le 6$

$\therefore$

$$\vec{r}(t) = \begin{cases} t\hat{i} \, ; & 0 \le t \le 2 \\ (4-t)\hat{i} + (t-2)\hat{j} \, ; & 2 \le t \le 4 \\ (6-t)\hat{j} \, ; & 4 \le t \le 6 \end{cases}$$

29. $\vec{r}(t) = \cos^3 t \, \hat{i} + \sin^3 t \, \hat{j}$
$t \in [0, 2\pi]$

$$L = \int_0^{2\pi} |\vec{r}\,'(t)| \, dt$$

$$\vec{r}\,'(t) = -3\cos^2 t \sin t \, \hat{i} + 3\sin^2 t \cos t \, \hat{j}$$

$$|\vec{r}\,'| = \sqrt{9\cos^4 t \sin^2 t + 9\sin^4 t \cos^2 t}$$

$$= \sqrt{9\cos^2 t \sin^2 t \underbrace{(\cos^2 t + \sin^2 t)}_{1}}$$

$$= \sqrt{9(\cos t \sin t)^2}$$

$$= \sqrt{9\left(\frac{\sin 2t}{2}\right)^2}$$

$$= \frac{3}{2}\sqrt{\sin^2 2t}$$

$$= \frac{3}{2}|\sin 2t|$$

$$\therefore \int_0^{2\pi} \frac{3}{2}|\sin 2t| \, dt$$

$\sin 2t \quad ; \quad 0 \le t \le 2\pi$

$0 \le 2t \le 4\pi$

$\sin 2t \geq 0 \; : \; 0 \le 2t \le \pi \Leftrightarrow 0 \le t \le \frac{\pi}{2}$

$2\pi \le 2t \le 3\pi$

$\pi \le t \le \frac{3\pi}{2}$

ισχύει

$$|\sin 2t| = \begin{cases} \sin 2t, & 0 \le t \le \frac{\pi}{2} \\ -\sin 2t, & \frac{\pi}{2} \le t \le \pi \\ \sin 2t, & \pi \le t \le \frac{3\pi}{2} \\ -\sin 2t, & \frac{3\pi}{2} \le t \le 2\pi \end{cases}$$

$\therefore$

$$\frac{3}{2}\int_0^{2\pi} |\sin 2t| \, dt =$$

$$= \underbrace{\frac{3}{2}\int_0^{\pi/2} \sin 2t \, dt}_{①} - \underbrace{\frac{3}{2}\int_{\frac{\pi}{2}}^{\pi} \sin 2t \, dt}_{②} + \underbrace{\frac{3}{2}\int_{\pi}^{\frac{3\pi}{2}} \sin 2t \, dt}_{③} - \underbrace{\frac{3}{2}\int_{3\pi/2}^{2\pi} \sin 2t \, dt}_{④}$$

$$① = \frac{3}{2}\int_0^{\pi/2} \sin 2t \, dt$$

$$= \frac{3}{2} \frac{-\cos 2t}{2} \Big]_0^{\pi/2}$$

$$= -\frac{3}{4}\left(\cos \pi - \cos 0\right)$$

$$= -\frac{3}{4}(-1-1)$$

$$= \frac{3}{2}$$

$$② = -\frac{3}{2}\int_{\frac{\pi}{2}}^{\pi} \sin 2t \, dt$$

$$= -\frac{3}{2}\left(-\frac{\cos 2t}{2}\right)\Big]_{\frac{\pi}{2}}^{\pi}$$

$$= \frac{3}{4}\left(\cos 2\pi - \cos \pi\right)$$

$$= \frac{3}{4}\left(1 - (-1)\right)$$

$$= \frac{6}{4} = \frac{3}{2}$$

$$③ = \frac{3}{2}\int_{\pi}^{\frac{3\pi}{2}} \sin 2t \, dt$$

$$= -\frac{3}{4} \cos 2t \Big]_{\pi}^{\frac{3\pi}{2}}$$

$$= -\frac{3}{4}\left(\cos 3\pi - \cos 2\pi\right)$$

$$= -\frac{3}{4}(-1-1) = \frac{3}{2}$$

(4) $= -\frac{3}{2}\int_{\frac{3\pi}{2}}^{2\pi} \sin 2t \, dt$

$= -\frac{3}{2} \left. \frac{(-)\cos 2t}{2} \right]_{\frac{3\pi}{2}}^{2\pi}$

$= \frac{3}{4}\left(\cos 4\pi - \cos 3\pi\right)$

$= \frac{3}{4}\left(1 - (-1)\right)$

$= \frac{3}{2}$

$\therefore$

$\frac{3}{2}\int_0^{2\pi} |\sin 2t| \, dt =$

$= \frac{3}{2} + \frac{3}{2} + \frac{3}{2} + \frac{3}{2}$

$= 6$ //

---

30. $\begin{cases} \vec{r}(t) = 2t\,\hat{i} + t^2\hat{j} + \ln t\,\hat{k} \\ 1 \le t \le 2 \end{cases}$

$L = \int_1^2 |\vec{r}\,'(t)| \, dt$

$\vec{r}\,' = 2\hat{i} + 2t\,\hat{j} + \frac{1}{t}\hat{k}$

$|\vec{r}\,'| = \sqrt{4 + 4t^2 + \frac{1}{t^2}}$

$L = \int_1^2 \sqrt{\frac{4t^4 + 4t^2 + 1}{t^2}} \, dt$

$= \int_1^2 \frac{1}{t}\sqrt{4t^4 + 4t^2 + 1} \, dt$

$= \int_1^2 \frac{1}{t}\sqrt{(2t^2+1)^2} \, dt$

$= \int_1^2 \frac{1}{t}(2t^2+1) \, dt$

$= \int_1^2 \left(2t + \frac{1}{t}\right) dt$

$= t^2 + \ln t \Big]_1^2$

$= 4 + \ln 2 - (1 + \ln 1)$

$= 4 + \ln 2 - 1$

$= 3 + \ln 2$

31.

$$\vec{r}(t) = \frac{1}{3}(1+t)^{3/2}\hat{i} + \frac{1}{3}(1-t)^{3/2}\hat{j} + \frac{1}{2}t\hat{k}$$

$$-1 \le t \le 1$$

$$\vec{r}'(t) = \frac{1}{2}(1+t)^{\frac{1}{2}}\hat{i} - \frac{1}{2}(1-t)^{\frac{1}{2}}\hat{j} + \frac{1}{2}\hat{k}$$

$$|\vec{r}'(t)| = \sqrt{\frac{1}{4}(1+\cancel{t}) + \frac{1}{4}(1-\cancel{t}) + \frac{1}{4}}$$

$$= \frac{\sqrt{3}}{2}$$

$$L = \int_{-1}^{1} |\vec{r}(t)|\, dt$$

$$= \frac{\sqrt{3}}{2}(1-(-1))$$

$$= \sqrt{3}$$

32.

$$\vec{r}(t) = e^{t}\hat{i} + e^{-t}\hat{j} + \sqrt{2}t\,\hat{k}$$

$$0 \le t \le 1$$

$$\vec{r}'(t) = e^{t}\hat{i} - e^{-t}\hat{j} + \sqrt{2}\,\hat{k}$$

$$|\vec{r}'| = \sqrt{e^{2t} + e^{-2t} + 2}$$

$$= \sqrt{(e^{t} + e^{-t})^2}$$

$$= e^{t} + e^{-t}$$

$$L = \int_0^1 |\vec{r}(t)|\, dt$$

$$= \int_0^1 (e^{t} + e^{-t})\, dt$$

$$= e^{t} - e^{-t}\Big]_0^1$$

$$= e - e^{-1} - (\cancel{1-1})^{0}$$

$$= e - \frac{1}{e}$$

33.

$$\vec{r}(t) = 2(t^2-1)^{3/2}\hat{i} + 3t^2\hat{j} + 3t^2\hat{k}$$

$$0 \le t \le \sqrt{8}$$

$$\vec{r}' = 6t(t^2-1)^{1/2}\hat{i} + 6t\hat{j} + 6t\hat{k}$$

$$|\vec{r}'| = \sqrt{36t^2(t^2-1) + 36t^2 + 36t^2}$$

$$= \sqrt{36t^4 + 36t^2}$$

$$= 6t\sqrt{t^2+1}$$

$$L = \int_0^{\sqrt{8}} 6t\sqrt{t^2+1}\, dt$$

$$= 2(t^2+1)^{3/2}\Big]_0^{\sqrt{8}}$$

$$= 2(8+1)^{3/2} - 2 =$$

$$= 54 - 2 = 52$$

34.

$\vec{r}(t) = (t^2+4)\hat{i} + 2t\hat{j}$

$\vec{r}'(t) = 2t\hat{i} + 2\hat{j}$

$|\vec{r}'(t)| = \sqrt{4t^2+4} = 2\sqrt{t^2+1}$

$$\vec{T}(t) = \frac{\vec{r}'(t)}{|\vec{r}(t)|} = \frac{2t\hat{i}+2\hat{j}}{2\sqrt{t^2+1}}$$

$$\boxed{\vec{T}(t) = \frac{t}{\sqrt{t^2+1}}\hat{i} + \frac{\hat{j}}{\sqrt{t^2+1}}}$$

$$\vec{T}'(t) = \left(\frac{t}{\sqrt{t^2+1}}\right)'\hat{i} + \left(\frac{1}{\sqrt{t^2+1}}\right)'\hat{j}$$

$$= \left(\frac{1}{\sqrt{t^2+1}} - \frac{t^2}{(t^2+1)^{3/2}}\right)\hat{i} +$$

$$+ -\frac{t}{(t^2+1)^{3/2}}\hat{j}$$

$$= \frac{1}{(t^2+1)^{3/2}}\hat{i} - \frac{t}{(t^2+1)^{3/2}}\hat{j}$$

$$= \frac{\hat{i}-t\hat{j}}{(t^2+1)^{3/2}}$$

$$|\vec{T}'| = \sqrt{\frac{1}{(t^2+1)^3} + \frac{t^2}{(t^2+1)^3}}$$

$$= \sqrt{\frac{t^2+1}{(t^2+1)^3}} = \frac{1}{\dots}$$

$$\vec{N}(t) = \frac{\vec{T}'(t)}{|\vec{T}'(t)|}$$

$$= \frac{\frac{\hat{i}-t\hat{j}}{(t^2+1)^{3/2}}}{\frac{1}{(t^2+1)}}$$

$$\boxed{\vec{N}(t) = \frac{\hat{i}-t\hat{j}}{\sqrt{t^2+1}}}$$

$$\kappa(t) = \frac{|\vec{T}'(t)|}{|\vec{r}'(t)|}$$

$$= \frac{\frac{1}{(t^2+1)}}{2\sqrt{t^2+1}}$$

$$\boxed{\kappa(t) = \frac{1}{2(t^2+1)^{3/2}}}$$

35.

$\vec{r}(t) = \cos t\,\hat{i} + \cos t\,\hat{j} + \sqrt{2}\sin t\,\hat{k}$

$\vec{r}\,'(t) = -\sin t\,\hat{i} - \sin t\,\hat{j} + \sqrt{2}\cos t\,\hat{k}$

$|\vec{r}\,'| = \sqrt{\sin^2 t + \sin^2 t + 2\cos^2 t}$

$= \sqrt{2}$

$\vec{T}(t) = \dfrac{\vec{r}\,'(t)}{|\vec{r}\,'(t)|} =$

$$\boxed{\vec{T}(t) = -\frac{\sin t}{\sqrt{2}}\,\hat{i} - \frac{\sin t}{\sqrt{2}}\,\hat{j} + \cos t\,\hat{k}}$$

$\vec{N}(t) = \vec{T}\,'(t) \big/ |\vec{T}\,'(t)|$

$\vec{T}\,'(t) = -\dfrac{\cos t}{\sqrt{2}}\,\hat{i} - \dfrac{\cos t}{\sqrt{2}}\,\hat{j} - \sin t\,\hat{k}$

$|\vec{T}\,'(t)| = \sqrt{\dfrac{\cos^2 t}{2} + \dfrac{\cos^2 t}{2} + \sin^2 t}$

$= 1$

$$\boxed{\vec{N}(t) = -\frac{\cos t}{\sqrt{2}}\,\hat{i} - \frac{\cos t}{\sqrt{2}}\,\hat{j} - \sin t\,\hat{k}}$$

$\kappa(t) = \dfrac{|\vec{T}\,'(t)|}{|\vec{r}\,'(t)|} = \dfrac{1}{\sqrt{2}}$

$$\boxed{\kappa(t) = \frac{1}{\sqrt{2}}}$$

36.

$$\vec{r}(t) = 2t\,\hat{i} + t^2\,\hat{j} + \frac{1}{3}t^3\,\hat{k}$$

$$\vec{r}\,'(t) = 2\,\hat{i} + 2t\,\hat{j} + t^2\,\hat{k}$$

$$|\vec{r}\,'| = \sqrt{4 + 4t^2 + t^4} = (t^2+2)$$

$$\vec{T}(t) = \frac{\vec{r}\,'(t)}{|\vec{r}(t)|}$$

$$\boxed{\vec{T}(t) = \frac{2}{t^2+2}\,\hat{i} + \frac{2t}{t^2+2}\,\hat{j} + \frac{t^2}{t^2+2}\,\hat{k}}$$

$$\vec{N}(t) = \frac{\vec{T}\,'(t)}{|\vec{T}\,'(t)|}$$

$$\vec{T}\,'(t) = \frac{-4t}{(t^2+2)^2}\,\hat{i} + \left(\frac{2}{t^2+2} - \frac{4t^2}{(t^2+2)^2}\right)\hat{j} + \left(\frac{2t}{t^2+2} - \frac{2t^3}{(t^2+2)^2}\right)\hat{k}$$

$$= \frac{-4t}{(t^2+2)^2}\,\hat{i} + \frac{2(t^2+2) - 4t^2}{(t^2+2)^2}\,\hat{j} + \frac{2t(t^2+2) - 2t^3}{(t^2+2)^2}\,\hat{k}$$

$$\vec{T}\,'(t) = \frac{-4t}{(t^2+2)^2}\,\hat{i} + \frac{4-2t^2}{(t^2+2)^2}\,\hat{j} + \frac{4t}{(t^2+2)^2}\,\hat{k}$$

$$|\vec{T}\,'(t)| = \left\{\frac{\cancel{16t^2}}{(t^2+2)^4} + \frac{16 - \cancel{16t^2} + 4t^4}{(t^2+2)^4} + \frac{16t^2}{(t^2+2)^4}\right\}^{1/2}$$

$$= \left\{\frac{16t^2 + 16 + 4t^4}{(t^2+2)^4}\right\}^{1/2}$$

$$= \left\{\frac{(2t^2+4)^2}{(t^2+2)^4}\right\}^{1/2}$$

$$= \left\{\frac{4\cancel{(t^2+2)^2}}{(t^2+2)^{\cancel{4}\,2}}\right\}^{1/2}$$

$$= \left\{\frac{4}{(t^2+2)^2}\right\}^{1/2} = \frac{2}{(t^2+2)}$$

$\therefore$

$$\vec{N}(t) = \frac{\vec{T}\,'(t)}{|\vec{T}\,'(t)|} =$$

$$= \frac{\dfrac{-4t}{(t^2+2)^{\cancel{2}}}}{\dfrac{2}{\cancel{(t^2+2)}}}\,\hat{i} + \frac{\dfrac{4-2t^2}{(t^2+2)^{\cancel{2}}}}{\dfrac{2}{\cancel{(t^2+2)}}}\,\hat{j} + \frac{4t}{(t^2+2)^2}\,\hat{k}$$

$$\boxed{\vec{N}(t) = \frac{-2t}{(t^2+2)}\hat{i} + \frac{2-t^2}{(t^2+2)}\hat{j} + \frac{+2t}{(t^2+2)}\hat{k}}$$

$$k(t) = \frac{|\vec{T}'(t)|}{|\vec{r}'(t)|}$$

$$= \frac{\frac{2}{(t^2+2)}}{(t^2+2)}$$

$$\boxed{k(t) = \frac{2}{(t^2+2)^2}}$$

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37. $r(t) = e^t\hat{i} + e^{-t}\hat{j} + \sqrt{2}t\hat{k}$

$$\vec{r}'(t) = e^t\hat{i} - e^{-t}\hat{j} + \sqrt{2}\hat{k}$$

$$|\vec{r}'(t)| = \sqrt{e^{2t} + e^{-2t} + 2}$$

$$= (e^t + e^{-t})$$

$$\vec{T}(t) = \vec{r}'(t) / |\vec{r}'(t)|$$

$$\boxed{\vec{T}(t) = \frac{e^t\hat{i} - e^{-t}\hat{j} + \sqrt{2}\hat{k}}{e^t + e^{-t}}}$$

$$\vec{N}(t) = \frac{\vec{T}'(t)}{|\vec{T}'(t)|}$$

$$\vec{T}(t) = \frac{e^t}{e^t+e^{-t}}\hat{i} - \frac{e^{-t}}{e^t+e^{-t}}\hat{j} + \frac{\sqrt{2}}{e^t+e^{-t}}\hat{k}$$

$$\Downarrow$$

$$\left(\frac{e^t}{e^t+e^{-t}}\right)' = \frac{e^t}{e^t+e^{-t}} - \frac{e^t(e^t-e^{-t})}{(e^t+e^{-t})^2}$$

$$= \frac{e^t(e^t+e^{-t}) - e^t(e^t-e^{-t})}{(e^t+e^{-t})^2}$$

$$= \frac{\cancel{e^{2t}}+1-\cancel{e^{2t}}+1}{(e^t+e^{-t})^2}$$

$$= \frac{2}{(e^t+e^{-t})^2}$$

$$\left(\frac{1}{e^t+e^{-t}}\right)' = \frac{-1(e^t-e^{-t})}{(e^t+e^{-t})^2}$$

37. Cont.

$$\left(\frac{e^{-t}}{e^{t}+e^{-t}}\right)' = \frac{-e^{-t}}{e^{t}+e^{-t}} - \frac{e^{-t}\,(e^{t}-e^{-t})}{(e^{t}+e^{-t})^2}$$

$$= \frac{-e^{-t}(e^{t}+e^{-t}) - e^{-t}(e^{t}-e^{-t})}{(e^{t}+e^{-t})^2}$$

$$= \frac{-1-\cancel{e^{-2t}}-1+\cancel{e^{-2t}}}{(e^{t}+e^{-t})^2}$$

$$= \frac{-2}{(e^{t}+e^{-t})^2}$$

$\therefore$

$$\vec{T}'(t) = \frac{2}{(e^{t}+e^{-t})^2}\,\hat{i} + \frac{2}{(e^{t}+e^{-t})^2}\,\hat{j} - \frac{(e^{t}-e^{-t})\sqrt{2}}{(e^{t}+e^{-t})^2}\,\hat{k}$$

$$|\vec{T}'(t)| = \left\{\frac{4}{(e^{t}+e^{-t})^4} + \frac{4}{(e^{t}+e^{-t})^4} + \frac{(e^{t}-e^{-t})^2\,2}{(e^{t}+e^{-t})^4}\right\}^{1/2}$$

$$|\vec{T}'(t)| = \left\{\frac{8}{(e^{t}+e^{-t})^4} + \frac{(e^{t}-e^{-t})^2\,2}{(e^{t}+e^{-t})^4}\right\}^{1/2}$$

$$= \left\{\frac{8+2e^{2t}-4+2e^{-2t}}{(e^{t}+e^{-t})^4}\right\}^{1/2}$$

$$= \left\{\frac{2e^{2t}+4+2e^{-2t}}{(e^{t}+e^{-t})^4}\right\}^{1/2}$$

$$= \left\{2\,\frac{(e^{2t}+2+e^{-2t})}{(e^{t}+e^{-t})^4}\right\}^{1/2}$$

$$= \left\{\frac{2\,\cancel{(e^{t}+e^{-t})^2}}{(e^{t}+e^{-t})^{\cancel{4}\,2}}\right\}^{1/2}$$

$$= \frac{\sqrt{2}}{e^{t}+e^{-t}}$$

$\therefore$

$$\vec{N}(t) = \vec{T}'(t)\,/\,|\vec{T}'(t)|$$

$$= \frac{\dfrac{2}{(e^{t}+e^{-t})^{\cancel{2}}}}{\dfrac{\sqrt{2}}{\cancel{e^{t}+e^{-t}}}}\,\hat{i} + \frac{\dfrac{2}{(e^{t}+e^{-t})^{\cancel{2}}}}{\dfrac{\sqrt{2}}{\cancel{e^{t}+e^{-t}}}}\,\hat{j} - \frac{\dfrac{(e^{t}-e^{-t})\sqrt{2}}{(e^{t}+e^{-t})^{\cancel{2}}}}{\dfrac{\sqrt{2}}{\cancel{e^{t}+e^{-t}}}}\,\hat{k}$$

$$\boxed{\vec{N}(t) = \frac{\sqrt{2}}{(e^{t}+e^{-t})}\hat{i} + \frac{\sqrt{2}}{(e^{t}+e^{-t})}\hat{j} - \frac{e^{t}-e^{-t}}{e^{t}+e^{-t}}\hat{k}}$$

$$K(t) = \frac{|\vec{T}'(t)|}{|\vec{r}'(t)|} = \frac{\frac{\sqrt{2}}{e^{t}+e^{-t}}}{\frac{e^{t}+e^{-t}}{1}}$$

$$\boxed{K(t) = \sqrt{2}/(e^{t}+e^{-t})^2}$$

38.
$$\begin{cases} \vec{r}(t) = 2t\,\hat{i} + t^2\hat{j} + \ln t\,\hat{k} \\ t \geq 0 \end{cases}$$

$$\vec{r}'(t) = 2\hat{i} + 2t\hat{j} + \frac{1}{t}\hat{k}$$

$$|\vec{r}'(t)| = \sqrt{4+4t^2+\frac{1}{t^2}} = \sqrt{\frac{4t^2+4t^4+1}{t^2}} = \frac{\sqrt{(2t^2+1)^2}}{t} = \frac{2t^2+1}{t}$$

$$\vec{T}(t) = \vec{r}'(t)/|\vec{r}'(t)|$$

$$= \frac{2}{\frac{2t^2+1}{t}}\hat{i} + \frac{2t}{\frac{2t^2+1}{t}}\hat{j} + \frac{\frac{1}{\cancel{t}}}{\frac{2t^2+1}{\cancel{t}}}\hat{k}$$

$$\boxed{\vec{T}(t) = \frac{2t}{2t^2+1}\hat{i} + \frac{2t^2}{2t^2+1}\hat{j} + \frac{1}{2t^2+1}\hat{k}}$$

$$\vec{T}'(t) = \left(\frac{2}{2t^2+1} - \frac{8t^2}{(2t^2+1)^2}\right)\hat{i} + \left(\frac{4t}{2t^2+1} - \frac{8t^3}{(2t^2+1)^2}\right)\hat{j} - \frac{4t}{(2t^2+1)^2}\hat{k}$$

$$= \frac{2(2t^2+1)-8t^2}{(2t^2+1)^2}\hat{i} + \frac{4t(2t^2+1)-8t^3}{(2t^2+1)^2}\hat{j} - \frac{4t}{(2t^2+1)^2}\hat{k}$$

$$\vec{T}'(t) = \frac{2-4t^2}{(2t^2+1)^2}\,\hat{\imath} + \frac{4t}{(2t^2+1)^2}\,\hat{\jmath} - \frac{4t}{(2t^2+1)^2}\,\hat{k}$$

$$|\vec{T}'(t)| = \left\{ \frac{4 - \cancel{16t^2} + 16t^4}{(2t^2+1)^4} + \frac{\cancel{16t^2}}{(2t^2+1)^4} + \frac{16t^2}{(2t^2+1)^4} \right\}^{1/2}$$

$$= \left(\frac{16t^4 + 16t^2 + 4}{(2t^2+1)^4} \right)^{1/2}$$

$$= \frac{(4t^2+2)}{(2t^2+1)^2}$$

$$= \frac{2\,\cancel{(2t^2+1)}}{(2t^2+1)^{\cancel{2}}}$$

$$= \frac{2}{2t^2+1}$$

$$\vec{N}(t) = \frac{\dfrac{2-4t^2}{(2t^2+1)^{\cancel{2}}}}{\dfrac{2}{\cancel{2t^2+1}}}\,\hat{\imath} + \frac{\dfrac{4t}{(2t^2+1)^{\cancel{2}}}}{\dfrac{2}{\cancel{2t^2+1}}}\,\hat{\jmath} - \frac{\dfrac{\cancel{4t}}{(2t^2+1)^2}}{\dfrac{\cancel{2}}{2t^2+1}}\,\hat{k}$$

$$\boxed{\vec{N}(t) = \frac{1-2t^2}{2t^2+1}\,\hat{\imath} + \frac{2t}{2t^2+1}\,\hat{\jmath} - \frac{2t}{2t^2+1}\,\hat{k}}$$

$$K(t) = |\vec{T}'(t)| \,/\, |\vec{r}'(t)|$$

$$= \frac{\dfrac{2}{2t^2+1}}{\dfrac{2t^2+1}{t}} = \frac{2t}{(2t^2+1)^2}$$