Cálculo C - Lista 7

Integrais de Superfície

Calcule $\int_\Omega g(x,y,z)dS$

1. $\int_\Omega x\,dS$ onde Ω é parte do plano $2x+3y+z=6$ situado no primeiro octante.

2. $\int_\Omega \sqrt{4x^2+4y^2+1}\,dS$ onde Ω é parte do parabolóide $z=x^2+y^2$ abaixo do plano $y=z$.

3. $\int_\Omega z(x^2+y^2)\,dS$ onde Ω é o hemisfério $\sqrt{x^2+y^2+z^2}=2$. , $z \geq 0$

4. $\int_\Omega \sqrt{4y+1}\,dS$ onde Ω é parte da superfície $y=x^2$ no primeiro octante e cortada pelo plano $2x+y+z=1$.

5. Use integral de superfície para determinar a área de um cone circular reto de raio R e altura h.

6. Calcule a integral de superfície $\int_\Omega (x^2-y^2)\,dS$ onde Ω é o hemisfério $z=\sqrt{9-x^2-y^2}$. Use coordenadas esféricas para parametrizar a superfície Ω.

Nos exercícios a seguir calcule $\int_\Omega \vec{F}\cdot d\vec{S}$

7. $\vec{F}=y\vec{i}-x\vec{j}+8\vec{k}$ e Ω é parte do parabolóide $z=9-x^2-y^2$ acima do plano xy e orientada com vetor normal $\vec{n}$ apontando para cima.

8. $\vec{F}=x\vec{i}+y\vec{j}+z\vec{k}$ e Ω é o cubo com vértices $(0,0,0),(1,0,0),(1,1,0),(0,1,0),(0,0,1)$, $(1,0,1),(1,1,1)$ e $(0,1,1)$ e orientada com vetor normal apontando para ~~cima~~. fora

9. $\vec{F}=yz^2\vec{i}+ye^x\vec{j}+x\vec{k}$ e Ω é a superfície $y=x^2$ com $0\le y\le 4$, $0\le z\le 1$ e orientada com vetor normal com componente y positiva.

10. $\vec{F}=yx\vec{i}+y^2\vec{j}+yz\vec{k}$ e Ω é o elipsóide $x^2+\frac{y^2}{4}+z^2=1$ orientado com vetor normal apontando para fora.

Respostas

1. $3\sqrt{14}$
2. $\frac{5\pi}{8}$
3. 16π
4. $(-61+44\sqrt{2})/5$
6. 0
7. 72π
8. 3
9. $2e^2-10e^{-2}$
10. 0

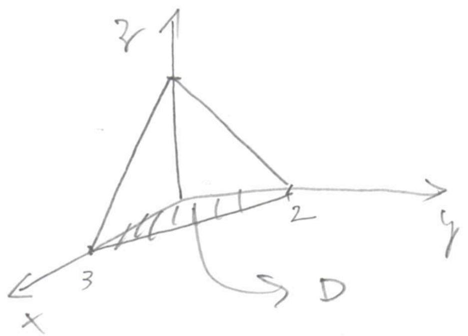
Cálculo C - Lista 7

1. $\int_\Omega x \, dS$; $\Omega = \begin{cases} 2x+3y+z=6 \\ \text{no primeiro octante} \end{cases}$

$z = 6 - 2x - 3y$

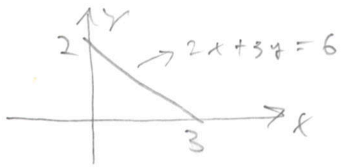
$dS = \sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2}\, dx\,dy = \sqrt{1 + (-2)^2 + (-3)^2}\, dx\,dy$

$= \sqrt{14}\, dx\,dy$



$z=0$: $2x+3y=6$

$x=0$: $3y+z=6$



$\therefore \int_\Omega x\, dS = \int_D x \sqrt{14}\, dx\,dy = \int_{x=0}^{3} dx \int_{y=0}^{\frac{6-2x}{3}} dy \sqrt{14}\, x$

$= \int_{x=0}^{3} dx \left. \sqrt{14}\, x\, y \right]_0^{\frac{6-2x}{3}}$

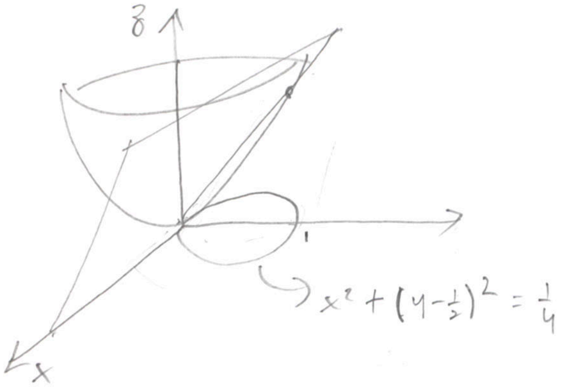
$= \int_{x=0}^{3} dx \sqrt{14}\, x \left(\frac{6-2x}{3}\right) = \int_{x=0}^{3} dx \frac{\sqrt{14}}{3}(6x - 2x^2)$

$= \left. \frac{\sqrt{14}}{3}\left(\frac{6x^2}{2} - \frac{2x^3}{3}\right) \right]_0^3 = \left. \frac{\sqrt{14}}{3}\left(3x^2 - \frac{2x^3}{3}\right) \right]_0^3$

$= \frac{\sqrt{14}}{3}\left(3 \cdot 9 - \frac{2}{3} 3^3\right) = \frac{\sqrt{14}}{3}(27 - 18)$

$= \frac{\sqrt{14}}{\cancel{3}} \cancel{9} = 3\sqrt{14}$

2. $\int_\Omega \sqrt{4x^2+4y^2+1}\, dS \quad ; \quad \begin{cases} \Omega : z = x^2+y^2 \\ \text{abaixo do plano } y = z \end{cases}$



$$\left.\begin{array}{l} z = x^2+y^2 \\ y = z \end{array}\right\} \therefore \quad y = x^2+y^2$$

$$x^2+y^2-y = 0$$

$$x^2+\left(y-\frac{1}{2}\right)^2 - \frac{1}{4} = 0$$

$$x^2+\left(y-\frac{1}{2}\right)^2 = \frac{1}{4}$$

$z = z(x,y)$

$$dS = \sqrt{1+\left(\frac{\partial z}{\partial x}\right)^2+\left(\frac{\partial z}{\partial y}\right)^2}\, dx\,dy = \sqrt{1+(2x)^2+(2y)^2}\, dx\,dy$$

$$= \sqrt{1+4x^2+4y^2}\, dx\,dy$$

$$\int_\Omega \sqrt{4x^2+4y^2+1}\, dS = \int_D \sqrt{4x^2+4y^2+1}\,\sqrt{1+4x^2+4y^2}\, dx\,dy$$

$$= \int_D (1+4x^2+4y^2)\, dx\,dy \quad (*)$$

$x^2+(y-\frac{1}{2})^2 = \frac{1}{4}$

coordenadas polares:

$$\begin{cases} x = r\cos\theta & 0 \le r \le \frac{1}{2} \\ y-\frac{1}{2} = r\sin\theta & 0 \le \theta \le 2\pi \end{cases}$$

$$\frac{\partial(x,y)}{\partial(r,\theta)} = r$$

$$(*) = \int_{r=0}^{\frac{1}{2}} \int_{\theta=0}^{2\pi} \left(1 + 4r^2\cos^2\theta + 4\left(\tfrac{1}{2} + r\sin\theta\right)^2\right) r\,dr\,d\theta$$

$$= \int_{r=0}^{\frac{1}{2}} \int_{\theta=0}^{2\pi} \left[1 + 4r^2\cos^2\theta + 4\left(\tfrac{1}{4} + r\sin\theta + r^2\sin^2\theta\right)\right] r\,dr\,d\theta$$

$$= \int_{r=0}^{\frac{1}{2}} \int_{\theta=0}^{2\pi} \left[1 + \underbrace{4r^2\cos^2\theta} + 1 + 4r\sin\theta + \underbrace{4r^2\sin^2\theta}\right] r\,dr\,d\theta$$

$$= \int_{r=0}^{\frac{1}{2}} \int_{\theta=0}^{2\pi} \left(2 + \underbrace{4r^2} + 4r\sin\theta\right) r\,dr\,d\theta$$

$$= \int_{r=0}^{\frac{1}{2}} \left(2\theta + 4r^2\theta - 4r\cos\theta\right)\Big]_0^{2\pi} r\,dr$$

$$= \int_{r=0}^{\frac{1}{2}} \left[4\pi + 8\pi r^2 - 4r(\cancel{\cos 2\pi} - \cancel{\cos 0})\right] r\,dr$$

$$= \int_{r=0}^{\frac{1}{2}} \left(4\pi r + 8\pi r^3\right) dr$$

$$= 4\pi\frac{r^2}{2} + 8\pi\frac{r^4}{4}\Big]_0^{\frac{1}{2}}$$

$$= 2\pi r^2 + 2\pi r^4\Big]_0^{1/2}$$

$$= 2\pi\frac{1}{4} + 2\pi\frac{1}{16} = \frac{\pi}{2} + \frac{\pi}{8} = \frac{5\pi}{8}$$

3. $\int_{\Omega} z(x^2+y^2)\,dS$; $\quad \Omega : \begin{cases} \sqrt{x^2+y^2+z^2} = 2 \\ z \geq 0 \end{cases}$

$\therefore \begin{cases} z^2+x^2+y^2 = 4 \\ z \geq 0 \end{cases} \Rightarrow z = \sqrt{4-x^2-y^2}$

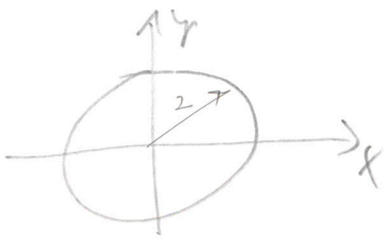
$$\frac{\partial z}{\partial x} = \frac{-x}{\sqrt{4-x^2-y^2}} \quad , \quad \frac{\partial z}{\partial y} = \frac{-y}{\sqrt{4-x^2-y^2}}$$

$$dS = \sqrt{1+\left(\frac{\partial z}{\partial x}\right)^2+\left(\frac{\partial z}{\partial y}\right)^2}\,dx\,dy = \sqrt{1+\frac{x^2}{4-x^2-y^2}+\frac{y^2}{4-x^2-y^2}}\,dx\,dy$$

$$= \sqrt{\frac{4}{4-x^2-y^2}}\,dx\,dy = \frac{2}{\sqrt{4-x^2-y^2}}\,dx\,dy$$

$$\therefore \int_{\Omega} \underline{z}(x^2+y^2)\,dS = \int_D \cancel{\sqrt{4-x^2-y^2}}\,(x^2+y^2)\,\frac{2}{\cancel{\sqrt{4-x^2-y^2}}}\,dx\,dy$$

$$= \int_D 2(x^2+y^2)\,\underline{dx\,dy}$$



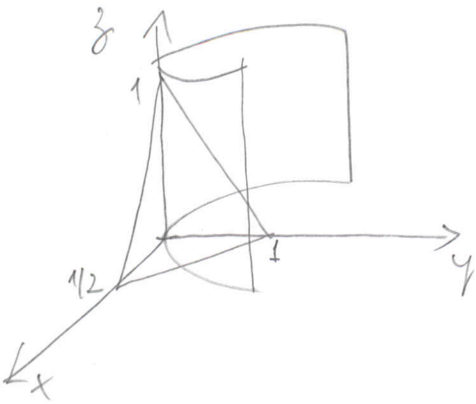
$$= \int_{r=0}^{2}\int_{\theta=0}^{2\pi} 2r^2\,\underline{r\,dr\,d\theta}$$

$$= \int_{r=0}^{2}\underline{\int_{\theta=0}^{2\pi} 2r^3\,dr\,d\theta} = \int_{r=0}^{2} 2r^3\,2\pi\,dr$$

$x = r\cos\theta$
$y = r\sin\theta$

$$= 4\pi\,\frac{r^4}{4}\Big]_0^2 = 16\pi$$

4. $\displaystyle\int_\Omega \sqrt{4y+1}\, dS$; $\Omega : \begin{cases} y = x^2 \\ 2x + y + z = 1 \end{cases}$



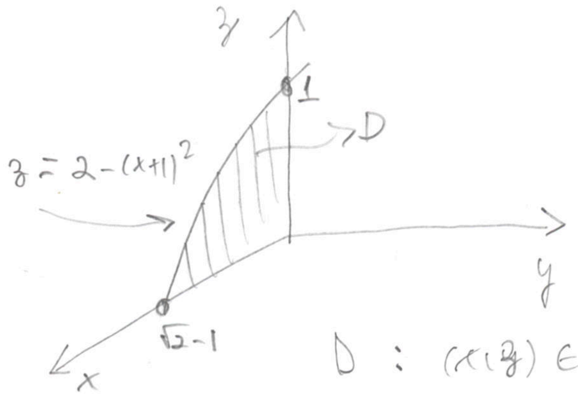
$y = y(x,z)$

γ : intersecção das superfícies

$\left.\begin{array}{l} y = x^2 \\ 2x + y + z = 1 \end{array}\right\} \Rightarrow 2x + x^2 + z = 1$

$(x+1)^2 - 1 + z = 1$

$z = 2 - (x+1)^2$



$D : (x,z) \in \mathbb{R}^2 : \begin{cases} 0 \leq x \leq \sqrt{2} - 1 \\ 0 \leq z \leq 2 - (x+1)^2 \end{cases}$

$$dS = \sqrt{1 + \left(\frac{\partial y}{\partial x}\right)^2 + \left(\frac{\partial y}{\partial z}\right)^2}\, dx\, dz = \sqrt{1 + 4x^2}\, dx\, dz$$

$\therefore$

$$\int_\Omega \sqrt{4y+1}\, dS = \int_D \sqrt{4x^2+1}\sqrt{1+4x^2}\, dx\, dz$$

$$= \int_D (1 + 4x^2)\, dx\, dz$$

$$= \int_{x=0}^{\sqrt{2}-1} \int_{z=0}^{2-(x+1)^2} (1+4x^2)\, dx\, dz$$

$$= \int_{x=0}^{\sqrt{2}-1} (1+4x^2)\, z \Big]_0^{2-(x+1)^2} dx$$

$= \int^{\sqrt{2}-1}$ ...

$$= \int_0^{\sqrt{2}-1} (1+4x^2)(1-x^2-2x)\,dx$$

$$= \int_0^{\sqrt{2}-1} (1-x^2-2x+4x^2-4x^4-8x^3)\,dx$$

$$= \int_0^{\sqrt{2}-1} (1+3x^2-2x-4x^4-8x^3)\,dx$$

$$= x + \frac{\cancel{3}x^3}{\cancel{3}} - \frac{\cancel{2}x^2}{\cancel{2}} - \frac{4x^5}{5} - \frac{\overset{2}{\cancel{8}}x^4}{\cancel{4}} \Big]_0^{\sqrt{2}-1}$$

$$= x\left(1+x^2-x-\frac{4}{5}x^4-2x^3\right) \Big]_0^{\sqrt{2}-1}$$

$$= (\sqrt{2}-1)\left(1+(\sqrt{2}-1)^2-(\sqrt{2}-1)-\frac{4}{5}(\sqrt{2}-1)^4-2(\sqrt{2}-1)^3\right)$$

$$= (\sqrt{2}-1)\Big[1+2-2\sqrt{2}+1 \quad -\sqrt{2}+1$$
$$-\frac{4}{5}(\underbrace{2-2\sqrt{2}+1}_{3-2\sqrt{2}})^2-2(2\sqrt{2}-3\cdot 2+3\sqrt{2}-1)\Big]$$

$$= (\sqrt{2}-1)\Big[5-3\sqrt{2}-\frac{4}{5}(9-12\sqrt{2}+8)$$
$$-2(5\sqrt{2}-7)\Big]$$

$$= (\sqrt{2}-1)\left[5-3\sqrt{2}-\frac{4}{5}(17-12\sqrt{2})-10\sqrt{2}+14\right]$$

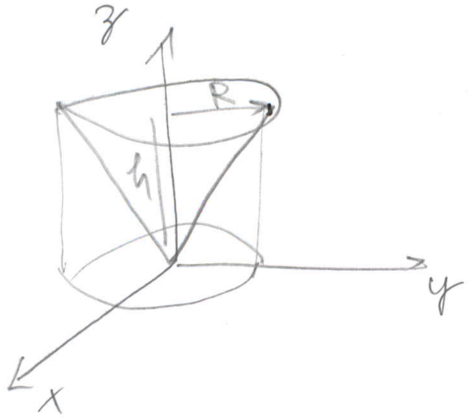
$$= (\sqrt{2}-1)\left[19-3\sqrt{2}-\frac{68}{5}+\frac{48\sqrt{2}}{5}-10\sqrt{2}\right]$$

$$= (\sqrt{2}-1)\left[19-13\sqrt{2}-\frac{68}{5}+\frac{48\sqrt{2}}{5}\right]$$

$$= (\sqrt{2}-1)\left[\frac{95-68}{5}+\frac{-65\sqrt{2}+48\sqrt{2}}{5}\right] = (\sqrt{2}-1)\left[\frac{27}{5}-\frac{17\sqrt{2}}{5}\right]$$

$$= \frac{44\sqrt{2}-61}{5}$$

5.



$$z^2 = ax^2 + by^2 \quad ; \quad z \geq 0$$

$y=0$: $z = \frac{h}{R}x$

$x=0$: $z = \frac{h}{R}y$

$$\therefore \begin{cases} z = \frac{h}{R}\sqrt{x^2+y^2} & \text{[Eq. Cone]} \\ z \geq 0 \end{cases}$$

$$S = \int_\Omega dS = \int_D \sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2}\, dx\, dy$$

$$\frac{\partial z}{\partial x} = \frac{h}{R}\frac{x}{\sqrt{x^2+y^2}}$$

$$\frac{\partial z}{\partial y} = \frac{h}{R}\frac{y}{\sqrt{x^2+y^2}}$$

$$= \int_D \sqrt{1 + \frac{h^2}{R^2}\frac{x^2}{x^2+y^2} + \frac{h^2}{R^2}\frac{y^2}{x^2+y^2}}\, dx\, dy$$

$$= \int_D \sqrt{\frac{R^2(x^2+y^2) + h^2x^2 + h^2y^2}{R^2(x^2+y^2)}}\, dx\, dy$$

$$= \int_D \sqrt{\frac{(R^2+h^2)\cancel{(x^2+y^2)}}{R^2\cancel{(x^2+y^2)}}}\, dx\, dy$$

$$= \int_D \sqrt{\frac{R^2+h^2}{R^2}}\, dx\, dy$$

$$= \sqrt{\frac{R^2+h^2}{R^2}} \underbrace{\int_D dx\, dy}_{\pi R^2}$$

$$\boxed{S = \pi R\sqrt{R^2+h^2}}$$

6. $\int_{\Omega} (x^2-y^2)\, dS \quad ; \quad \Omega : z = \sqrt{9-x^2-y^2}$

Em coordenadas esféricas temos:

$$\begin{cases} x = 3 \sin\theta \cos\varphi \\ y = 3 \sin\theta \sin\varphi \\ z = 3 \cos\theta \\ 0 \le \theta \le \frac{\pi}{2} \quad , \quad 0 \le \varphi \le 2\pi \end{cases}$$

$$\begin{cases} dS = R^2 \sin\theta \, d\theta \, d\varphi \quad \text{[Já visto]} \\ \quad\; = 9 \sin\theta \, d\theta \, d\varphi \end{cases}$$

$\therefore$

$$\int_{\Omega} (x^2-y^2)\, \underset{\sim}{dS} = \int_{\theta=0}^{\frac{\pi}{2}} \int_{\varphi=0}^{2\pi} (9\sin^2\theta \cos^2\varphi - 9\sin^2\theta \sin^2\varphi) \cdot 9 \sin\theta \, d\theta \, d\varphi$$

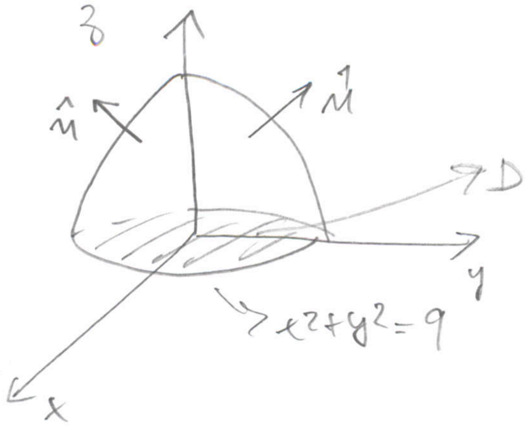
$$= \int_{\theta=0}^{\frac{\pi}{2}} \int_{\varphi=0}^{2\pi} 9 \sin^2\theta \, \underbrace{(\cos^2\varphi - \sin^2\varphi)} \, 9 \sin\theta \, d\theta \, d\varphi$$

$$= \int_{\theta=0}^{\frac{\pi}{2}} \int_{\varphi=0}^{2\pi} 81 \sin^3\theta \, (\underline{\cos 2\varphi}) \, d\theta \, d\varphi$$

$$= \int_{\theta=0}^{\frac{\pi}{2}} 81 \sin^3\theta \left. \frac{\sin 2\varphi}{2} \right]_0^{2\pi} d\theta$$

$$= \int_{\theta=0}^{\frac{\pi}{2}} \frac{81}{2} \sin^3\theta \left[\frac{\cancelto{0}{\sin 4\pi - \sin 0}}{2} \right] d\theta = 0$$

7. $\vec{F} = y\hat{i} - x\hat{j} + 8\hat{k}$; $\Omega : \begin{cases} z = 9 - x^2 - y^2 \\ z \geq 0 \\ \vec{n} \text{ aponta para cima} \end{cases}$



$$d\vec{S} = \hat{n}\, dS$$

$$\hat{n} = \frac{\left(-\frac{\partial z}{\partial x}, -\frac{\partial z}{\partial y}, 1\right)}{\sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2}} = \frac{(2x, 2y, 1)}{\sqrt{4x^2 + 4y^2 + 1}}$$

$$dS = \sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2}\, dx\,dy$$

$\therefore$

$$// \; d\vec{S} = (2x, 2y, 1)\, dx\,dy \; //$$

$$\int_\Omega \vec{F} \cdot d\vec{S} = \int_\Omega (y\hat{i} - x\hat{j} + 8\hat{k}) \cdot (2x\hat{i} + 2y\hat{j} + \hat{k})\, dx\,dy$$

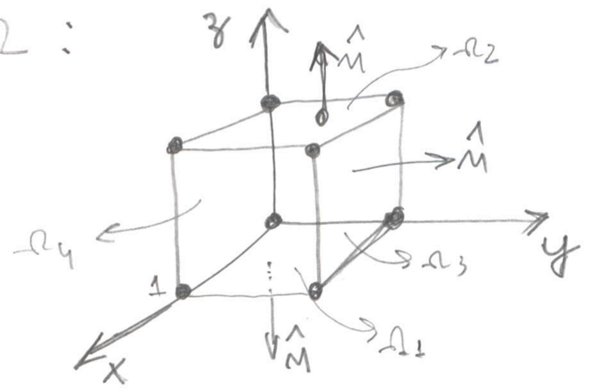
$$= \int_D (\cancel{2xy} - \cancel{2yx} + 8)\, dx\,dy$$

$$= \int_D 8\, dx\,dy = 8 \int_D dx\,dy = 8\pi 9$$

$$= 72\pi \; //$$

8. $\vec{F} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

Ω :



$\Omega_1 : \begin{cases} z = 0 \\ 0 \le x \le 1 \ , \ 0 \le y \le 1 \\ \hat{n}_1 \equiv (0,0,-1) \end{cases}$

$\Omega_2 : \begin{cases} z = 1 \\ 0 \le x \le 1 \ , \ 0 \le y \le 1 \\ \hat{n}_2 = (0,0,1) \end{cases}$

$\Omega_3 : \begin{cases} y = 1 \\ 0 \le x \le 1 \ , \ 0 \le z \le 1 \\ \hat{n}_3 = (0,1,0) \end{cases}$

$\Omega_4 : \begin{cases} y = 0 \\ 0 \le x \le 1 \ , \ 0 \le z \le 1 \\ \hat{n}_4 = (0,-1,0) \end{cases}$

$\Omega_5 : \begin{cases} x = 1 \\ 0 \le y \le 1 \ , \ 0 \le z \le 1 \\ \hat{n}_5 = (1,0,0) \end{cases}$

$\Omega_6 : \begin{cases} x = 0 \\ 0 \le y \le 1 \ , \ 0 \le z \le 1 \\ \hat{n}_6 = (-1,0,0) \end{cases}$

Daí

$$\oint_{\Omega} \vec{F} \cdot d\vec{S} = \int_{\Omega_1} \vec{F} \cdot \hat{n}_1 \, dS_1 + \int_{\Omega_2} \vec{F} \cdot \hat{n}_2 \, dS_2 + \int_{\Omega_3} \vec{F} \cdot \hat{n}_3 \, dS_3 +$$

$$+ \int_{\Omega_4} \vec{F} \cdot \hat{n}_4 \, dS_4 + \int_{\Omega_5} \vec{F} \cdot \hat{n}_5 \, dS_5 + \int_{\Omega_6} \vec{F} \cdot \hat{n}_6 \, dS_6$$

$\therefore$

$$\int_{\Omega_1} \vec{F} \cdot \hat{n}_1 \, dS_1 = \int_{\Omega_1} (x\hat{\imath} + y\hat{\jmath} + z\hat{k}) \cdot (-\hat{k}) \, dx\,dy$$

$$= \int_{\Omega_1} \underbrace{-z}_{} \, dx\,dy = \int 0 \, dy\,dy = 0 //$$

$$\int_{\Omega_2} \vec{F}\cdot\hat{n}_2 \, dS_2 = \int_{\Omega_2} (x\hat{i}+y\hat{j}+z\hat{k})\cdot\hat{k} \; dx\,dy$$

$$= \int_{\Omega_2} \underbrace{z}\, dx\,dy = \underbrace{\int 1\, dx\,dy}$$

$$= 1$$

$$\int_{\Omega_3} \vec{F}\cdot\hat{n}_3 \, dS_3 = \int_{\Omega_3} (x\hat{i}+y\hat{j}+z\hat{k})\cdot\hat{j} \; dx\,dz$$

$$= \int_{\Omega_3} y \; dx\,dz = \int_{\Omega_3} dx\,dz = 1$$

$$\int_{\Omega_4} \vec{F}\cdot\hat{n}_4 \, dS_4 = \int_{\Omega_4} (x\hat{i}+y\hat{j}+z\hat{k})\cdot(-\hat{j}) \; dx\,dz$$

$$= \int_{\Omega_4} -\underbrace{y}_{=0}\, dx\,dz = 0$$

$$\int_{\Omega_5} \vec{F}\cdot\hat{n}_5 \, dS_5 = \int_{\Omega_5} (x\hat{i}+y\hat{j}+z\hat{k})\cdot\hat{i} \; dy\,dz$$

$$= \int_{\Omega_5} \underbrace{x}\, dy\,dz = \int_{\Omega_5} 1\, dy\,dz = 1$$

$$\int_{\Omega_6} \vec{F}\cdot\hat{n}_6 \, dS_6 = \int_{\Omega_6} (x\hat{i}+y\hat{j}+z\hat{k})\cdot(-\hat{i})\, dy\,dz = \int -\underbrace{x}\, dy\,dz$$

$$= \int 0\, dy\,dz = 0$$

$$\oint_{\Omega} \vec{F}\cdot d\vec{S} = 0 + 1 + 1 + 0 + 1 + 0 = \underline{\underline{3}}$$

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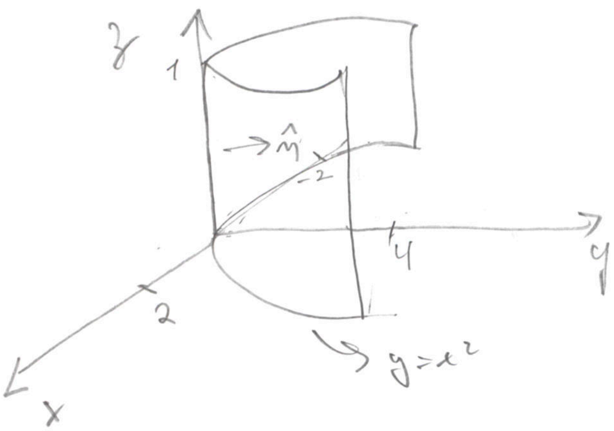
9. $\vec{F} = yz^2\,\hat{\imath} + ye^x\,\hat{\jmath} + x\,\hat{k}$

$$\Omega : \begin{cases} y = x^2 \\ 0 \le y \le 4 \ , \ 0 \le z \le 1 \\ \hat{n} \text{ tem componente } y \text{ positiva} \end{cases}$$



$y = y(x,z)$

$$\hat{n} = \frac{\left(-\frac{\partial y}{\partial x}, 1, -\frac{\partial y}{\partial z}\right)}{\sqrt{1 + \left(\frac{\partial y}{\partial x}\right)^2 + \left(\frac{\partial y}{\partial z}\right)^2}} = \frac{(-2x, 1, 0)}{\sqrt{1 + 4x^2}}$$

$$\begin{cases} d\vec{S} = \hat{n}\, dS = (-2x\,\hat{\imath} + \hat{\jmath})\, dx\, dz \\ (x,y) \in D : \ -2 \le x \le 2 , \quad 0 \le z \le 1 \end{cases}$$

$$\therefore \int_{\Omega} \vec{F}\cdot d\vec{S} = \int_D \left(yz^2\,\hat{\imath} + ye^x\,\hat{\jmath} + x\,\hat{k}\right)\cdot\left(-2x\,\hat{\imath} + \hat{\jmath}\right) dx\, dz$$

$$= \int_D \left(-2x\underbrace{y}\,z^2 + \underbrace{y}\,e^x\right) dx\, dz$$

$$\int_{\Omega} \vec{F}\cdot d\vec{S} = \int_D \left(-2x\,x^2\,z^2 + x^2e^x\right) dx\,dz$$

$$= \int_{x=-2}^{2}\int_{z=0}^{1} \left(-2x^3z^2 + x^2e^x\right) dx\,dz$$

$$= \int_{x=-2}^{2} -2x^3\frac{z^3}{3} + x^2e^x z \Big]_0^1 dx$$

$$(*) = \int_{x=-2}^{2}\left(-\frac{2x^3}{3}1 + x^2e^x\right) dx$$

Mas

$$\int x^2e^x\,dx = x^2e^x - \int 2xe^x\,dx$$

$u = x^2 \to du = 2x\,dx$ | $u = x \to du = dx$

$dv = e^x dx \to v = e^x$ | $dv = e^x dx \to v = e^x$

$$= x^2e^x - 2\left(xe^x - \int e^x dx\right)$$

$$= x^2e^x - 2(xe^x - e^x)$$

$$= x^2e^x - 2xe^x + 2e^x$$

$\therefore$

$$(*) = -\frac{2}{3}\frac{x^4}{4} + x^2e^x - 2xe^x + 2e^x\Big]_{-2}^{2} =$$

$$= -\frac{1}{6}(2^4) + 4e^2 - 2.2e^2 + 2e^2 -$$

$$- \left[-\frac{1}{6}(-2)^4 + (-2)^2 e^{-2} - 2(-2)e^{-2} + 2e^{-2} \right]$$

$$= -\frac{1}{6}16 + \cancel{4e^2} - \cancel{4e^2} + 2e^2$$

$$- \left[-\frac{1}{6}16 + 4e^{-2} + 4e^{-2} + 2e^{-2} \right]$$

$$= \cancel{-\frac{8}{3}} + 2e^2 + \cancel{\frac{8}{3}} - 10e^{-2}$$

$$= 2e^2 - 10e^{-2}$$

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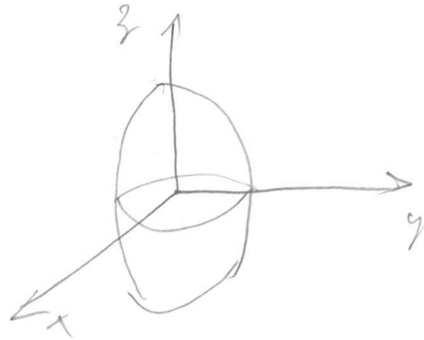
10. $\vec{F} = yx\,\hat{i} + y^2\,\hat{j} + yz\,\hat{k}$



$\Omega : \begin{cases} x^2 + \frac{y^2}{4} + z^2 = 1 \\ \hat{n} \text{ vetor normal p/ fora} \end{cases}$

$\Omega = \Omega_1 \cup \Omega_2$

$\Omega_1 : \begin{cases} z = \sqrt{1 - x^2 - \frac{y^2}{4}} \quad , \quad z \geq 0 \\ \hat{n}_1 = \left(-\frac{\partial z}{\partial x}, -\frac{\partial z}{\partial y}, 1 \right) \Big/ \sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2} \\ d\vec{S}_1 = \left(\frac{x}{\sqrt{1 - x^2 - \frac{y^2}{4}}}\,\hat{i} + \frac{y/4}{\sqrt{1 - x^2 - \frac{y^2}{4}}}\,\hat{j} + \hat{k} \right) dx\,dy \end{cases}$

$$\Omega_2 : \begin{cases} z = -\sqrt{1-x^2-\frac{y^2}{4}} \; ; \; z \le 0 \\ \hat{n}_2 = \left(\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}, -1 \right) \Big/ \sqrt{1+\left(\frac{\partial z}{\partial x}\right)^2+\left(\frac{\partial z}{\partial y}\right)^2} \\ d\vec{S}_2 = \left(\frac{+x}{\sqrt{1-x^2-\frac{y^2}{4}}}\,\hat{\imath} + \frac{y/4}{\sqrt{1-x^2-\frac{y^2}{4}}}\,\hat{\jmath} - \hat{k} \right) dx\,dy \end{cases}$$

$$\Omega = \underbrace{\int_{\Omega_1} \vec{F}\cdot d\vec{S}_1}_{(*)} + \underbrace{\int_{\Omega_2} \vec{F}\cdot d\vec{S}_2}_{(**)}$$

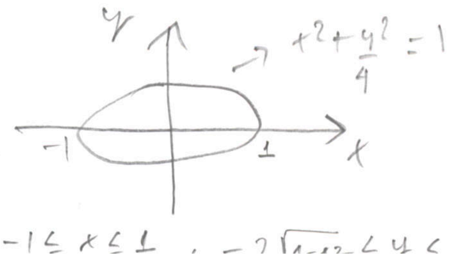
$$(*) = \int_{D_1} (yx\,\hat{\imath} + y^2\,\hat{\jmath} + yz\,\hat{k}) \cdot \left(\frac{x}{\sqrt{1-x^2-\frac{y^2}{4}}}\,\hat{\imath} + \frac{y/4}{\sqrt{1-x^2-\frac{y^2}{4}}}\,\hat{\jmath} + \hat{k} \right) dx\,dy$$

$$= \int_{D_1} \left(\frac{yx^2}{\sqrt{1-x^2-\frac{y^2}{4}}} + \frac{\frac{y^3}{4}}{\sqrt{1-x^2-\frac{y^2}{4}}} + y\,z \right) dx\,dy$$

$$= \int_{D_1} \left(\frac{yx^2 + \frac{y^3}{4}}{\sqrt{1-x^2-\frac{y^2}{4}}} + y\sqrt{1-x^2-\frac{y^2}{4}} \right) dx\,dy$$

$$= \int_{D_1} \frac{\cancel{yx^2} + \cancel{\frac{y^3}{4}} + y\left(1 \cancel{- x^2 - \frac{y^2}{4}}\right)}{\sqrt{1-x^2-\frac{y^2}{4}}}\, dx\,dy$$

$$= \int_{D_1} \frac{y}{\sqrt{1-x^2-y^2}}\, dx\,dy =$$



$-1 \le x \le 1 \; , \; -2\sqrt{1-x^2} \le y \le 2\sqrt{1-x^2}$

$$= \int_{x=-1}^{1} \int_{y=-2\sqrt{1-x^2}}^{2\sqrt{1-x^2}} \frac{y}{\sqrt{1-x^2-\frac{y^2}{4}}} \, dx\,dy =$$

$$= \int_{x=-1}^{1} -4\sqrt{1-x^2-\frac{y^2}{4}} \,\Bigg]_{y=-2\sqrt{1-x^2}}^{2\sqrt{1-x^2}} dx$$

$$= \int_{x=-1}^{1} \left(-4\sqrt{1-x^2-\frac{\cancel{4}(1-x^2)}{\cancel{4}}} + 4\sqrt{1-x^2-\frac{\cancel{4}(1-x^2)}{\cancel{4}}} \right) dx$$

$$= \int_{x=-1}^{1} \left(-4\sqrt{\cancel{1}-\cancel{x^2}-\cancel{1}+\cancel{x^2}} + 4\sqrt{\cancel{1}-\cancel{x^2}-\cancel{1}+\cancel{x^2}} \right) dx$$

$$= 0$$

también

$$(**) = \int_{\Omega_2} \vec{F} \cdot d\vec{S}_2 = \int_{\Omega_2} \left(yx\,\hat{i} + y^2\,\hat{j} + yz\,\hat{k} \right) \cdot \left(\frac{+x}{\sqrt{1-x^2-\frac{y^2}{4}}}\,\hat{i} + \frac{+\frac{y}{4}}{\sqrt{1-x^2-\frac{y^2}{4}}}\,\hat{j} - \hat{k} \right) dx\,dy$$

$$= \int_{\Omega_2} \left(\frac{+yx^2}{\sqrt{1-x^2-\frac{y^2}{4}}} + \frac{\frac{y^3}{4}}{\sqrt{1-x^2-\frac{y^2}{4}}} - yz \right) dx\,dy$$

$$= \int_{\Omega_2} \left(\frac{+yx^2 + \frac{y^3}{4}}{\sqrt{1-x^2-\frac{y^2}{4}}} - y(-)\sqrt{1-x^2-\frac{y^2}{4}} \right) dx\,dy$$

$$= \int_{\Omega_2} \frac{+\cancel{yx^2} \cancel{+\frac{y^3}{4}} + y\left(1-\cancel{x^2}-\cancel{\frac{y^2}{4}}\right)}{\sqrt{1-x^2-\frac{y^2}{4}}} \, dx\,dy$$

$$= \int_{\Omega_2} \frac{y}{\sqrt{1-x^2-\frac{y^2}{4}}}\, dx\, dy = 0 \quad \text{(Mesmo cálculo)}$$

$$\therefore \quad \boxed{\oint_{\Omega} \vec{F} \cdot d\vec{S} = 0}$$

$$\int \frac{x^2}{1-x}\,dx = -x^2 \ln(1-x) + \int 2x \ln(1-x)\,dx$$

$u = x^2 \rightarrow du = 2x\,dx$

$dv = \frac{1}{1-x}\,dx \qquad v = -\ln(1-x)$

$u = $ [illegible] $dv = $ [illegible]

$du = $ [illegible]

$$\int \frac{1}{[illegible]}$$