Lista 14

(I)

1. $y''' - y'' - 12y' = 0$

$y = e^{\lambda x}$

$\lambda^3 - \lambda^2 - 12\lambda = 0$

$\lambda(\lambda^2 - \lambda - 12) = 0$

$\Rightarrow \underline{\underline{\lambda = 0}}$, $\lambda^2 - \lambda - 12 = 0 \therefore \begin{cases} \lambda = 4 \\ \lambda = -3 \end{cases}$

3 raízes reais distintos:

$y = A e^{0x} + B e^{4x} + C e^{-3x}$

$\therefore \boxed{y = A + B e^{4x} + C e^{-3x}}$

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2. $y''' + 2y'' - 5y' - 6y = 0$

$y = e^{\lambda x}$: $\lambda^3 + 2\lambda^2 - 5\lambda - 6 = 0$

$\rightarrow \underline{\lambda = -1}$ é raiz

$$\begin{array}{l|l} \lambda^3 + 2\lambda^2 - 5\lambda - 6 & \lambda + 1 \\ \hline -\lambda^3 - \lambda^2 & \lambda^2 + \lambda - 6 \\ \quad \lambda^2 - 5\lambda - 6 & \\ \quad -\lambda^2 - \lambda & \\ \quad\quad -6\lambda - 6 & \\ \quad\quad 6\lambda + 6 & \end{array}$$

$\lambda = -1 \pm \sqrt{1 + 24} / 2$

$= \dfrac{-1 \pm 5}{2} \nearrow 2 \searrow -3$

$\lambda = -3, -1, 2$ raizes

$$\boxed{y = Ae^{-3x} + Be^{-x} + Ce^{2x}}$$

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3. $y''' - 3y'' + 3y' - y = 0$

$y = e^{\lambda x}$: $\lambda^3 - 3\lambda^2 + 3\lambda - 1 = 0$

$\underline{\underline{\lambda = 1}}$

$$\begin{array}{l|l} \lambda^3 - 3\lambda^2 + 3\lambda - 1 & \lambda - 1 \\ \cline{2-2} -\lambda^3 + \lambda^2 & \lambda^2 - 2\lambda + 1 \\ \hline -\cancel{2\lambda^2} + 3\lambda - 1 & \\ +\cancel{2\lambda^2} - 2\lambda & \\ \hline \lambda - 1 & \\ -\lambda + 1 & \\ \hline 0 & \end{array}$$

$\lambda^2 - 2\lambda + 1 = 0$

$\lambda = +1$ (dupla)

Raizes:

$\lambda = 1$ (Mult. 3)

$$\boxed{y = Ae^{x} + Bxe^{x} + Cx^2e^{x}}$$

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4. $y^{(4)} + 6y''' + 5y'' - 24y' - 36y = 0$

$\lambda^4 + 6\lambda^3 + 5\lambda^2 - 24\lambda - 36 = 0$

$\underline{\underline{\lambda = 2 \text{ é raiz}}}$: $\left\{ \begin{array}{l} 2^4 + 6.2^3 + 5.2^2 - 24.2 - 36 = \\ = 16 + \cancel{48} + 20 - \cancel{48} - 36 \\ = 0 \end{array} \right.$

$$
\begin{array}{l|l}
\lambda^4 + 6\lambda^3 + 5\lambda^2 - 24\lambda - 36 & \lambda - 2 \\ \hline
-\lambda^4 + 2\lambda^3 & \lambda^3 + 8\lambda^2 + 21\lambda + 18 \\
\quad 8\lambda^3 + 5\lambda^2 - 24\lambda - 36 & \\
\quad -8\lambda^3 + 16\lambda^2 & \\
\qquad 21\lambda^2 - 24\lambda - 36 & \\
\qquad -21\lambda^2 + 42\lambda & \\
\qquad\qquad 18\lambda - 36 & \\
\qquad\qquad -18\lambda + 36 & \\
\qquad\qquad\quad 0 &
\end{array}
$$

$\lambda^3 + 8\lambda^2 + 21\lambda + 18$

$\underline{\lambda = -2}$:

$-8 + 8 \cdot 4 + 21 \cdot (-2) + 18 =$

$= -8 + 32 - 42 + 18$

$= 0$

$\therefore$ $\underline{\lambda = -2 \text{ é raiz}}$

$$
\begin{array}{l|l}
\lambda^3 + 8\lambda^2 + 21\lambda + 18 & \lambda + 2 \\ \hline
-\lambda^3 - 2\lambda^2 & \lambda^2 + 6\lambda + 9 \\
\quad 6\lambda^2 + 21\lambda + 18 & \\
\quad -6\lambda^2 - 12\lambda & \\
\qquad 9\lambda + 18 & \\
\qquad -9\lambda - 18 & \\
\qquad\quad 0 &
\end{array}
$$

$\lambda^2 + 6\lambda + 9 = 0$

$\underline{\lambda = -3}$ (dupla)

Raízes:

$$
\left.\begin{array}{l}
\lambda = 2 \text{ (simples)} \\
\lambda = -2 \text{ (simples)} \\
\lambda = -3 \text{ (dupla)}
\end{array}\right\}
\quad \boxed{y = Ae^{2x} + Be^{-2x} + Ce^{-3x} + Dxe^{-3x}}
$$

5. $y^{(4)} - y''' - 9y'' - 11y' - 4y = 0$

$\lambda^4 - \lambda^3 - 9\lambda^2 - 11\lambda - 4 = 0$

$\underline{\lambda = -1}$: $(-1)^4 - (-1)^3 - 9(-1)^2 - 11(-1) - 4 = 0$

$= 1 + 1 - 9 + 11 - 4$

$= 0$

$$\cancel{\lambda^4} - \lambda^3 - 9\lambda^2 - 11\lambda - 4 : (\lambda + 1) = \lambda^3 - 2\lambda^2 - 7\lambda - 4$$

$$-\cancel{\lambda^4} - \lambda^3$$

$$-\cancel{2\lambda^3} - 9\lambda^2 - 11\lambda - 4$$

$$+\cancel{2\lambda^3} + 2\lambda^2$$

$$-7\lambda^2 - 11\lambda - 4$$

$$7\lambda^2 + 7\lambda$$

$$-4\lambda - 4$$

$$4\lambda + 4$$

$$0$$

$\underline{\lambda = -1}$

$(-1)^3 - 2(-1)^2 - 7(-1) - 4 =$

$= -1 - 2 + 7 - 4$

$= 0$

$$\cancel{\lambda^3} - 2\lambda^2 - 7\lambda - 4 : (\lambda + 1) = \lambda^2 - 3\lambda -$$

$$-\cancel{\lambda^3} - \lambda^2$$

$$-\cancel{3\lambda^2} - 7\lambda - 4$$

$$+\cancel{3\lambda^2} + 3\lambda$$

$$-4\lambda - 4$$

$$+4\lambda + 4$$

$$0$$

$\lambda^2 - 3\lambda - 4 = 0$

$\lambda = 4$; $\lambda = -1$

Raízes :

$\left.\begin{array}{l}\lambda = -1 \ (\text{mult. } 3) \\ \lambda = 4 \ (\text{simples})\end{array}\right\}$ $\boxed{y = Ae^{-x} + Bxe^{-x} + Cx^2e^{-x} + + De^{4x}}$

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6. $y''' + 4y' = 0$

$\lambda^3 + 4\lambda = 0$

$\lambda(\lambda^2 + 4) = 0 \Rightarrow \left.\begin{array}{l}\lambda = 0 \\ \lambda = \pm 2i\end{array}\right]$

$\lambda = 0 \longrightarrow A$

$\lambda = \pm 2i \longrightarrow B\cos 2x + C\sin 2x$

$\therefore \boxed{y = A + B\cos 2x + C\sin 2x}$

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7. $y^{(4)} + y''' + 2y'' - y' + 3y = 0$

$\lambda^4 + \lambda^3 + 2\lambda^2 - \lambda + 3 = 0$

$\lambda_1 = -1 \pm i\sqrt{2}$ $\longrightarrow$ $e^{-x}(A\cos\sqrt{2}x + B\sin\sqrt{2}x)$

raízes

$\lambda_2 = \frac{1}{2} \pm \frac{i}{2}\sqrt{3}$ $\longrightarrow$ $e^{+\frac{x}{2}}(C\cos\sqrt{3}x + D\sin\sqrt{3}x)$

$$\boxed{y = e^{-x}\left(A\cos\sqrt{2}x + B\sin\sqrt{2}x\right) + e^{\frac{x}{2}}\left(C\cos\frac{\sqrt{3}}{2}x + D\sin\frac{\sqrt{3}}{2}x\right)}$$

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8. $y^{(4)} + 5y'' - 36y = 0$

$\lambda^4 + 5\lambda^2 - 36 = 0$

$(\lambda^2) = -5 \pm \sqrt{25+144}/2$

$= \dfrac{-5 \pm \sqrt{169}}{2} = \dfrac{-5 \pm 13}{2} \to -9, \; 4$

$\lambda^2 = -9 \Rightarrow \lambda = \pm 3i \to A\cos 3x + B\sin 3x$

$\lambda^2 = 4 \Rightarrow \lambda = \pm 2 \to Ce^{2x} + De^{-2x}$

$\therefore \boxed{y = A\cos 3x + B\sin 3x + Ce^{2x} + De^{-2x}}$

(II)

9. $y''' + 2y'' - y' - 2y = e^x + x^2$

y_h

$$\lambda^3 + 2\lambda^2 - \lambda - 2 = 0$$

$\lambda = 1$:

$$\begin{array}{r|l} \cancel{\lambda^3} + 2\lambda^2 - \lambda - 2 & \lambda - 1 \\ \cancel{-\lambda^3} + \lambda^2 & \lambda^2 + 3\lambda + 2 \\ \hline \cancel{3\lambda^2} - \lambda - 2 & \\ \cancel{-3\lambda^2} + 3\lambda & \\ \hline 2\lambda - 2 & \\ -2\lambda + 2 & \\ \hline 0 & \end{array}$$

$\lambda^2 + 3\lambda + 2 = 0$

$\begin{cases} \lambda = -1 \\ \lambda = -2 \end{cases}$

Raízes

$\left.\begin{array}{l} \lambda_1 = 1 \\ \lambda_2 = -1 \\ \lambda = -2 \end{array}\right\}$ $y_h = Ae^{x} + Be^{-x} + Ce^{-2x}$

y_p

$r(x) = e^x + x^2$

Regra Básica $\begin{cases} e^x \to Ae^x \xrightarrow[y_h]{e^x \text{ está em}} Axe^x \text{ (Modificada)} \\ x^2 \to B_2x^2 + B_1x + B_0 \end{cases}$

$\therefore$ $y_p = Axe^x + B_2x^2 + B_1x + B_0$

$$y'_p = Ae^x + Axe^x + 2B_2x + B_1$$

$$\begin{cases} y''_p = Ae^x + Ae^x + Axe^x + 2B_2 \\ \quad = 2Ae^x + Axe^x + 2B_2 \end{cases}$$

$$y'''_p = 2Ae^x + Ae^x + Axe^x$$

$$\therefore \quad y''' + 2y'' - y' - 2y = e^x + x^2$$

$$\begin{cases} 2Ae^x + Ae^x + Axe^x + 4Ae^x + 2Axe^x + 4B_2 \\ - Ae^x - Axe^x - 2B_2x - B_1 \\ - 2Axe^x - 2B_2x^2 - 2B_1x - 2B_0 = e^x + x^2 \end{cases}$$

$$6Ae^x + \underbrace{4B_2 - B_1 - 2B_0} + (-2B_2 - 2B_1)x - 2B_2x^2 =$$

$$= e^x + x^2$$

$$\therefore \begin{cases} 6A = 1 \implies A = 1/6 \\ -2B_2 - 2B_1 = 0 \longrightarrow B_1 = -B_2 = \frac{1}{2} \\ -2B_2 = 1 \implies B_2 = -\frac{1}{2} \\ 4B_2 - B_1 - 2B_0 = 0 \longrightarrow B_0 = \frac{4B_2 - B_1}{2} \end{cases}$$

$$B_0 = \frac{-2 - \frac{1}{2}}{2} = -\frac{5}{4}$$

$\therefore$

$A = \frac{1}{6}$, $B_0 = -\frac{5}{4}$, $B_1 = \frac{1}{2}$, $B_2 = -\frac{1}{2}$.

$$y_p = \frac{x}{6}e^x - \frac{x^2}{2} + \frac{x}{2} - \frac{5}{4}$$

$\therefore$

$y = y_H + y_p$

$$\boxed{y = Ae^x + Be^{-x} + Ce^{-2x} + \frac{x}{6}e^x - \frac{x^2}{2} + \frac{x}{2} - \frac{5}{4}}$$

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10. $y''' + 3y'' + 2y' = x^2 + 4x + 8$

$\underline{y_H}$ $\quad \lambda^3 + 3\lambda^2 + 2\lambda = 0$

$\underline{\lambda = -1}$: $(-1)^3 + 3(-1)^2 + 2(-1) = -1 + 3 - 2 = 0$

$$\begin{array}{l|l} \lambda^3 + 3\lambda^2 + 2\lambda & \lambda + 1 \\ \hline -\lambda^3 - \lambda^2 & \lambda^2 + 2\lambda \\ \hline \quad 2\lambda^2 + 2\lambda & \\ \quad -2\lambda^2 - 2\lambda & \\ \hline \quad\quad 0 & \end{array}$$

$\lambda^2 + 2\lambda = 0$

$\lambda(\lambda + 2) = 0 \rightarrow \begin{cases} \lambda = 0 \\ \lambda = -2 \end{cases}$

Raízes:

$$\left.\begin{aligned} r_1 &= -1 \\ r_2 &= 0 \\ r_3 &= -2 \end{aligned}\right\} \begin{aligned} &\longrightarrow e^{-x} \\ &\longrightarrow e^{0x} = 1 \\ &\longrightarrow e^{-2x} \end{aligned}$$

$$\therefore\; y_h = C_0 + C_1 e^{-x} + C_2 e^{-2x}$$

y_p:

$$r(x) = x^2 + 4x + 8$$

Regra básica: $y_p = A_2 x^2 + A_1 x + A_0$

Aqui A_0 está presente em y_h

logo modificamos y_p para

$$\tilde{y}_p = x\,y_p = A_2 x^3 + A_1 x^2 + A_0 x$$

$$\begin{cases} y'_p = 3A_2 x^2 + 2A_1 x + A_0 \\ y''_p = 6A_2 x + 2A_1 \\ y'''_p = 6A_2 \end{cases}$$

$$y'''_p + 3y''_p + 2y'_p = x^2 + 4x + 8$$

$$6A_2 + 18A_2x + 6A_1 + 6A_2x^2 + 4A_1x + 2A_0 = x^2+4x+8$$

$$6A_2x^2 + (4A_1 + 18A_2)x + 6A_2 + 6A_1 + 2A_0 = x^2+4x+8$$

$$\Rightarrow \begin{cases} 6A_2 = 1 \Rightarrow A_2 = \frac{1}{6} \\ 4A_1 + 18A_2 = 4 \Rightarrow 4A_1 + 18\frac{1}{6} = 4 \\ \qquad 4A_1 + 3 = 4 \Rightarrow A_1 = \frac{1}{4} \\ 6A_2 + 6A_1 + 2A_0 = 8 \end{cases}$$

$$\therefore \quad 6\frac{1}{6} + 6\frac{1}{4} + 2A_0 = 8$$

$$1 + \frac{3}{2} + 2A_0 = 8$$

$$2A_0 = 7 - \frac{3}{2}$$

$$2A_0 = \frac{11}{2}$$

$$A_0 = \frac{11}{4}$$

$$\therefore \quad // \; y_p = \frac{x^3}{6} + \frac{x^2}{4} + \frac{11}{4}x \; //$$

$$\boxed{y = y_h + y_p = C_0 + C_1e^{-x} + C_2e^{-2x} + \frac{11}{4}x + \frac{x^2}{4} + \frac{x^3}{6}}$$

$+5xe^{2x}+e^{2x}$

$\underline{y_h}$: $\lambda^3-\lambda^2-4\lambda+4=0$

$\underline{\lambda=1}$: $1-1-4+4=0$

$$\begin{array}{r|l} \lambda^3-\lambda^2-4\lambda+4 & \lambda-1 \\ \cline{2-2} -\lambda^3+\lambda^2 & \lambda^2-4 \\ \hline -4\lambda+4 & \\ +4\lambda-4 & \\ \hline 0\ 0 & \end{array}$$

$\lambda^2-4=0$

$\underline{\lambda=\pm 2}$

Raízes

$\lambda=1 \longrightarrow e^{x}$

$\lambda=-2 \longrightarrow e^{-2x}$

$\lambda=2 \longrightarrow e^{2x}$

$\therefore \; // \; y_h = k_0e^{x}+k_1e^{-2x}+k_2e^{2x} \; //$

$\underline{y_p}$: $r(x) = \underbrace{2x^2-4x-1} + 2x^2e^{2x}+5xe^{2x}+e^{2x}$

$= 2x^2-4x-1 + (2x^2+5x+1)e^{2x}$

Logo bastaria

$2x^2-4x-1 \longrightarrow y_{p1} = A_2x^2+A_1x+A_0$

$(2x^2+5x+1)e^{2x} \longrightarrow y_{p2} = (B_2x^2+B_1x+B_0)e^{2x}$

$$\therefore \left\{ \begin{aligned} y_p''' &= 6B_2 e^{2x} + 6(6B_2 x + 2B_1) e^{2x} + \\ &+ 12(3B_2 x^2 + 2B_1 x + B_0) e^{2x} \\ &+ 8(B_2 x^3 + B_1 x^2 + B_0 x) e^{2x} \end{aligned} \right.$$

$$\therefore \quad y_p''' - y_p'' - 4y_p' + 4y_p = 2x^2 - 4x - 1 + 2x^2 e^{2x} + 5x e^{2x} + e^{2x}$$

$$6B_2 e^{2x} + (36B_2 x + 12B_1) e^{2x} + (36B_2 x^2 + 24 B_1 x + 12B_0) e^{2x}$$

$$+ (8B_2 x^3 + 8B_1 x^2 + 8B_0 x) e^{2x}$$

$$- 2A_2 - (6B_2 x + 2B_1) e^{2x} -$$

$$- (12B_2 x^2 + 8B_1 x + 4B_0) e^{2x}$$

$$- (4B_2 x^3 + 4B_1 x^2 + 4B_0 x) e^{2x}$$

$$- 8A_2 x - 4A_1 - 4(3B_2 x^2 + 2B_1 x + B_0) e^{2x}$$

$$- 8(B_2 x^3 + B_1 x^2 + B_0 x) e^{2x}$$

$$+ 4A_2 x^2 + 4A_1 x + 4A_0 + 4(B_2 x^3 + B_1 x^2 + B_0 x) e^{2x}$$

$$= 2x^2 - 4x - 1 + 2x^2 e^{2x} + 5x e^{2x} + e^{2x}$$

O termo $B_0 e^{2x}$ presente em y_{p2} está presente também em y_h.

Logo modificamos y_{p2} pondo:

$$\tilde{y}_{p2} = x\, y_{p2} = (B_2 x^3 + B_1 x^2 + B_0 x) e^{2x}$$

$$\therefore \quad y_p = y_{p1} + \tilde{y}_{p2}$$

$$= A_2 x^2 + A_1 x + A_0 + (B_2 x^3 + B_1 x^2 + B_0 x) e^{2x}$$

$$y'_p = 2A_2 x + A_1 + (3B_2 x^2 + 2B_1 x + B_0) e^{2x}$$
$$+ 2(B_2 x^3 + B_1 x^2 + B_0 x) e^{2x}$$

$$y''_p = 2A_2 + (6B_2 x + 2B_1) e^{2x} + 2(3B_2 x^2 + 2B_1 x + B_0) e^{2x}$$
$$+ 2(3B_2 x^2 + 2B_1 x + B_0) e^{2x} + 4(B_2 x^3 + B_1 x^2 + B_0 x) e^{2x}$$

$\therefore$

$$y''_p = 2A_2 + (6B_2 x + 2B_1) e^{2x} + 4(3B_2 x^2 + 2B_1 x + B_0) e^{2x}$$
$$+ 4(B_2 x^3 + B_1 x^2 + B_0 x) e^{2x}$$

$$y'''_p = 6B_2 e^{2x} + 2(6B_2 x + 2B_1) e^{2x} +$$
$$+ 4(6B_2 x + 2B_1) e^{2x} + 8(3B_2 x^2 + 2B_1 x + B_0) e^{2x}$$
$$+ 4(3B_2 x^2 + 2B_1 x + B_0) e^{2x}$$
$$+ 8(B_2 x^3 + B_1 x^2 + B_0 x) e^{2x}$$

$$e^{2x}\left(\underline{6B_2 + 12B_1 + 12B_0 - 2B_1 - 4B_0 - 4B_0}\right)$$

$$+\, x e^{2x}\left(36B_2 + 24B_1 + \cancel{8B_0} - 6B_2 - 8B_1 - \cancel{4B_0} - 8B_1 - \cancel{8B_0} + \cancel{4B_0}\right)$$

$$+\, x^2 e^{2x}\left(36B_2 + \cancel{8B_1} - 12B_2 - \cancel{4B_1} - 12B_2 - \cancel{8B_1} + \cancel{4B_1}\right)$$

$$+\, x^3 e^{2x}\left(\cancel{8B_2} - \cancel{4B_2} - \cancel{8B_2} + \cancel{4B_2}\right) \quad \text{Ok!}$$

$$\left(-2A_2 - 4A_1 + 4A_0\right)$$

$$+\left(-8A_2 x + 4A_1 x\right)$$

$$+\, 4A_2 x^2 \equiv 2x^2 - 4x - 1 + 2x^2 e^{2x} + 5x e^{2x} + e^{2x}$$

$$\Rightarrow$$

$$6B_2 + 10B_1 + 4B_0 = 1 \Longrightarrow 1 + 4B_0 = 1 \Rightarrow \boxed{B_0 = 0}$$

$$30B_2 + 8B_1 = 5 \Longrightarrow \frac{30}{6} + 8B_1 = 5 \Rightarrow \boxed{B_1 = 0}$$

$$12B_2 = 2 \Longrightarrow \boxed{B_2 = \frac{1}{6}}$$

$$-2A_2 - 4A_1 + 4A_0 = -1 \Longrightarrow \cancel{-1} + 4A_0 = \cancel{-1} \quad \boxed{A_0 = 0}$$

$$-8A_2 + 4A_1 = -4 \Longrightarrow -2A_2 + A_1 = -1$$

$$-1 + A_1 = -1 \quad \therefore \boxed{A_1 = 0}$$

$$4A_2 = 2 \Longrightarrow \boxed{A_2 = \frac{1}{2}}$$

$\therefore \; y_p = A_2x^2 + A_1x + A_0 + (B_2x^3 + B_1x^2 + B_0x)e^{2x}$

$$// \; y_p = \frac{x^2}{2} + \frac{x^3}{6}e^{2x} \; //$$

$$\boxed{y = y_h + y_p = k_0 e^{x} + k_1 e^{-2x} + k_2 e^{2x} + \frac{x^2}{2} + \frac{x^3}{6}e^{2x}}$$

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12. $y''' + 4y'' + 9y' + 10y = -e^{x}$

$\underline{\underline{y_h}}$ $\quad \lambda^3 + 4\lambda^2 + 9\lambda + 10 = 0$

$\underline{\underline{\lambda = -2}}$ $\quad (-2)^3 + 4(-2)^2 + 9(-2) + 10 =$

$= -8 + 16 - 18 + 10$

$= 0$

$$\begin{array}{l|l}
\lambda^3 + 4\lambda^2 + 9\lambda + 10 & \lambda + 2 \\ \hline
-\lambda^3 - 2\lambda^2 & \lambda^2 + 2\lambda + 5 \\
\quad 2\lambda^2 + 9\lambda + 10 & \\
\quad -2\lambda^2 - 4\lambda & \\
\qquad 5\lambda + 10 & \\
\qquad -5\lambda - 10 & \\
\qquad\quad 0 &
\end{array}$$

$\lambda^2 + 2\lambda + 5 = 0$

$\lambda = -2 \pm \sqrt{4 - 20} \,/\, 2$

$= \frac{-2 \pm 4i}{2} \rightarrow -1 \pm 2i$

Raízes:

$k_1 = -2 \longrightarrow e^{-2x}$

$k_2 = -1 \pm 2i \longrightarrow e^{-x}\cos 2x \quad , \quad e^{-x}\sin 2x$

$$\| y_h = k_0 e^{-2x} + k_1 e^{-x}\cos 2x + k_2 e^{-x}\sin 2x \|$$

y_p : $y''' + 4y'' + 9y' + 10y = -e^x$

$r(x) = -e^x \longrightarrow y_p = Ae^x \rightarrow$ não está presente em y_h

$$\left.\begin{array}{l} y_p' = Ae^x \\ y_p'' = Ae^x \\ y_p''' = Ae^x \end{array}\right\} \Rightarrow Ae^x + 4Ae^x + 9Ae^x + 10Ae^x = -e^x$$

$$24Ae^x = -e^x$$

$$\therefore A = -\frac{1}{24}$$

$$\therefore \| y_p = -\frac{1}{24} e^x \|$$

$$\therefore \boxed{y = y_h + y_p = k_0 e^{-2x} + k_1 e^{-x}\cos 2x + k_2 e^{-x}\sin 2x - \frac{1}{24}e^x}$$

13. $y''' + y'' - 2y = x^2 + 10\cos 2x$

y_h: $\lambda^3 + \lambda^2 - 2 = 0$

$\lambda = 1$

$$\begin{array}{l|l} \cancel{\lambda^3} + \lambda^2 - 2 & \lambda - 1 \\ \hline -\lambda^3 + \lambda^2 & \lambda^2 + 2\lambda + 2 \\ \hline \cancel{2\lambda^2} - 2 & \\ -2\lambda^2 + 2\lambda & \\ \hline 2\lambda - 2 & \\ -2\lambda + 2 & \\ \hline 0 & \end{array}$$

$\lambda^2 + 2\lambda + 2 = 0$

$\lambda = -2 \pm \sqrt{4-8} / 2$

$\lambda = \dfrac{-2 \pm 2i}{2} = -1 \pm i$

Raízes

$\lambda = 1 \longrightarrow e^x$

$\lambda = -1 \pm i \longrightarrow e^{-x}\cos x \; ; \; e^{-x}\sin x$

$$// \; y_h = k_0 e^x + k_1 e^{-x}\cos x + k_2 e^{-x}\sin x \; //$$

y_p $\quad r(x) = x^2 + 10\cos 2x$

Forma Básica

$x^2 \longrightarrow y_{p1} \equiv Ax^2 + Bx + C$

$10\cos 2x \longrightarrow y_{p2} = D\cos 2x + E\sin 2x$

$\hookrightarrow$ não está presente em y_h

$$y_p = Ax^2 + Bx + C + D\cos 2x + E\sin 2x$$

$$y_p' = 2Ax + B - 2D\sin 2x + 2E\cos 2x$$

$$y_p'' = 2A - 4D\cos 2x - 4E\sin 2x$$

$$y_p''' = 8D\sin 2x - 8E\cos 2x$$

$$\therefore \quad y_p''' + y_p'' - 2y_p = x^2 + 10\cos 2x$$

$$\left\{\begin{array}{l} 8D\sin 2x - 8E\cos 2x + 2A - 4D\cos 2x - 4E\sin 2x \\ - 2Ax^2 - 2Bx - 2C - 2D\cos 2x - 2E\sin 2x = \\ = x^2 + 10\cos 2x \end{array}\right.$$

$$\sin 2x\,(8D - 6E) + \cos 2x\,(-8E - 6D) +$$
$$+ (2A - 2C) - 2Bx - 2Ax^2 = x^2 + 10\cos 2x$$

$$\left.\begin{array}{l} 8D - 6E = 0 \\ -8E - 6D = 10 \end{array}\right\}$$

$$2A - 2C = 0 \implies -1 - 2C = 0 \implies C = -\frac{1}{2}$$

$$-2B = 0 \implies B = 0$$

$$-2A = 1 \implies A = -\frac{1}{2}$$

$$-8E - 6D = 10$$

$$\Longrightarrow \quad -8E - \cancel{6}^{3} \cdot \frac{3E}{\cancel{4}_{2}} = 10$$

$$-8E - \frac{9E}{2} = 10$$

$$-16E - 9E = 20$$

$$-25E = 20$$

$$E = \frac{-20}{25} = \frac{-4}{5}$$

$$D = \frac{3}{4}\left(-\frac{4}{5}\right) = -\frac{3}{5}$$

$$\therefore \quad y_p = -\frac{x^2}{2} - \frac{1}{2} - \frac{5}{9}\cos 2x - \frac{20}{27}\sin 2x$$

$$y = y_h + y_p$$

$$\boxed{\begin{aligned} y = {} & k_0 e^{x} + k_1 e^{-x}\cos x + k_2 e^{-x}\sin x + \\ & -\frac{1}{2} - \frac{x^2}{2} - \frac{3}{5}\cos 2x - \frac{4}{5}\sin 2x \end{aligned}}$$

Lista 14 – Respostas

1. $A + Be^{4x} + Ce^{-3x}$

2. $y = Ae^{-3x} + Be^{-x} + Ce^{2x}$

3. $y = Ae^{x} + Bxe^{x} + Cx^2e^{x}$

4. $y = Ae^{2x} + Be^{-2x} + Ce^{-3x} + Dxe^{-3x}$

5. $y = Ae^{-x} + Bxe^{-x} + Cx^2e^{-x} + De^{4x}$

6. $y = A + B\cos 2x + C \sin 2x$

7. $y = e^{-x}(A\cos\sqrt{2}x + B\sin\sqrt{2}x) +$
$+ e^{\frac{x}{2}}(C\cos\frac{\sqrt{3}}{2}x + D\sin\frac{\sqrt{3}}{2}x)$

8. $y = A\cos 3x + B\sin 3x + Ce^{2x} + De^{-2x}$

9. $y = Ae^{x} + Be^{-x} + Ce^{-2x} + \frac{x}{6}e^{x} - \frac{x^2}{2} + \frac{x}{2} - \frac{5}{4}$

10. $y = C_0 + C_1e^{-x} + C_2e^{-2x} + \frac{11}{4}x + \frac{x^2}{4} + \frac{x^3}{6}$

11. $y = k_0e^{x} + k_1e^{-2x} + k_2e^{2x} + \frac{x^2}{2} + \frac{x^3}{6}e^{2x}$

12. $y = k_0e^{-2x} + k_1e^{-x}\cos 2x + k_2e^{-x}\sin 2x - \frac{1}{24}e^{x}$

13. $y = k_0e^{x} + k_1e^{-x}\cos x + k_2e^{-x}\sin x - \frac{1}{2} -$
$-\frac{x^2}{2} - \frac{3}{5}\cos 2x - \frac{4}{5}\sin 2x$

1. $y'' - y = 0$

$\lambda^2 - 1 = 0 \quad \therefore \quad \lambda = \pm 1$: 2 raizes reais distintas

$$\| y = c_1 e^{x} + c_2 e^{-x} \|$$

2. $4y'' - 9y = 0$

$4\lambda^2 - 9 = 0 \quad \therefore \quad \lambda^2 = \frac{9}{4} \quad \therefore \quad \lambda = \pm \frac{3}{2}$

$$\| y = c_1 e^{\frac{3}{2}x} + c_2 e^{-\frac{3}{2}x} \|$$

3. $y'' - 9y = 0$

$\lambda^2 - 9 = 0 \quad \therefore \quad \lambda = \pm 3$

$$\| y = c_1 e^{3x} + c_2 e^{-3x} \|$$

4. $y'' - 4y' + 3y = 0$

$\lambda^2 - 4\lambda + 3 = 0$

$\lambda = \frac{4 \pm \sqrt{16 - 12}}{2} = \frac{4 \pm 2}{2}$ $\nearrow 3$, $\searrow 1$

$y = C_1 e^{3x} + C_2 e^{x}$

Cálculo C - Lista 14

Equações diferenciais lineares de ordem n

(I) Equações homogêneas com coeficientes constantes

1. $y''' - y'' - 12y' = 0$ correção
2. $y''' + 2y'' - 5y' - 6y = 0$
3. $y''' - 3y'' + 3y' - y = 0$
4. $y^{(4)} + 6y''' + 5y'' - 24y' - 36y = 0$
5. $y^{(4)} - y''' - 9y'' - 11y' - 4y = 0$
6. $y''' + 4y' = 0$
7. $y^{(4)} + y''' + 2y'' - y' + 3y = 0$
8. $y^{(4)} + 5y'' - 36y = 0$

(II) Métodos dos coeficientes a determinar

9. $y''' + 2y'' - y' - 2y = e^x + x^2$
10. $y''' + 3y'' + 2y' = x^2 + 4x + 8$
11. $y''' - y'' - 4y' + 4y = 2x^2 - 4x - 1 + 2x^2e^{2x} + 5xe^{2x} + e^{2x}$
12. $y''' + 4y'' + 9y' + 10y = -e^x$
13. $y''' + y'' - 2y = x^2 + 10\cos 2x$
14. O que é o princípio da superposição? Ele se aplica a equações não lineares? Ele se aplica a equações lineares não-homogêneas? Ele se aplica a equações lineares homogêneas?
15. Quantas constantes arbitrárias estão presentes na solução geral da equação linear não-homogênea de ordem n? Quantas condições adicionais são necessárias para se determinar estas constantes?
16. Como determinar se duas soluções de uma equação diferencial linear são linearmente independentes?