Cálculo C - Lista 15

Transformada de Laplace (I)

Propriedades:

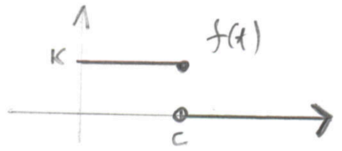
$\mathcal{L}(f') = s\mathcal{L}(f) - f(0)$ (1)

$\mathcal{L}(f^{(n)}) = s^n\mathcal{L}(f) - s^{n-1}f(0) - s^{n-2}f'(0) - - \dots - f^{(n-1)}(0)$ (2)

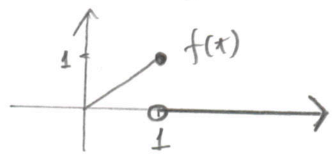
$\mathcal{L}(\int_0^t f(u)du) = \frac{1}{s}\mathcal{L}(f)$ (3)

Encontre a transformada de Laplace das seguintes funções

1. $t + 4$
2. $a + bt + ct^2$
3.



4.



5.

$$f(t) = \begin{cases} 0, & 0 \le t < 1 \\ t, & 1 \le t < 2 \\ 1, & 2 \le t \end{cases}$$

6.

$$f(t) = \begin{cases} 0, & 0 \le t \le 5 \\ e^{-t}, & 5 < t \end{cases}$$

7. $(t^2 + \frac{1}{2})^2$
8. $\sin \pi t$
9. $\cos(\omega t + \theta)$
10. $\cos^2 t$
11. $\cos^2 \omega t$
12. $\sinh^2 2t$

Use os teoremas da derivada (1) e (2) e obtenha as transformadas a seguir

13. $\mathcal{L}(t\cos\omega t) = \frac{s^2-\omega^2}{(s^2+\omega^2)^2}$
14. $\mathcal{L}(t\sin\omega t) = \frac{2\omega s}{(s^2+\omega^2)^2}$
15. $\mathcal{L}(t\cosh at) = \frac{s^2+a^2}{(s^2-a^2)^2}$
16. $\mathcal{L}(t\sinh at) = \frac{2as}{(s^2-a^2)^2}$

Use o teorema da integral (3) para calcular $f(t)$ sendo dado $\mathcal{L}(f)$

17. $\frac{1}{s^2+s}$
18. $\frac{10}{s(s^2+9)}$
19. $\frac{1}{s^2(s+1)}$
20. $\frac{1}{s^2}\left(\frac{s-1}{s+1}\right)$
21. $\frac{54}{s^3(s-3)}$
22. $\frac{1}{s^2}\left(\frac{s+1}{s^2+1}\right)$

Lista 15

1.

$$\mathcal{L}(t+4) = \mathcal{L}(t) + 4\mathcal{L}(1)$$

$$= \frac{1}{s} + 4 \; ; \quad \left.\begin{matrix} \\ s>0 \quad s>0 \end{matrix}\right\} \text{ para } s>0$$

$$\boxed{\mathcal{L}(t+4) = \frac{1+4s}{s} \; ; \; s>0}$$

2. $$\mathcal{L}(a+bt+ct^2) = a\mathcal{L}(1) + b\mathcal{L}(t) + c\mathcal{L}(t^2)$$

$$= \frac{a}{s} + \frac{b}{s^2} + \frac{2c}{s^3}$$

3. $$f(t) = \begin{cases} k, & 0 \leq t \leq c \\ 0, & t > c \end{cases}$$

$$\mathcal{L}(f(t)) = \int_0^{+\infty} e^{-st} f(t)\, dt$$

$$= \int_0^c e^{-st} k \, dt$$

$$= \left. \frac{e^{-st}k}{-s} \right|_0^c = \frac{e^{-sc}k}{-s} + \frac{k}{s}$$

$$\boxed{\mathcal{L}(f(t)) = \frac{k}{s}\left(1 - e^{-sc}\right)}$$

4.
$$f(t)=\begin{cases} t, & 0\le t\le 1 \\ 0, & t>1 \end{cases}$$

$$\mathcal{L}(f(t)) = \int_0^\infty e^{-st} f(t)\,dt$$

$$= \int_0^1 e^{-st}\,t\,dt$$

$$\int e^{-st}t\,dt = -\frac{t}{s}e^{-st} + \int \frac{e^{-st}}{s}\,dt$$

$f=t \rightarrow df=dt$

$dg=e^{-st}dt \rightarrow g=\dfrac{e^{-st}}{-s}$

$$= -\frac{t}{s}e^{-st} - \frac{e^{-st}}{s^2}$$

$$= -\frac{t}{s}e^{-st} - \frac{e^{-st}}{s^2}\Bigg]_0^1$$

$$= -\frac{1}{s}e^{-s} - \frac{e^{-s}}{s^2} + \frac{1}{s^2}$$

$$\boxed{\mathcal{L}(f(t)) = -\frac{e^{-s}}{s} + \frac{1}{s^2}\left(1-e^{-s}\right)}$$

5. $$f(x) = \begin{cases} 0, & 0 \leq t < 1 \\ t, & 1 \leq t < 2 \\ 1, & 2 \leq t \end{cases}$$

$$\mathcal{L}(f(t)) = \int_0^\infty e^{-st} f(t)\, dt$$

$$= \int_0^1 e^{-st}\,\cancel{0}^{\,0}\, dt + \int_1^2 e^{-st}\, t\, dt +$$

$$+ \int_2^\infty e^{-st}\, t\, dt$$

$$= \int_1^2 e^{-st}\, t\, dt + \int_2^\infty e^{-st}\, dt$$

Mas

$$\int e^{-st}\, t\, dt = -\frac{t e^{-st}}{s} + \int \frac{e^{-st}}{s}\, dt$$

$$u = t \longrightarrow du = dt$$

$$dv = e^{-st} dt \longrightarrow v = \frac{e^{-st}}{-s}$$

$$= -\frac{t e^{-st}}{s} - \frac{e^{-st}}{s^2}$$

$\therefore$

$$\int_1^2 e^{-st}\,t\,dt = -\frac{t e^{-st}}{s}\Big]_1^2 - \frac{e^{-st}}{s^2}\Big]_1^2$$

$$= -\frac{2e^{-2s}}{s} + \frac{e^{-s}}{s} - \frac{e^{-2s}}{s^2} + \frac{e^{-s}}{s^2}$$

$$= \frac{e^{-2s}}{s}\left[-2-\frac{1}{s}\right] + \frac{e^{-s}}{s}\left(1+\frac{1}{s}\right)$$

$$= \frac{e^{-2s}}{s}\,\frac{(-2s-1)}{s} + \frac{e^{-s}}{s}\left(\frac{s+1}{s}\right)$$

$$= -\frac{2s+1}{s^2}e^{-2s} + \frac{s+1}{s^2}e^{-s} \quad //$$

$$\int_2^\infty e^{-st}\,dt = \lim_{k\to\infty} \frac{e^{-st}}{-s}\Big]_2^k$$

$$= \lim_{k\to\infty}\left[\underbrace{\frac{e^{-sk}}{-s}}_{\to 0} + \frac{e^{-2s}}{s}\right]$$

& $s>0$

$$= \frac{e^{-2s}}{s}, \quad \underline{s>0}$$

$\therefore$

$$\mathcal{L}(f(t)) = -\frac{2s+1}{s^2}e^{-2s} + \frac{s+1}{s^2}e^{-s} + \frac{e^{-2s}}{s}, \quad s>$$

$$\boxed{\mathcal{L}(f(t)) = -\frac{s+1}{s^2}e^{-2s} + \frac{s+1}{s^2}e^{-s}, \quad s>0}$$

6. $$f(t) = \begin{cases} 0, & 0 \le t \le 5 \\ e^{-t}, & 5 < t \end{cases}$$

$$\mathcal{L}(f(t)) = \int_0^\infty e^{-st} f(t)\,dt$$

$$= \int_0^5 \cancelto{0}{e^{-st}\,0}\,dt + \int_5^\infty e^{-st} e^{-t}\,dt$$

$$= \int_5^\infty e^{-(s+1)t}\,dt$$

$$= \lim_{k\to\infty} \frac{-1}{(s+1)} e^{-(s+1)t}\Big]_5^k$$

$$= \lim_{k\to\infty}\left[\frac{-1}{s+1} \underbrace{e^{-(s+1)k}}_{\to 0} + \frac{1}{(s+1)} e^{-(s+1)5}\right]$$

0 le $s+1 > 0$

$\therefore$

$$\boxed{\mathcal{L}(f(t)) = \frac{1}{s+1} e^{-5(s+1)}, \quad s > -1}$$

7. $\mathcal{L}((t^2+\frac{1}{2})^2) = \mathcal{L}(t^4+t^2+\frac{1}{4})$

$= \mathcal{L}(t^4) + \mathcal{L}(t^2) + \frac{1}{4}\mathcal{L}(1)$

$= \frac{4!}{s^5} + \frac{1\,2!}{s^3} + \frac{1}{4}\frac{1}{s}\ ;\ s>0$

$$\boxed{\mathcal{L}((t^2+\tfrac{1}{2})^2) = \frac{24}{s^5} + \frac{2}{s^3} + \frac{1}{4s}\ ;\ s>0}$$

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8. $\mathcal{L}(\sin\pi t) = \int_0^\infty e^{-st}\sin\pi t\,dt$

$u = \sin\pi t \to du = \pi\cos\pi t\,dt$

$dv = e^{-st}\,dt \to v = \frac{e^{-st}}{-s}$

$\int e^{-st}\sin\pi t\,dt = -\sin\pi t\,\frac{e^{-st}}{s} - \int \pi\cos\pi t\,\frac{e^{-st}}{-s}\,dt$

$= -\sin\pi t\,\frac{e^{-st}}{s} + \frac{\pi}{s}\int \cos\pi t\, e^{-st}\,dt$ (*)

Mas

$\int \cos\pi t\, e^{-st}\,dt$

$u = \cos\pi t \to du = -\pi\sin\pi t\,dt$

$dv = e^{-st}\,dt \to v = \frac{e^{-st}}{-s}$

8. Cont.

$$\int \cos \pi t \, e^{-st} dt = -\frac{e^{-st}}{s} \cos \pi t - \int \pi \sin \pi t \frac{e^{-st}}{s} dt$$

$$= -\frac{e^{-st}}{s} \cos \pi t - \frac{\pi}{s} \int \sin \pi t \, e^{-st} dt \quad (**)$$

$\therefore$ (**) $\to$ (*) :

$$\int e^{-st} \sin \pi t \, dt = -\sin \pi t \frac{e^{-st}}{s} + \frac{\pi}{s} \left[-\frac{e^{-st}}{s} \cos \pi t - \frac{\pi}{s} \int \sin \pi t \, e^{-st} dt \right]$$

$\therefore$

$$\int e^{-st} \sin \pi t \, dt = -\sin \pi t \frac{e^{-st}}{s} - \pi \frac{e^{-st}}{s^2} \cos \pi t - \frac{\pi^2}{s^2} \int \sin \pi t \, e^{-st} dt$$

$\therefore$

$$\left(1 + \frac{\pi^2}{s^2}\right) \int e^{-st} \sin \pi t \, dt = -\sin \pi t \frac{e^{-st}}{s} - \pi \cos \pi t \frac{e^{-st}}{s^2}$$

$$\int e^{-st} \sin \pi t \, dt = -\frac{s^2}{s^2 + \pi^2} \left(\sin \pi t \frac{e^{-st}}{s} + \pi \cos \pi t \frac{e^{-st}}{s^2} \right)$$

$\therefore$

$$\int_0^\infty e^{-st}\sin\pi t\,dt =$$

$$= \lim_{k\to\infty} \frac{-s^2}{s^2+\pi^2}\left(\sin\pi t\,\frac{e^{-st}}{s} + \pi\cos\pi t\,\frac{e^{-st}}{s^2}\right)\Bigg]_0^k$$

$$= \lim_{k\to\infty} \frac{-s^2}{s^2+\pi^2}\left(\underbrace{\sin\pi k\,\frac{e^{-sk}}{s}}_{\to 0 \text{ se } s>0} + \underbrace{\pi\cos\pi k\,\frac{e^{-sk}}{s^2}}_{\to 0 \text{ se } s>0} - \frac{\pi}{s^2}\right)$$

$$= \frac{-\cancel{s^2}}{s^2+\pi^2}\left(-\frac{\pi}{\cancel{s^2}}\right)$$

$$= \frac{\pi}{s^2+\pi^2}$$

$$\therefore \boxed{\mathcal{L}(\sin\pi t) = \frac{\pi}{s^2+\pi^2}} \quad ; (s>0)$$

9. $\mathcal{L}(\cos(\omega t+\theta)) = \mathcal{L}\left(\frac{e^{i(\omega t+\theta)} + e^{-i(\omega t+\theta)}}{2}\right)$

$$= \frac{1}{2}\mathcal{L}(e^{i\theta}e^{i\omega t}) + \frac{1}{2}\mathcal{L}(e^{-i\theta}e^{-i\omega t})$$

$$= \frac{1}{2}e^{i\theta}\mathcal{L}(e^{i\omega t}) + \frac{1}{2}e^{-i\theta}\mathcal{L}(e^{-i\omega t})$$

$$= \frac{e^{i\theta}}{2}\frac{1}{s-i\omega} + \frac{e^{-i\theta}}{2}\frac{1}{s+i\omega}$$

$$= \frac{e^{i\theta}}{2}\frac{s+i\omega}{s^2+\omega^2} + \frac{e^{-i\theta}}{2}\frac{s-i\omega}{s^2+\omega^2}$$

$$= \frac{1}{s^2+\omega^2}\left(e^{i\theta}s + i\omega e^{i\theta} + e^{-i\theta}s - i\omega e^{-i\theta}\right)$$

$$= \frac{1}{2}\frac{1}{(s^2+\omega^2)}\left[s\,(e^{i\theta}+e^{-i\theta}) + i\omega\,(e^{i\theta}-e^{-i\theta})\right]$$

$$= \frac{1}{2}\frac{1}{(s^2+\omega^2)}\left[s\,2\cos\theta + i\omega\,2i\,\mathrm{sen}\,\theta\right]$$

$$= \frac{1}{\cancel{2}}\frac{1}{(s^2+\omega^2)}\left[\cancel{2}s\cos\theta - \cancel{2}\omega\,\mathrm{sen}\,\theta\right]$$

$$= \boxed{\frac{1}{s^2+\omega^2}\left[s\cos\theta - \omega\,\mathrm{sen}\,\theta\right]}$$

10.

$\to \mathcal{L}(\cos^2 t) = \frac{1}{4}\mathcal{L}(e^{2it}) + \frac{1}{4}\mathcal{L}(e^{-2it}) + \frac{1}{2}\mathcal{L}(1)$

$$\cos t = \frac{e^{it}+e^{-it}}{2}$$

$$\cos^2 t = \frac{e^{2it}+2+e^{-2it}}{4}$$

$$= \frac{e^{2it}}{4} + \frac{e^{-2it}}{4} + \frac{1}{2}$$

$$\to = \frac{1}{4}\frac{1}{s-2i} + \frac{1}{4}\frac{1}{s+2i} + \frac{1}{2s}$$

$$= \frac{1}{4}\frac{s+2i}{s^2+4} + \frac{1}{4}\frac{s-2i}{s^2+4} + \frac{1}{2s}$$

$$= \frac{1}{4(s^2+4)}\left[s+\cancel{2i}+s-\cancel{2i}\right] + \frac{1}{2s}$$

$$= \frac{1}{4(s^2+4)}2s + \frac{1}{2s}$$

$$\boxed{\mathcal{L}(\cos^2 t) = \frac{s}{2(s^2+4)} + \frac{1}{2s}}$$

11. $\mathcal{L}(\cos^2 \omega t) = \mathcal{L}\left(\frac{e^{2i\omega t}}{4}\right) + \mathcal{L}\left(\frac{e^{-2i\omega t}}{4}\right) + \mathcal{L}\left(\frac{1}{2}\right)$

$$\cos \omega t = \frac{e^{i\omega t} + e^{-i\omega t}}{2}$$

$$\cos^2 \omega t = \frac{e^{2i\omega t} + 2 + e^{-2i\omega t}}{4}$$

$$= \frac{e^{2i\omega t}}{4} + \frac{e^{-2i\omega t}}{4} + \frac{1}{2}$$

$$= \frac{1}{4}\frac{1}{s-2i\omega} + \frac{1}{4}\frac{1}{s+2i\omega} + \frac{1}{2s}$$

$$= \underbrace{\frac{1}{4}\frac{s+2i\omega}{s^2+4\omega^2} + \frac{1}{4}\frac{s-2i\omega}{s^2+4\omega^2}} + \frac{1}{2s}$$

$$= \frac{1}{\cancel{4}}\frac{\cancel{2}s}{s^2+4\omega^2} + \frac{1}{2s}$$

$$\boxed{\mathcal{L}(\cos^2 \omega t) = \frac{s}{2s^2+8\omega^2} + \frac{1}{2s}}$$

12. $\mathcal{L}(\sinh^2 2t) = \frac{1}{4}\mathcal{L}(e^{4t}) + \frac{1}{4}\mathcal{L}(e^{-4t}) - \frac{1}{2}\mathcal{L}(1)$

$\sinh 2t = \dfrac{e^{2t} - e^{-2t}}{2}$

$\sinh^2 2t = \dfrac{e^{4t} - 2 + e^{-4t}}{4}$

$$= \frac{1}{4}\frac{1}{s-4} + \frac{1}{4}\frac{1}{s+4} - \frac{1}{2}\frac{1}{s}$$

$$= \frac{1}{4}\left(\frac{1}{s-4} + \frac{1}{s+4}\right) - \frac{1}{2s}$$

$$= \frac{1}{4}\frac{s\cancel{+4} + s\cancel{-4}}{s^2-16} - \frac{1}{2s}$$

$$= \frac{1}{\cancel{4}_{2}}\frac{\cancel{2}s}{s^2-16} - \frac{1}{2s}$$

$$= \frac{1}{2}\frac{s}{s^2-16} - \frac{1}{2s}$$

$$= \boxed{\frac{1}{2}\left[\frac{s}{s^2-16} - \frac{1}{s}\right]}$$

13 ; 14

Seja $\begin{cases} f(t) = t\cos\omega t \\ g(t) = t\sin\omega t \end{cases}$

Então

$$f'(t) = \cos\omega t - t\omega\sin\omega t$$

$$g'(t) = \sin\omega t + t\omega\cos\omega t$$

$\rightarrow \mathcal{L}(f') = s\,\mathcal{L}(f) - f(0)$

$$\mathcal{L}(\cos\omega t - t\omega\sin\omega t) = s\,\mathcal{L}(t\cos\omega t) -$$

$$\underbrace{\mathcal{L}(\cos\omega t)} - \omega\,\mathcal{L}(t\sin\omega t) = s\,\mathcal{L}(t\cos\omega t)$$

$$\therefore \quad \omega\,\mathcal{L}(t\sin\omega t) + s\,\mathcal{L}(t\cos\omega t) = \underbrace{\frac{s}{s^2+\omega^2}}$$

(*)

$\rightarrow \mathcal{L}(g') = s\,\mathcal{L}(g) - g(0)$

$$\mathcal{L}(\sin\omega t + t\omega\cos\omega t) = s\,\mathcal{L}(t\sin\omega t)$$

$$\underbrace{\mathcal{L}(\sin\omega t)} + \omega\,\mathcal{L}(t\cos\omega t) = s\,\mathcal{L}(t\sin\omega t)$$

$$\omega\,\mathcal{L}(t\cos\omega t) - s\,\mathcal{L}(t\sin\omega t) = -\frac{\omega}{s^2+\omega^2}$$

(*1)

$$(*) \times s : \quad \omega s \, \mathcal{L}(t \sin \omega t) + s^2 \mathcal{L}(t \cos \omega t) = \frac{s^2}{s^2+\omega^2}$$

$$(*1) \times \omega : \quad \omega^2 \mathcal{L}(t \cos \omega t) - s\omega \, \mathcal{L}(t \sin \omega t) = \frac{-\omega^2}{s^2+\omega^2}$$

$$\Rightarrow \quad (\omega^2 + s^2) \, \mathcal{L}(t \cos \omega t) = \frac{s^2-\omega^2}{s^2+\omega^2}$$

$$\boxed{\mathcal{L}(t \cos \omega t) = \frac{s^2-\omega^2}{(s^2+\omega^2)^2}}$$

$$(*) : \quad \omega \, \mathcal{L}(t \sin \omega t) + s \, \mathcal{L}(t \cos \omega t) = \frac{s}{s^2+\omega^2}$$

$$\omega \mathcal{L}(t \sin \omega t) + s \frac{s^2-\omega^2}{(s^2+\omega^2)^2} = \frac{s}{s^2+\omega^2}$$

$$\omega \mathcal{L}(t \sin \omega t) = \frac{s}{s^2+\omega^2} - s \frac{s^2-\omega^2}{(s^2+\omega^2)^2}$$

$$= \frac{s}{s^2+\omega^2}\left[1 - \frac{(s^2-\omega^2)}{s^2+\omega^2}\right]$$

$$= \frac{s}{s^2+\omega^2}\left[\frac{\cancel{s^2}+\omega^2-\cancel{s^2}+\omega^2}{s^2+\omega^2}\right]$$

$$\cancel{\omega} \, \mathcal{L}(t \sin \omega t) = \frac{2s\cancel{\omega^2}}{(s^2+\omega^2)^2} \quad \therefore \quad \boxed{\mathcal{L}(t \sin \omega t) = \frac{2s\omega}{(s^2+\omega^2)^2}}$$

15; 16

$$\begin{cases} f(t) = t\cosh at \\ g(t) = t\sinh at \end{cases}$$

Então

$$f'(t) = \cosh at + at\sinh at$$

$$g'(t) = \sinh at + at\cosh at$$

$\rightarrow \mathcal{L}(f') = s\,\mathcal{L}(f) - f(0)$

$$\mathcal{L}(\cosh at + at\sinh at) = s\,\mathcal{L}(t\cosh at)$$

$$\mathcal{L}(\cosh at) + a\,\mathcal{L}(t\sinh at) = s\,\mathcal{L}(t\cosh at)$$

$$\frac{s}{s^2-a^2} + a\,\mathcal{L}(t\sinh at) = s\,\mathcal{L}(t\cosh at)$$

$$// \; a\,\mathcal{L}(t\sinh at) - s\,\mathcal{L}(t\cosh at) = \frac{-s}{s^2-a^2} \; //$$

$\rightarrow \mathcal{L}(g') = s\,\mathcal{L}(g) - g(0)$

$$\mathcal{L}(\sinh at + at\cosh at) = s\,\mathcal{L}(t\sinh at)$$

$$\mathcal{L}(\sinh at) + a\,\mathcal{L}(t\cosh at) = s\,\mathcal{L}(t\sinh at)$$

$$\frac{a}{s^2-a^2} + a\,\mathcal{L}(t\cosh at) = s\,\mathcal{L}(t\sinh at)$$

$$// \; a\,\mathcal{L}(t\cosh at) - s\,\mathcal{L}(t\sinh at) = \frac{-a}{s^2-a^2} \; //$$

$$a\mathcal{L}(t\sinh at) - s\mathcal{L}(t\cosh at) = \frac{-s}{s^2-a^2}$$

$$a\mathcal{L}(t\cosh at) - s\mathcal{L}(t\sinh at) = \frac{-a}{s^2-a^2}$$

$$as\mathcal{L}(t\sinh at) - s^2\mathcal{L}(t\cosh at) = \frac{-s^2}{s^2-a^2}$$

$$a^2\mathcal{L}(t\cosh at) - as\mathcal{L}(t\sinh at) = \frac{-a^2}{s^2-a^2}$$

$$(a^2-s^2)\mathcal{L}(t\cosh at) = -\frac{s^2+a^2}{s^2-a^2}$$

$$\mathcal{L}(t\cosh at) = -\frac{s^2+a^2}{(a^2-s^2)(s^2-a^2)}$$

$$\boxed{\mathcal{L}(t\cosh at) = \frac{s^2+a^2}{(s^2-a^2)^2}}$$

$$\rightarrow a\mathcal{L}(t\sinh at) - s\mathcal{L}(t\cosh at) = \frac{-s}{s^2-a^2}$$

$$a\mathcal{L}(t\sinh at) - \frac{s(s^2+a^2)}{(s^2-a^2)^2} = \frac{-s}{s^2-a^2}$$

$$a\mathcal{L}(t\sinh at) = \frac{s(s^2+a^2)}{(s^2-a^2)^2} - \frac{s}{(s^2-a^2)}$$

$$= \frac{s(s^2+a^2) - s(s^2-a^2)}{(s^2-a^2)^2}$$

$$a\,\mathcal{L}(t\sinh at) = \frac{\cancel{s^3} + sa^2 - \cancel{s^3} + sa^2}{(s^2-a^2)^2}$$

$$\cancel{a}\,\mathcal{L}(t\sinh at) = \frac{2sa^{\cancel{2}}}{(s^2-a^2)^2}$$

$$\boxed{\mathcal{L}(t\sinh at) = \frac{2as}{(s^2-a^2)^2}}$$

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17. $$\mathcal{L}^{-1}\left(\frac{1}{s^2+s}\right) = \mathcal{L}^{-1}\left(\frac{1}{s(s+1)}\right)$$

$$\mathcal{L}\left(\int_0^t f(u)\,du\right) = \frac{1}{s}\mathcal{L}(f)$$

$$\Leftrightarrow$$

$$\int_0^t f(u)\,du = \mathcal{L}^{-1}\left(\frac{1}{s}\mathcal{L}(f)\right) \doteq \mathcal{L}^{-1}\left(\frac{1}{s(s+1)}\right)$$

$$\therefore \quad \mathcal{L}(f) = \frac{1}{s+1} \Rightarrow f = e^{-t}$$

$$\therefore \quad \int_0^t e^{-u}\,du = \mathcal{L}^{-1}\left(\frac{1}{s(s+1)}\right)$$

$$\left.\frac{e^{-u}}{-1}\right]_0^t = \mathcal{L}^{-1}\left(\frac{1}{s(s+1)}\right)$$

$$-e^{-t} + 1 = \mathcal{L}^{-1}\left(\frac{1}{s(s+1)}\right)$$

$$\therefore \quad \boxed{\mathcal{L}^{-1}\left(\frac{1}{s(s+1)}\right) = 1 - e^{-t}}$$

18. $\frac{10}{s(s^2+9)}$

$$\mathcal{L}\left(\int_0^t f(u)\,du\right) = \frac{1}{s}\mathcal{L}(f)$$

$$\int_0^t f(u)\,du = \mathcal{L}^{-1}\left(\frac{1}{s}\mathcal{L}(f)\right) \doteq \mathcal{L}^{-1}\left(\frac{10}{s(s^2+9)}\right)$$

$$\therefore \quad \mathcal{L}(f) = \frac{10}{s^2+9} =$$

$$\therefore \quad f = \mathcal{L}^{-1}\left(\frac{10}{s^2+9}\right)$$

$$= 10\,\mathcal{L}^{-1}\left(\frac{1}{s^2+9}\right)$$

$$= 10\,\frac{1}{3}\sin 3t$$

$$= \frac{10}{3}\sin 3t$$

$$\therefore \int_0^t \frac{10}{3}\sin 3t\,dt = \frac{10}{3}\,\frac{(-)}{3}\cos 3t\Big]_0^t$$

$$= -\frac{10}{9}\cos 3t + \frac{10}{9}$$

$$\therefore \boxed{\mathcal{L}^{-1}\left(\frac{10}{s(s^2+9)}\right) = \frac{10}{9}(1-\cos 3t)}$$

19. $\mathcal{L}^{-1}\left(\dfrac{1}{s^2(s+1)}\right)$

• $\mathcal{L}\left(\int_0^t f(u)\,du\right) = \dfrac{1}{s}\mathcal{L}(f)$

$(\star) = \int_0^t f(u)\,du = \mathcal{L}^{-1}\left(\dfrac{1}{s}\mathcal{L}(f)\right) \doteq \mathcal{L}^{-1}\left(\dfrac{1}{s^2(s+1)}\right)$

$\therefore \begin{cases} \mathcal{L}(f) = \dfrac{1}{s(s+1)} \\ f = \mathcal{L}^{-1}\left(\dfrac{1}{s(s+1)}\right) \end{cases}$

Também

• $\int_0^t g(u)\,du = \mathcal{L}^{-1}\left(\dfrac{1}{s}\mathcal{L}(g)\right) \doteq \mathcal{L}^{-1}\left(\dfrac{1}{s(s+1)}\right)$

$\therefore \mathcal{L}(g) = \dfrac{1}{s+1}$

$\therefore g = e^{-t}$

$\therefore \int_0^t g(u)\,du = \int_0^t e^{-u}\,du = \left.\dfrac{e^{-u}}{-1}\right|_0^t$

$= -e^{-t} + 1$

$\therefore$

$(\star\star) = f(t) = \mathcal{L}^{-1}\left(\dfrac{1}{s(s+1)}\right) = 1 - e^{-t}$

$\therefore$

$(**) \to (*)$:

$$\mathcal{L}^{-1}\left(\frac{1}{s^2(s+1)}\right) = \int_0^t (1-e^{-u})\,du$$

$$= u + e^{-u}\Big]_0^t$$

$$\boxed{\mathcal{L}^{-1}\left(\frac{1}{s^2(s+1)}\right) = t + e^{-t} - 1}$$

---

20. $\frac{1}{s^2}\left(\frac{s-1}{s+1}\right)$

$$\mathcal{L}\left(\int_0^t f(u)\,du\right) = \frac{1}{s}\mathcal{L}(f)$$

$$\int_0^t f(u)\,du = \mathcal{L}^{-1}\left(\frac{1}{s}\mathcal{L}(f)\right) \doteq \mathcal{L}^{-1}\left(\frac{1}{s}\,\frac{s-1}{s(s+1)}\right)$$

$$\therefore \quad \mathcal{L}(f) = \frac{s-1}{s(s+1)}$$

$(*)$ $$f = \mathcal{L}^{-1}\left(\frac{s-1}{s(s+1)}\right)$$

$(**)$ $$\int_0^t g(u)\,du = \mathcal{L}^{-1}\left(\frac{1}{s}\mathcal{L}(g)\right) \doteq \mathcal{L}^{-1}\left(\frac{1}{s}\,\frac{s-1}{s+1}\right)$$

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Não melhoram

$$\therefore\ \mathcal{L}(g) = \frac{s-1}{s+1} = \frac{s+1-2}{s+1}$$

$$= 1 - \frac{2}{s+1}$$

$$g = \mathcal{L}^{-1}(1) - 2\,\mathcal{L}^{-1}\left(\frac{1}{s+1}\right)\ ?$$

Aqui, tentaremos:

$$\rightarrow f = \mathcal{L}^{-1}\left(\frac{s-1}{s(s+1)}\right) = \mathcal{L}^{-1}\left(\frac{s-1}{s^2+s}\right)$$

$$= \mathcal{L}^{-1}\left(\frac{s-1}{\left(s+\frac{1}{2}\right)^2-\frac{1}{4}}\right)$$

$$= \mathcal{L}^{-1}\left(\frac{s+\frac{1}{2}-\frac{3}{2}}{\left(s+\frac{1}{2}\right)^2-\frac{1}{4}}\right)$$

$$= \mathcal{L}^{-1}\left(\frac{s+\frac{1}{2}}{\left(s+\frac{1}{2}\right)^2-\frac{1}{4}}\right) - \frac{3}{2}\,\mathcal{L}^{-1}\left(\frac{1}{\left(s+\frac{1}{2}\right)^2-\frac{1}{4}}\right)$$

$$= e^{-\frac{1}{2}t}\cosh\frac{t}{2} - \frac{3}{2}\,e^{-\frac{1}{2}t}\,2\sinh\frac{t}{2}$$

$$//\ f(t) = e^{-\frac{1}{2}t}\left(\cosh\frac{t}{2} - 3\sinh\frac{t}{2}\right)\ //$$

$$\therefore\ \mathcal{L}^{-1}\left(\frac{1}{s}\,\frac{s-1}{s(s+1)}\right) = \int_0^t f(u)\,du = \int_0^t \left(e^{-\frac{1}{2}u}\left(\cosh\frac{u}{2} - 3\sinh\frac{u}{2}\right)\right)du$$

$$\mathcal{L}^{-1}\left(\frac{1}{s}\,\frac{s-1}{s(s+1)}\right) = \int_0^t e^{-\frac{1}{2}u}\left(\cosh\frac{u}{2} - 3\sinh\frac{u}{2}\right) du$$

$$= \int_0^t e^{-\frac{1}{2}u}\left(\frac{e^{+\frac{u}{2}}+e^{-\frac{u}{2}}}{2} - 3\,\frac{e^{\frac{u}{2}}-e^{-\frac{u}{2}}}{2}\right) du$$

$$= \int_0^t \left[\frac{1+e^{-u}}{2} - \frac{3}{2}\left(1-e^{-u}\right)\right] du$$

$$= \frac{1}{2}\left(u - e^{-u}\right) - \frac{3}{2}\left(u + e^{-u}\right)\Big]_0^t$$

$$= \frac{1}{2}\left(t - e^{-t}\right) - \frac{3}{2}\left(t + e^{-t}\right) - \frac{1}{2}(-1) + \frac{3}{2}(1)$$

$$= \frac{1}{2}t \underbrace{- \frac{1}{2}e^{-t}} - \frac{3}{2}t \underbrace{- \frac{3}{2}e^{-t}} + \frac{1}{2} + \frac{3}{2}$$

$$= -t - 2e^{-t} + 2$$

$$= \boxed{2\left(1-e^{-t}\right) - t}$$

21.

$$\frac{54}{s^3(s-3)}$$

$$(*) \quad \int_0^t f(u)\,du = \mathcal{L}^{-1}\left(\frac{1}{s}\mathcal{L}(f)\right) \doteq \mathcal{L}^{-1}\left(\frac{54}{s^3(s-3)}\right)$$

$$= \mathcal{L}^{-1}\left(\frac{54}{s\, s^2(s-3)}\right)$$

$$\therefore \quad \mathcal{L}(f) = \frac{54}{s^2(s-3)}$$

$$(**) \quad f = \mathcal{L}^{-1}\left(\frac{54}{s\, s(s-3)}\right)$$

$$(***) \quad \int_0^t g(u)\,du = \mathcal{L}^{-1}\left(\frac{1}{s}\mathcal{L}(g)\right) \doteq \mathcal{L}^{-1}\left(\frac{54}{s\, s(s-3)}\right)$$

$$\therefore \quad \mathcal{L}(g) = \frac{54}{s(s-3)}$$

$$= 54 \, \frac{1}{s^2 - 3s}$$

$$= 54 \, \frac{1}{\left(s-\frac{3}{2}\right)^2 - \frac{9}{4}}$$

$$g = \mathcal{L}^{-1}\left(54 \, \frac{1}{\left(s-\frac{3}{2}\right)^2 - \frac{9}{4}}\right)$$

$$= 54 \, \mathcal{L}^{-1}\left(\frac{1}{\left(s-\frac{3}{2}\right)^2 - \frac{9}{4}}\right) = 54\, e^{\frac{3}{2}t} \frac{2}{3} \sinh \frac{3}{2}t$$

$$= 36\, e^{\frac{3}{2}t} \sinh \frac{3}{2} t$$

$g = 36 e^{3/2 t} \sinh \frac{3}{2} t$

(*AA)

$$\int_0^t 36 e^{\frac{3}{2}u} \sinh \frac{3}{2} u \, du = L^{-1}\left(\frac{54}{s \, s(s-3)} \right)$$

$$= \int_0^t 36 e^{\frac{3}{2}u} \frac{e^{\frac{3}{2}u} - e^{-\frac{3}{2}u}}{2} du$$

$$= \int_0^t 18 \left(e^{3u} - 1 \right) du$$

$$= 18 \frac{e^{3u}}{3} - 18 u \Big]_0^t$$

$$= 6 e^{3t} - 18 t - 6$$

$$L^{-1}\left(\frac{54}{s \, s(s-3)} \right) = 6 e^{3t} - 18 t - 6 = f(t)$$

$$f(t) = 6 e^{3t} - 18 t - 6$$

(*) $$L^{-1}\left(\frac{54}{s \, s^2(s-3)} \right) = \int_0^t f(u) \, du = \int_0^t (6 e^{3u} - 18u - 6) du$$

$$= \frac{6 e^{3u}}{3} - \frac{18 u^2}{2} - 6u \Big]_0^t$$

$$= \boxed{2 e^{3t} - 9 t^2 - 6 t - 2}$$

22.

$$\mathcal{L}^{-1}\left(\frac{1}{s^2}\left(\frac{s+1}{s^2+1}\right)\right)$$

$$\int_0^t f(u)\,du = \mathcal{L}^{-1}\left(\frac{1}{s}\mathcal{L}(f)\right)$$

$$\mathcal{L}^{-1}\left(\frac{1}{s}\frac{s+1}{s(s^2+1)}\right) = \mathcal{L}^{-1}\left(\frac{1}{s}\mathcal{L}(f)\right) = \int_0^t f(u)\,du$$

$$\therefore \mathcal{L}(f) = \frac{s+1}{s(s^2+1)} \Rightarrow f = \mathcal{L}^{-1}\left(\frac{s+1}{s(s^2+1)}\right) = \mathcal{L}^{-1}\left(\frac{1}{s}\mathcal{L}(g)\right)$$

$$= \int_0^t g(u)\,du$$

$$\therefore \mathcal{L}(g) = \frac{s+1}{s^2+1}$$

$$g = \mathcal{L}^{-1}\left(\frac{s+1}{s^2+1}\right)$$

$$= \mathcal{L}^{-1}\left(\frac{s}{s^2+1}\right) + \mathcal{L}^{-1}\left(\frac{1}{s^2+1}\right)$$

$$g = \cos t + \sin t$$

$$f = \int_0^t g(u)\,du = \int_0^t (\cos u + \sin u)\,du$$

$$= \sin u - \cos u \Big]_0^t$$

$$= \sin t - \cos t + 1$$

$$\mathcal{L}^{-1}\left(\frac{1}{s}\,\frac{s+1}{s(s^2+1)}\right) = \int_0^t f(u)\,du$$

$$= \int_0^t (\sin u - \cos u + 1)\,du$$

$$= -\cos u - \sin u + u \Big]_0^t$$

$$= -\cos t - \sin t + t - \left[-\cos 0 - \sin 0 + 0\right]$$

$$= \boxed{-\cos t - \sin t + t + 1}$$

9. $\mathcal{L}(\cos(\omega t+\theta))$

$$\mathcal{L}(\cos(\omega t+\theta)) = \int_0^{+\infty} e^{-st}\cos(\omega t+\theta)\,dt$$

Seja $z = \omega t+\theta \rightarrow dz = \omega\,dt$

Seja $\omega > 0$

$$\int_0^{+\infty} e^{-st}\cos(\omega t+\theta)\,dt =$$

$$= \int_\theta^{\infty} e^{-s\left(\frac{z-\theta}{\omega}\right)}\cos z\,\frac{1}{\omega}\,dz$$

$$= \frac{e^{\frac{s\theta}{\omega}}}{\omega}\int_\theta^{\infty} e^{-\frac{s}{\omega}z}\cos z\,dz$$

Mas

$$\int e^{-\frac{s}{\omega}z}\cos z\,dz =$$

$u = e^{-\frac{s}{\omega}z} \rightarrow du = -\frac{s}{\omega}e^{-\frac{s}{\omega}z}\,dz$

$dv = \cos z\,dz \rightarrow v = +\sin z$

$$= +\sin z\,e^{-\frac{s}{\omega}z} + \int \frac{s}{\omega}\sin z\,e^{-\frac{s}{\omega}z}\,dz$$

$$= +\sin z\,e^{-\frac{s}{\omega}z} + \frac{s}{\omega}\int \sin z\,e^{-\frac{s}{\omega}z}\,dz$$

(*)

$$(*1) = \int \sin z \; e^{-\frac{1}{\omega} z} \, dz =$$

$$u = e^{-\frac{1}{\omega} z} \rightarrow du = -\frac{1}{\omega} e^{-\frac{1}{\omega} z} \, dz$$

$$dv = \sin z \, dz \rightarrow v = -\cos z$$

$$= -\cos z \; e^{-\frac{1}{\omega} z} - \int \frac{1}{\omega} \cos z \; e^{-\frac{1}{\omega} z} \, dz$$

$$= -\cos z \; e^{-\frac{1}{\omega} z} - \frac{1}{\omega} \int \cos z \; e^{-\frac{1}{\omega} z} \, dz$$

$(*1) \longrightarrow (*)$:

$$\int e^{-\frac{1}{\omega} z} \cos z \, dz = + \sin z \; e^{-\frac{1}{\omega} z} + \\ + \frac{1}{\omega} \left(-\cos z \; e^{-\frac{1}{\omega} z} - \frac{1}{\omega} \int \cos z \; e^{-\frac{1}{\omega} z} \, dz \right)$$

$$= + \sin z \; e^{-\frac{1}{\omega} z} - \frac{1}{\omega} \cos z \; e^{-\frac{1}{\omega} z} \\ - \frac{1^2}{\omega^2} \int \cos z \; e^{-\frac{1}{\omega} z} \, dz$$

$\therefore$

$$\left(1 + \frac{1^2}{\omega^2}\right) \int e^{-\frac{1}{\omega} z} \cos z \, dz = \left(\sin z - \frac{1}{\omega} \cos z \right) e^{-\frac{1}{\omega} z}$$

$$\int e^{-\frac{1}{\omega} z} \cos z \, dz = \frac{\omega^2}{\omega^2 + 1^2} \left(\sin z \; e^{-\frac{1}{\omega} z} - \\ - \frac{1}{\omega} \cos z \; e^{-\frac{1}{\omega} z} \right)$$

$$\int_0^\infty e^{-st}\cos(\omega t+\theta)\,dt =$$

$$= \frac{e^{\frac{s\theta}{\omega}}}{\omega}\lim_{k\to\infty}\left[\frac{\omega^2}{\omega^2+s^2}\left(\sin z\, e^{-\frac{s}{\omega}z} - \frac{s}{\omega}\cos z\, e^{-\frac{s}{\omega}z}\right)\right]_{z=\theta}^{k}$$

$$= \frac{e^{\frac{s\theta}{\omega}}}{\omega}\lim_{k\to\infty}\left[\frac{\omega^2}{\omega^2+s^2}\left(\sin k\, e^{-\frac{s}{\omega}k} - \frac{s}{\omega}\cos k\, e^{-\frac{s}{\omega}k} - \sin\theta\, e^{-\frac{s}{\omega}\theta} + \frac{s}{\omega}\cos\theta\, e^{-\frac{s}{\omega}\theta}\right)\right]$$

Aqui $\omega > 0$, logo

$$\lim_{k\to\infty}\sin k\, e^{-\frac{s}{\omega}k} = 0 \quad \text{se} \quad s>0$$

$$\lim_{k\to\infty}\cos k\, e^{-\frac{s}{\omega}k} = 0 \quad \text{se} \quad s>0$$

$$\therefore = \frac{\omega^{\cancel{2}}}{\omega^2+s^2}\,\frac{e^{\frac{s\theta}{\omega}}}{\cancel{\omega}}\left[-\sin\theta\, e^{-\frac{s}{\omega}\theta} + \frac{s}{\omega}\cos\theta\, e^{-\frac{s}{\omega}\theta}\right]$$

$$= \frac{\omega}{\omega^2+s^2}\left[\frac{s}{\omega}\cos\theta - \sin\theta\right] \quad (s>0)$$

(1)

<u>Se $\omega < 0$</u> : Se $\omega < 0$ então $\omega = -|\omega|$

$$\int_0^{+\infty} e^{-st}\cos(\omega t+\theta)\,dt = \int_\theta^{-\infty} e^{-s\frac{z-\theta}{-|\omega|}}\cos(z)\,\frac{-dz}{|\omega|}$$

$z = -|\omega|t+\theta$
$$= \frac{-1}{|\omega|}\int_\theta^{-\infty} e^{s\frac{z-\theta}{|\omega|}}\cos z\,dz$$

$t = \dfrac{z-\theta}{-|\omega|}$
$$= -\frac{1}{|\omega|}e^{-\frac{s\theta}{|\omega|}}\int_\theta^{-\infty} e^{\frac{sz}{|\omega|}}\cos z\,dz$$

Mas

$$\int e^{\frac{sz}{|\omega|}}\cos z\,dz =$$

$$\begin{cases} u = e^{\frac{sz}{|\omega|}} \rightarrow du = \frac{s}{|\omega|}e^{\frac{sz}{|\omega|}}dz \\ dv = \cos z\,dz \rightarrow v = \sin z \end{cases}$$

$$= \sin z\, e^{\frac{sz}{|\omega|}} - \frac{s}{|\omega|}\int e^{\frac{sz}{|\omega|}}\sin z\,dz$$
(*)

$$(*) = \int e^{\frac{sz}{|\omega|}}\sin z\,dz = -e^{\frac{sz}{|\omega|}}\cos z + \int \frac{s}{|\omega|}\cos z\, e^{\frac{sz}{|\omega|}}dz$$

$$\begin{cases} u = e^{\frac{sz}{|\omega|}} \rightarrow du = \frac{s}{|\omega|}e^{\frac{sz}{|\omega|}}dz \\ dv = \sin z\,dz \rightarrow v = -\cos z \end{cases}$$

$$= -e^{\frac{sz}{|\omega|}}\cos z + \frac{s}{|\omega|}\int \cos z\, e^{\frac{sz}{|\omega|}}dz$$
(**)

(8') $\to$ (7) :

$$\int e^{\frac{s z}{|w|}} \cos z \, dz = \sin z \, e^{\frac{s z}{|w|}} -$$

$$- \frac{s}{|w|} \left(- e^{\frac{s z}{|w|}} \cos z + \frac{s}{|w|} \int \cos z \, e^{\frac{s z}{|w|}} dz \right)$$

$$= \sin z \, e^{\frac{s z}{|w|}} + \frac{s}{|w|} e^{\frac{s z}{|w|}} \cos z -$$

$$- \frac{s^2}{|w|^2} \int \cos z \, e^{\frac{s z}{|w|}} dz$$

$$\left(1 + \frac{s^2}{|w|^2}\right) \int e^{\frac{s z}{|w|}} \cos z \, dz =$$

$$= e^{\frac{s z}{|w|}} \left(\sin z + \frac{s}{|w|} \cos z \right)$$

$$\therefore \quad \int e^{\frac{s z}{|w|}} \cos z \, dz = \frac{e^{\frac{s z}{|w|}}}{\left(1 + \frac{s^2}{|w|^2}\right)} \left(\sin z + \frac{s}{|w|} \cos z \right)$$

$\therefore$

$$\int_{\theta}^{\infty} e^{-st} \cos(wt + \theta) \, dt = - \frac{1}{|w|} e^{-\frac{s\theta}{|w|}} \cdot \int_{\theta}^{-\infty} e^{\frac{s z}{|w|}} \cos z \, dz$$

$$= - \frac{1}{|w|} e^{-\frac{s\theta}{|w|}} \lim_{k \to -\infty} \frac{e^{\frac{s z}{|w|}}}{\left(1 + \frac{s^2}{|w|^2}\right)} \left(\sin z + \frac{s}{|w|} \cos z \right) \Big|_{\theta}^{k}$$

$$= -\frac{1}{|\omega|} e^{-\frac{s\theta}{|\omega|}} \cdot \lim_{k \to -\infty} \left(\frac{\sin k \, e^{\frac{sk}{|\omega|}}}{\left(1+\frac{s^2}{|\omega|^2}\right)} \xrightarrow{} 0 \quad s>0 \right. +$$

$$+ \frac{s}{|\omega|} e^{\frac{s}{|\omega|}k} \cos k \xrightarrow{} 0 \quad s>0$$

$$\left. - \frac{e^{\frac{s\theta}{|\omega|}}}{\left(1+\frac{s^2}{|\omega|^2}\right)} \left(\sin\theta + \frac{s}{|\omega|}\cos\theta\right) \right)$$

$$= \frac{1}{|\omega|} e^{-\frac{s\theta}{|\omega|}} (-) \frac{e^{\frac{s\theta}{|\omega|}}}{\left(1+\frac{s^2}{|\omega|^2}\right)} \left(\sin\theta + \frac{s}{|\omega|}\cos\theta\right)$$

$$= \frac{1}{\cancel{|\omega|}} \frac{|\omega|^{\cancel{2}}}{|\omega|^2+s^2} \left(\sin\theta + \frac{s}{|\omega|}\cos\theta\right)$$

$$= \frac{|\omega|}{\omega^2+s^2} \left(\sin\theta + \frac{s}{|\omega|}\cos\theta\right)$$

se $\omega < 0$ então $|\omega| = -\omega$

$$= \frac{-\omega}{\omega^2+s^2} \left(\sin\theta + \frac{s}{-\omega}\cos\theta\right)$$

$$= \frac{\omega}{\omega^2+s^2} \left(\frac{s}{\omega}\cos\theta - \sin\theta\right) \qquad s>0$$

(2)

De (1) e (2) tem-se:

$$\boxed{\mathcal{L}\left(\cos(\omega t+\theta)\right) = \frac{\omega}{\omega^2+s^2}\left(\frac{s}{\omega}\cos\theta - \sin\theta\right), \quad s>0}$$

$$u(t)\,f(t) = \begin{cases} 0, & t<0 \\ \cancel{f(t)}\ k & t>0 \end{cases}$$

$$\boxed{0 \cdot u(t-c) = 0}$$

$$k\,[u(t-0) - u(t-c)] + c$$

$$\int_0^{\infty}$$

$$\mathcal{L}\left(\int_0^t f(u)\,du\right) = \frac{1}{s}F(s)$$

$$\mathcal{L}\left(\int_0^t \underbrace{\left[\int_0^t f(u)\,du\,dv\right]}_{g(u)}\right) =$$

$$\frac{1}{s}\,B(s)$$