Cálculo C - Lista 16

Transformada de Laplace (II):

Transformada de Laplace inversa

Encontre a transformada de Laplace inversa das seguintes funções

1. $\frac{1}{s^2+9}$
2. $\frac{s-4}{s^2-4}$
3. $\frac{2}{s} + \frac{1}{s+2}$
4. $\frac{1}{s^5} + \frac{1}{s^2}$
5. $\frac{1}{s(s+1)}$
6. $\frac{1}{s^2+3s}$
7. $\frac{1}{(s-a)(s-b)}$, $(a \neq b)$
8. $\frac{5}{(s-2)^7}$
9. $\frac{s+2}{(s+2)^2+1}$
10. $\frac{1}{s}\left(\frac{\omega}{s^2+\omega^2}\right)$
11. $\frac{1}{s^2+3s+2}$
12. $\frac{1}{s^2+4s+4}$
13. $\frac{s}{s^2+4s+2}$
14. $\frac{s}{(s+a)^2+b^2}$
15. $\frac{s^2}{(s+1)(s+2)(s+3)}$
16. $\frac{1}{(s-1)^2(s+2)}$
17. $\frac{s^2-3s}{(s-2)(s-1)^2}$
18. $\frac{3s^2+4}{s^4+s^2}$

Lista 16 - Respostas

1. $\frac{1}{3}\sin 3t$

2. $\cosh 2t - 2\sinh 2t$

3. $2 + e^{-2t}$

4. $\frac{t^4}{4!} + t$

5. $1 - e^{-t}$

6. $\frac{1}{3}(1 - e^{-3t})$

7. $\frac{e^{at} - e^{bt}}{a-b}$

8. $\frac{e^{2t}t^6}{144}$

9. $e^{-2t}\cos t$

10. $\frac{1}{\omega}(1 - \cos \omega t)$

11. $e^{-t} - e^{-2t}$

2. te^{-2t}

3. $e^{-2t}(\cosh\sqrt{2}t - \sqrt{2}\sinh\sqrt{2}t)$

4. $e^{-at}(\cos bt - \frac{a}{b}\sin bt)$

5. $\frac{1}{2}e^{-t} - 4e^{-2t} + \frac{9}{2}e^{-3t}$

6. $-\frac{1}{9}e^{t} + \frac{1}{3}e^{t}t + \frac{1}{9}e^{-2t}$

17. $-2e^{2t} + 3e^{t} + 2te^{t}$

18. $4t - \sin t$

Lista 16 - Cálculo C

1. $\dfrac{1}{s^2+9}$; $\mathcal{L}(\sin \omega t) = \dfrac{\omega}{s^2+\omega^2}$

$$\frac{1}{\omega}\mathcal{L}(\sin \omega t) = \frac{1}{s^2+\omega^2}$$

$$\therefore \frac{1}{s^2+9} = \frac{1}{3}\mathcal{L}(\sin 3t)$$

$$\therefore \mathcal{L}^{-1}\left(\frac{1}{s^2+9}\right) = \mathcal{L}^{-1}\left(\frac{1}{3}\mathcal{L}(\sin 3t)\right)$$

$$= \frac{1}{3}\mathcal{L}^{-1}(\mathcal{L}(\sin 3t))$$

$$\boxed{\mathcal{L}^{-1}\left(\frac{1}{s^2+9}\right) = \frac{1}{3}\sin 3t}$$

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2. $\dfrac{s-4}{s^2-4}$; $\dfrac{s-4}{s^2-4} = \dfrac{s}{s^2-4} - \dfrac{4}{s^2-4}$

$$\mathcal{L}(\cosh at) = \frac{s}{s^2-a^2}$$

$$\mathcal{L}(\sinh at) = \frac{a}{s^2-a^2}$$

$$\mathcal{L}^{-1}\left(\frac{s-4}{s^2-4}\right) = \mathcal{L}^{-1}\left(\frac{s}{s^2-4}\right) - 2\mathcal{L}^{-1}\left(\frac{2}{s^2-4}\right)$$

$$= \cosh 2t - 2\sinh 2t$$

$$\therefore \boxed{\mathcal{L}^{-1}\left(\frac{1}{s^2+9}\right) = \cosh 2t - 2\sinh 2t}$$

3. $\frac{2}{s} + \frac{1}{s+2}$; $\mathcal{L}^{-1}\left(\frac{2}{s} + \frac{1}{s+2}\right) =$

$$= \mathcal{L}^{-1}\left(\frac{2}{s}\right) + \underbrace{\mathcal{L}^{-1}\left(\frac{1}{s+2}\right)}$$

$\mathcal{L}(1) = \frac{1}{s}$

$\mathcal{L}(e^{at}) = \frac{1}{s-a}$

$$= \underbrace{2\mathcal{L}^{-1}\left(\frac{1}{s}\right)} +$$

$$= 2 + e^{-2t}$$

$$\therefore \boxed{\mathcal{L}^{-1}\left(\frac{2}{s} + \frac{1}{s+2}\right) = 2 + e^{-2t}}$$

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4. $\frac{1}{s^5} + \frac{1}{s^2}$; $\mathcal{L}^{-1}\left(\frac{1}{s^5} + \frac{1}{s^2}\right) = \mathcal{L}^{-1}\left(\frac{1}{s^5}\right) + \mathcal{L}^{-1}\left(\frac{1}{s^2}\right)$

$\mathcal{L}(t^n) = \frac{n!}{s^{n+1}}$;

$$= \frac{1}{4!}t^4 + t$$

$$\therefore \boxed{\mathcal{L}^{-1}\left(\frac{1}{s^5} + \frac{1}{s^2}\right) = \frac{t^4}{4!} + t}$$

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5. $\frac{1}{s(s+1)}$; $\frac{1}{s(s+1)} = \frac{A}{s} + \frac{B}{s+1}$

$$\frac{1}{s(s+1)} = \frac{A(s+1) + Bs}{s(s+1)}$$

$$\Rightarrow 1 = As + A + Bs$$

$$1 = (A+B)s + A \quad \Rightarrow$$

5. Cont

$$A+B=0 \quad \Rightarrow \quad \underline{\underline{B=-1}}$$
$$\underline{\underline{A=1}}$$

$$\therefore \quad \frac{1}{s(s+1)} = \frac{1}{s} - \frac{1}{s+1}$$

$$\therefore \quad \mathcal{L}^{-1}\left(\frac{1}{s(s+1)}\right) = \mathcal{L}^{-1}\left(\frac{1}{s}\right) - \mathcal{L}^{-1}\left(\frac{1}{s+1}\right)$$

$$\left[\mathcal{L}(1) = \frac{1}{s}\right] \qquad = 1 - e^{-t}$$

$$\therefore \quad \boxed{\mathcal{L}^{-1}\left(\frac{1}{s(s+1)}\right) = 1 - e^{-t}}$$

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6. $$\frac{1}{s^2+3s} = \frac{1}{s(s+3)} = \frac{A}{s} + \frac{B}{s+3}$$

$$\frac{1}{s(s+3)} = \frac{As+3A+Bs}{s(s+3)}$$

$$\therefore \quad A+B=0 \qquad \therefore \quad \| B = -\tfrac{1}{3} \|$$
$$3A=1 \Rightarrow \| A = \tfrac{1}{3} \|$$

$$\therefore \quad \frac{1}{s^2+3s} = \frac{1}{3s} + \frac{-1}{3(s+3)}$$

$$\mathcal{L}^{-1}\left(\frac{1}{s^2+3s}\right) = \mathcal{L}^{-1}\left(\frac{1}{3s}\right) + \mathcal{L}^{-1}\left(\frac{-1}{3(s+3)}\right)$$

$$= \frac{1}{3}\underbrace{\mathcal{L}^{-1}\left(\frac{1}{s}\right)} - \frac{1}{3}\underbrace{\mathcal{L}^{-1}\left(\frac{1}{s+3}\right)}$$

$$= \frac{1}{3}\cdot 1 - \frac{1}{3}e^{-3t}$$

$$\therefore \boxed{\mathcal{L}^{-1}\left(\frac{1}{s^2+3s}\right) = \frac{1}{3}\left(1 - e^{-3t}\right)}$$

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7. $$\frac{1}{(s-a)(s-b)} = \frac{A}{s-a} + \frac{B}{s-b}$$

$$\frac{1}{(s-a)(s-b)} = \frac{As - Ab + Bs - aB}{(s-a)(s-b)}$$

$$\Rightarrow \quad A+B = 0 \quad \Rightarrow \quad \underline{\underline{A = -B}}$$

$$\left\{ \begin{array}{l} -Ab - aB = 1 \\ \therefore \quad bB - aB = 1 \\ \quad \| B = \frac{1}{b-a} \| \quad \therefore \| A = \frac{1}{a-b} \| \end{array} \right.$$

$$\therefore \quad \frac{1}{(s-a)(s-b)} = \frac{1}{(a-b)(s-a)} + \frac{1}{(b-a)(s-b)}$$

$$\mathcal{L}^{-1}\left(\frac{1}{(s-a)(s-b)}\right) = \frac{1}{a-b}\mathcal{L}^{-1}\left(\frac{1}{s-a}\right) + \frac{1}{b-a}\mathcal{L}^{-1}\left(\frac{1}{s-b}\right)$$

$$= \frac{1}{a-b}e^{at} + \frac{1}{b-a}e^{bt}$$

$$\therefore \mathcal{L}^{-1}\left(\frac{1}{(s-a)(s-b)}\right) = \frac{e^{at}}{a-b} + \frac{e^{bt}}{b-a}$$

$$\boxed{\mathcal{L}^{-1}\left(\frac{1}{(s-a)(s-b)}\right) = \frac{e^{at}-e^{bt}}{a-b}}$$

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8. $\dfrac{5}{(s-2)^7}$; $\mathcal{L}^{-1}\left(\dfrac{5}{(s-2)^7}\right) = 5\,\mathcal{L}^{-1}\left(\dfrac{1}{(s-2)^7}\right)$

$$\left.\begin{aligned} \mathcal{L}(t^6) &= \frac{6!}{t^7} \\ \mathcal{L}(e^{at}) &= \frac{1}{(s-a)^2} \end{aligned}\right\} \quad \mathcal{L}\left(\frac{e^{2t}t^6}{6!}\right) = \frac{1}{6!}\mathcal{L}(e^{2t}t^6)$$

$$= \frac{1}{\cancel{6!}}\,\frac{\cancel{6!}}{(s-2)^7} = \frac{1}{(s-2)^7}$$

$$\therefore \mathcal{L}^{-1}\left(\frac{5}{(s-2)^7}\right) = 5\,\underbrace{\mathcal{L}^{-1}\left(\frac{1}{(s-2)^7}\right)}$$

$$= 5\,\frac{e^{2t}t^6}{6!}$$

$$\boxed{\mathcal{L}^{-1}\left(\frac{5}{(s-2)^7}\right) = \frac{e^{2t}t^6}{144}}$$

$$\frac{5}{6!} = \frac{\cancel{5}}{6\cancel{5}4321} = \frac{1}{6\times 24} = \frac{1}{144}$$

$$\left\{\begin{aligned} \mathcal{L}(e^{at}f(x)) &= F(s-a) \\ F &= \mathcal{L}^{-1}(f) \end{aligned}\right.$$

9. $\dfrac{s+2}{(s+2)^2+1}$

$$\left.\begin{aligned} \mathcal{L}(\cos t) &= \frac{s}{s^2+1} \\ \mathcal{L}(e^{-2t}\cos t) &= \frac{s+2}{(s+2)^2+1} \end{aligned}\right\} \Longrightarrow$$

$$\Longrightarrow \boxed{\mathcal{L}^{-1}\left(\frac{s+2}{(s+2)^2+1}\right) = e^{-2t}\cos t}$$

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10. $\dfrac{1}{s}\left(\dfrac{\omega}{s^2+\omega^2}\right) \longrightarrow$ Podia-se também decompor em frações parciais

$$\left.\begin{aligned} \mathcal{L}\left(\int_0^t f(u)\,du\right) &= \frac{1}{s}\mathcal{L}(f(t)) \\ \mathcal{L}(\sin \omega t) &= \frac{\omega}{s^2+\omega^2} \end{aligned}\right\}$$

$$\mathcal{L}\left(\int_0^t f(u)\,du\right) = \frac{1}{s}\mathcal{L}(f(t)) = \frac{1}{s}\underbrace{\frac{\omega}{(s^2+\omega^2)}}$$

$$= \frac{1}{s}\mathcal{L}(\sin \omega t)$$

$$\therefore \quad f(t) = \sin \omega t$$

$$\therefore \quad \mathcal{L}\left(\int_0^t \sin \omega u\,du\right) = \frac{1}{s}\frac{\omega}{s^2+\omega^2} \Longrightarrow$$

$$\int_0^t \sin \omega u \, du = \mathcal{L}^{-1}\left(\frac{1}{s} \frac{\omega}{s^2+\omega^2} \right)$$

$$-\frac{\cos \omega u}{\omega} \Big]_0^t = \mathcal{L}^{-1}\left(\frac{1}{s} \frac{\omega}{s^2+\omega^2} \right)$$

$$\boxed{-\frac{\cos \omega t}{\omega} + \frac{1}{\omega} = \mathcal{L}^{-1}\left(\frac{1}{s} \frac{\omega}{s^2+\omega^2} \right)}$$

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11. $$\frac{1}{s^2+3s+2} = \frac{1}{(s+1)(s+2)} = \frac{A}{s+1} + \frac{B}{s+2}$$

$$\frac{1}{(s+1)(s+2)} = \frac{As+2A+Bs+B}{(s+1)(s+2)}$$

$$\frac{1}{(s+1)(s+2)} = \frac{(A+B)s+2A+B}{(s+1)(s+2)}$$

$$\Rightarrow \quad A+B=0 \quad \Rightarrow \quad A=-B$$

$$+2A+B=1 \quad \Rightarrow \quad -2B+B=1$$

$$//B=-1// \quad \therefore //A=+1//$$

$\therefore$

$$\frac{1}{(s+1)(s+2)} = \frac{+1}{s+1} - \frac{1}{s+2} \Rightarrow$$

$$\mathcal{L}^{-1}\left(\frac{1}{(s+1)(s+2)}\right) = \mathcal{L}^{-1}\left(\frac{+1}{s+1}\right) - \mathcal{L}^{-1}\left(\frac{1}{s+2}\right)$$

$$= +1\, \underbrace{\mathcal{L}^{-1}\left(\frac{1}{s+1}\right)} - e^{-2t}$$

$$= e^{-t} - e^{-2t}$$

$$\therefore \boxed{\mathcal{L}^{-1}\left(\frac{1}{(s+1)(s+2)}\right) = e^{-t} - e^{-2t}}$$

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12. $$\frac{1}{s^2+4s+4} = \frac{1}{(s+2)^2}$$

$$\left.\begin{aligned} \mathcal{L}(t) &= \frac{1}{s^2} \\ \mathcal{L}(e^{-2t}t) &= \frac{1}{(s+2)^2} \end{aligned}\right\} \quad \begin{aligned} \mathcal{L}^{-1}\left(\frac{1}{s^2+4s+4}\right) &= \mathcal{L}^{-1}\left(\frac{1}{(s+2)^2}\right) \\ &= e^{-2t}t \end{aligned}$$

$$\therefore \boxed{\mathcal{L}^{-1}\left(\frac{1}{s^2+4s+4}\right) = e^{-2t}t}$$

13. $$\frac{s}{s^2+4s+2} = \frac{s}{[s-(-2-\sqrt{2})]\,[s-(-2+\sqrt{2})]}$$

$$s^2+4s+2=0$$

$$s = -4 \pm \sqrt{16-8}/2$$

$$= \frac{-4 \pm 2\sqrt{2}}{2}$$

$$= -2 \pm \sqrt{2}$$

$$= \frac{A}{[s-(-2-\sqrt{2})]} + \frac{B}{s-(-2+\sqrt{2})}$$

$$= \frac{As - A(-2+\sqrt{2}) + Bs - B(-2-\sqrt{2})}{(s^2+4s+2)}$$

$$\Rightarrow \begin{cases} A+B=1 \Longrightarrow \underline{\underline{A = 1-B}} \\ -A(-2+\sqrt{2}) - B(-2-\sqrt{2}) = 0 \end{cases}$$

$$-(1-B)(-2+\sqrt{2}) - B(-2-\sqrt{2}) = 0$$

$$2 - \sqrt{2} \cancel{-2B} + B\sqrt{2} \cancel{+2B} + B\sqrt{2} = 0$$

$$2B\sqrt{2} = \sqrt{2} - 2$$

$$B = \frac{\sqrt{2}-2}{2\sqrt{2}} = \frac{1}{2} - \frac{\sqrt{2}}{2}$$

$$A = 1 - B = 1 - \frac{1}{2} + \frac{\sqrt{2}}{2} = \frac{1+\sqrt{2}}{2}$$

$$\frac{1}{s^2+4s+2} = \frac{\frac{1+\sqrt{2}}{2}}{s-(-2-\sqrt{2})} + \frac{\frac{1-\sqrt{2}}{2}}{s-(-2+\sqrt{2})}$$

$$\mathcal{L}^{-1}\left(\frac{1}{s^2+4s+2}\right) = \frac{1+\sqrt{2}}{2}\,\mathcal{L}^{-1}\left(\frac{1}{s-(-2-\sqrt{2})}\right) +$$

$$+ \frac{1-\sqrt{2}}{2}\,\mathcal{L}^{-1}\left(\frac{1}{s-(-2+\sqrt{2})}\right)$$

$$= \frac{1+\sqrt{2}}{2}\, e^{(-2-\sqrt{2})t} + \frac{1-\sqrt{2}}{2}\, e^{(-2+\sqrt{2})t}$$

$$= e^{-2t}\left(\left(\frac{1+\sqrt{2}}{2}\right) e^{-\sqrt{2}t} + \frac{1-\sqrt{2}}{2}\, e^{\sqrt{2}t}\right)$$

$$= e^{-2t}\left[\frac{e^{-\sqrt{2}t}+e^{\sqrt{2}t}}{2} - \sqrt{2}\,\frac{e^{\sqrt{2}t}-e^{-\sqrt{2}t}}{2}\right]$$

$$= \boxed{e^{-2t}\left(\cosh\sqrt{2}t - \sqrt{2}\sinh\sqrt{2}t\right)}$$

14. $\dfrac{s}{(s+a)^2+b^2}$

$\mathcal{L}(\cos bt) = \dfrac{s}{s^2+b^2}$; $\mathcal{L}(\sin bt) = \dfrac{b}{s^2+b^2}$

$\mathcal{L}(e^{-at}\cos bt) = \dfrac{s+a}{(s+a)^2+b^2}$; $\mathcal{L}(e^{-at}\sin bt) = \dfrac{b}{(s+a)^2+b^2}$

$\therefore \quad \dfrac{s}{(s+a)^2+b^2} = \dfrac{s+a}{(s+a)^2+b^2} - \dfrac{a}{(s+a)^2+b^2}$

$$\mathcal{L}^{-1}\left(\frac{s}{(s+a)^2+b^2}\right) = \mathcal{L}^{-1}\left(\frac{s+a}{(s+a)^2+b^2}\right) - a\,\mathcal{L}^{-1}\left(\frac{1}{(s+a)^2+b^2}\right)$$

$$= e^{-at}\cos bt - a\,\frac{1}{b}\,e^{-at}\sin bt$$

$$= \boxed{e^{-at}\left(\cos bt - \frac{a}{b}\sin bt\right)}$$

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15. $\dfrac{s^2}{(s+1)(s+2)(s+3)} = \dfrac{A}{(s+1)} + \dfrac{B}{(s+2)} + \dfrac{C}{(s+3)}$

$$\frac{s^2}{(s+1)(s+2)(s+3)} = \frac{A(s+2)(s+3) + B(s+1)(s+3) + C(s+1)(s+2)}{(s+1)(s+2)(s+3)}$$

$$\Longrightarrow \quad s^2 = A(s+2)(s+3) + B(s+1)(s+3) + C(s+1)(s+2)$$

$\underline{\underline{s=-2}}$: $(-2)^2 = B(-1)(1) \Longrightarrow \underline{\underline{B=-4}}$

$\underline{\underline{s=(-1)}}$: $(-1)^2 = A(1)(2) \Longrightarrow \underline{\underline{A=\frac{1}{2}}}$

$\underline{\underline{s=-3}}$: $(-3)^2 = C(-2)(-1) \Longrightarrow \underline{\underline{C=9/2}}$

$$\mathcal{L}^{-1}\left(\frac{1}{(s-1)^2(s+2)}\right) = \frac{1}{2}\mathcal{L}^{-1}\left(\frac{1}{s+1}\right) - 4\,\mathcal{L}^{-1}\left(\frac{1}{s+2}\right) + \frac{9}{2}\mathcal{L}^{-1}\left(\frac{1}{s+3}\right)$$

$$= \boxed{\frac{1}{2}e^{-t} - 4e^{-2t} + \frac{9}{2}e^{-3t}}$$

18. $\dfrac{1}{(s-1)^2(s+2)} = \dfrac{A}{(s-1)} + \dfrac{B}{(s-1)^2} + \dfrac{C}{(s+2)}$

$= \dfrac{A(s-1)(s+2) + B(s+2) + C(s-1)^2}{(s-1)^2(s+2)}$

$1 = \dfrac{As^2 + As - 2A + Bs + 2B + Cs^2 - 2Cs + C}{(s-1)^2(s+2)}$

$\Rightarrow \begin{cases} A + C = 0 \Rightarrow A = -C \\ A + B - 2C = 0 \rightarrow B - 3C = 0 \\ -2A + 2B + C = 1 \rightarrow 2B + 3C = 1 \end{cases} \quad 3B = 1 \quad \| B = \frac{1}{3} \|$

$\| C = +\frac{B}{3} = +\frac{1}{9} \|$

$\| A = -\frac{1}{9} \|$

$\therefore$

$\dfrac{1}{(s-1)^2(s+2)} = -\dfrac{1}{9(s-1)} + \dfrac{1}{3(s-1)^2} + \dfrac{1}{9(s+2)}$

$\mathcal{L}^{-1}\left(\dfrac{1}{(s-1)^2(s+2)}\right) = -\dfrac{1}{9}\mathcal{L}^{-1}\left(\dfrac{1}{s-1}\right) + \dfrac{1}{3}\mathcal{L}^{-1}\left(\dfrac{1}{(s-1)^2}\right) - \dfrac{1}{9}\mathcal{L}^{-1}\left(\dfrac{1}{s+2}\right)$

$\boxed{= -\dfrac{1}{9}e^{1t} + \dfrac{1}{3}e^{t}t + \dfrac{1}{9}e^{-2t}}$

17. $$\frac{s^2-3s}{(s-2)(s-1)^2} = \frac{A}{s-2} + \frac{B}{s-1} + \frac{C}{(s-1)^2}$$

$$\frac{s^2-3s}{(s-2)(s-1)^2} = \frac{A(s-1)^2 + B(s-2)(s-1) + C(s-2)}{(s-2)\ (s-1)\ (s-1)^2}$$

$$\Rightarrow \quad s^2-3s = A(s-1)^2 + B(s-2)(s-1) + C(s-2)$$

$\underline{s=1}$: $1-3 = C(-1) \Rightarrow //C=2//$

$\underline{\underline{s=2}}$: $4-6 = A \Rightarrow //A=-2//$

$\underline{\underline{s=0}}$ $0 = A + B(-2)(-1) + C(-2)$

$0 = -2 + 2B - 4$

$\Rightarrow //B = 3//$

$$\frac{s^2-3s}{(s-2)(s-1)^2} = \frac{-2}{s-2} + \frac{3}{2(s-1)} + \frac{1}{2(s-1)^2}$$

$$\mathcal{L}^{-1}\left(\frac{s^2-3s}{(s-2)(s-1)^2}\right) = -2\mathcal{L}^{-1}\left(\frac{1}{s-2}\right) + 3\mathcal{L}^{-1}\left(\frac{1}{s-1}\right) + 2\mathcal{L}^{-1}\left(\frac{1}{(s-1)^2}\right)$$

$$= \boxed{-2e^{2t} + 3e^{t} + 2e^{t}t}$$

18. $$\frac{3s^2+4}{s^4+s^2} = \frac{3s^2+4}{s^2(s^2+1)} = \frac{A}{s} + \frac{B}{s^2} + \frac{Cs+D}{s^2+1}$$

$$= \frac{As(s^2+1) + B(s^2+1) + (Cs+D)s^2}{s^2(s^2+1)}$$

$$= \frac{As^3 + As + Bs^2 + B + Cs^3 + Ds^2}{s^2(s^2+1)}$$

$$\frac{3s^2+4}{s^2(s^2+1)} = \frac{(A+C)s^3 + (B+D)s^2 + As + B}{s^2(s^2+1)}$$

$$\begin{cases} A+C = 0 \Rightarrow \underline{\underline{C=0}} \\ B+D = 3 \Rightarrow \underline{D=-1} \\ \underline{A=0} \\ \underline{\underline{B=4}} \end{cases}$$

$$\frac{3s^2+4}{s^4+s^2} = \frac{4}{s^2} - \frac{1}{s^2+1}$$

$$L^{-1}\left(\frac{3s^2+4}{s^4+s^2}\right) = L^{-1}\left(\frac{4}{s^2}\right) - \underbrace{L^{-1}\left(\frac{1}{s^2+1}\right)}$$

$$= \boxed{4t - \sin t}$$

$\mathcal{L}\left(e^{at} f(t)\right) = F(s-a)$

$\Leftrightarrow$

$$\boxed{e^{at} f(t) = \mathcal{L}^{-1}\left(F(s-a)\right)}$$

$$\mathcal{L}^{-1}\left(\frac{5}{(s-2)^7}\right) = 5\,\boxed{\mathcal{L}^{-1}\left(\frac{1}{(s-2)^7}\right)} = \mathcal{L}^{-1}\left(F(s-a)\right)$$

$f(t) = \mathcal{L}^{-1}\left(\frac{1}{s^7}\right)$

$a = 2$

$\mathcal{L}^{-1}\left(\frac{1}{s^7}\right)$

$= 5\,\frac{t^6}{6!}\,e^{+2t}$