Exercícios Resolvidos : Integração por Substit.

Calcule as integrais usando o método da substituição :

1. $\int (4x-3)^5 \, dx$

2. $\int x\sqrt{x^2-1} \, dx$

3. $\int x\sqrt{2-3x^2} \, dx$

4. $\int 3t(1-2t^2)^{10} \, dt$

5. $\int \frac{x^2}{(x^3+5)^4} \, dx$

6. $\int \frac{t \, dt}{\sqrt{2t^2+1}}$

7. $\int x^2 \sqrt[3]{2-4x^3} \, dx$

8. $\int \frac{(x+1)}{(x^2+2x+5)^2} \, dx$

9. $\int \operatorname{sen} \frac{t}{3} \, dt$

10. $\int \frac{\cos\sqrt{x}}{\sqrt{x}} \, dx$

11. $\int \frac{x^3-1}{(x^4-4x)^{2/3}} \, dx$

12. $\int \frac{1}{x^3}\left(1+\frac{1}{x^2}\right)^{5/3} dx$

13. $\int (2-t^2)\sqrt[4]{6t-t^3} \, dt$

14. $\int \frac{2-x^2}{(x^3-6x+1)^5} \, dx$

15. $\int \left(1-\frac{1}{x^2}\right)\left(x+\frac{1}{x}\right)^{-3} dx$

16. $\int \frac{x}{\sqrt{x+3}} \, dx$

17. $\int x\sqrt{x+2} \, dx$

18. $\int x^2\sqrt{x+4} \, dx$

19. $\int \frac{\sqrt{x}}{\sqrt{1+\sqrt{x}}}\, dx$

20. $\int \frac{t^2}{\sqrt{t+2}}\, dt$

21. $\int \cos 7x\, dx$

22. $\int t^9 \sin t^{10}\, dt$

23. $\int (\sin t)^6 \cos t\, dt$

24. $\int \frac{\sin t}{(\cos t)^3}\, dt$

25. $\int \sqrt{\sin 2z}\, \cos 2z\, dz$

26. $\int \sin 3z \sqrt{1-\cos 3z}\, dz$

27. $\int \frac{\sin z}{\cos^2 z}\, dz$

28. $\int \frac{1}{\sqrt{z}} \sec^2 \sqrt{z}\, dz$

29. $\int \frac{1}{z^2} \operatorname{cosec}^2 \frac{1}{z}\, dz$

30. $\int \frac{dx}{x-1}$

31. $\int \frac{2}{1-4x}\, dx$

32. $\int \frac{x}{x^2+4}\, dx$

33. $\int \frac{x^2}{1-x^3}\, dx$

34. $\int \frac{\cos x}{1-\sin x}\, dx$

35. $\int \frac{\sin x}{1-3\cos x}\, dx$

36. $\int \frac{1}{x(1+\ln x)}\, dx$

37. $\int \frac{1}{\sqrt{x}(1+\sqrt{x})}$

38. $\int \frac{x+2}{x^2+4x-1}\, dx$

39. $\int \frac{\ln z}{z}\, dz$

41. $\int \frac{1}{1+x^{1/3}}\, dx$

40. $\int \frac{\ln(\ln t)}{t \ln t}\, dt$

Solução

1. $\int (4x-3)^5 \, dx$

Seja $u = 4x-3 \rightarrow du = 4dx$

$\therefore$

$$\int (4x-3)^5 dx = \int u^5 \frac{du}{4} = \frac{1}{4}\int u^5 du$$

$$= \frac{1}{4}\frac{u^6}{6} = \frac{u^6}{24} = \frac{(4x-3)^6}{24}$$

$\therefore$

$$\int (4x-3)^5 \, dx = \frac{(4x-3)^6}{24} + C$$

2. $\int x\sqrt{x^2-1}\, dx$

Seja $u = x^2-1$; $du = 2x\,dx$

$$\frac{du}{2} = x\,dx$$

$\therefore$

$$\int x\sqrt{x^2-1}\, dx = \int \sqrt{u}\,\frac{du}{2} = \frac{1}{2}\int \sqrt{u}\, du$$

$$= \frac{1}{2}\frac{u^{3/2}}{3/2} = \frac{1}{3}u^{3/2}$$

$$= \frac{1}{3}(x^2-1)^{3/2}$$

$\therefore$

$$\int x\sqrt{x^2-1}\, dx = \frac{1}{3}(x^2-1)^{3/2} + C$$

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3. $\int x\sqrt{2-3x^2}\,dx$

$u = 2-3x^2$, $du = -6x\,dx$

$-\frac{1}{6}du = x\,dx$

$$\int x\sqrt{2-3x^2}\,dx = \int \sqrt{u}\,\frac{-1}{6}du = \frac{-1}{6}\int \sqrt{u}\,du$$

$$= -\frac{1}{6}\,\frac{u^{3/2}}{3/2} + C$$

$$\int x\sqrt{2-3x^2}\,dx = -\frac{1}{9}(2-3x^2)^{3/2} + C$$

4. $\int 3t(1-2t^2)^{10}\,dt$

$u = 1-2t^2$, $du = -4t\,dt$

$\frac{du}{-4} = t\,dt$

$$\int 3t(1-2t^2)^{10}\,dt = \int u^{10}\,\frac{3du}{-4}$$

$$= -\frac{3}{4}\int u^{10}\,du$$

$$= -\frac{3}{4}\,\frac{u^{11}}{11}$$

$$\int 3t(1-2t^2)^{10}\,dt = -\frac{3}{44}(1-2t^2)^{11} + C$$

5. $\displaystyle\int \frac{x^2\,dx}{(x^3+5)^4}$

$u = x^3+5 \quad , \quad du = 3x^2\,dx$

$\qquad\qquad\qquad\qquad \dfrac{du}{3} = x^2\,dx$

$$\int \frac{x^2dx}{(x^3+5)^4} = \int \frac{1}{u^4}\,\frac{du}{3} = \frac{1}{3}\int \frac{du}{u^4}$$

$$= \frac{1}{3}\,\frac{u^{-3}}{-3} = -\frac{1}{9}\,(x^3+5)^{-3}$$

$\therefore$

$$\int \frac{x^2dx}{(x^3+5)^4} = -\frac{1}{9\,(x^3+5)^3} + C$$

6. $\displaystyle\int \frac{t\,dt}{\sqrt{2t^2+1}}$

$u = 2t^2+1 \quad , \quad du = 4t\,dt$

$\qquad\qquad\qquad\qquad \dfrac{du}{4} = t\,dt$

$$\int \frac{t\,dt}{\sqrt{2t^2+1}} = \int \frac{1}{\sqrt{u}}\,\frac{du}{4} = \frac{1}{4}\int \frac{du}{\sqrt{u}} = \frac{1}{4}\,\frac{u^{1/2}}{1/2}$$

$$= \frac{1}{2}\sqrt{u} = \frac{1}{2}\sqrt{2t^2+1}$$

$\therefore$

$$\int \frac{t\,dt}{\sqrt{2t^2+1}} = \frac{1}{2}\sqrt{2t^2+1} + C$$

7. $\int x^2 \sqrt[3]{2-4x^3}\, dx$

$u = 2-4x^3$, $du = -12x^2\,dx$

$\frac{du}{-12} = x^2 dx$

$$\int x^2 \sqrt[3]{2-4x^3}\, dx = \int \sqrt[3]{u}\, \frac{du}{-12} = \frac{-1}{12}\int u^{1/3}\, du$$

$$= -\frac{1}{12}\,\frac{u^{4/3}}{4/3} = -\frac{1}{16}\,u^{4/3}$$

$$= -\frac{1}{16}\,(2-4x^3)^{4/3}$$

$\therefore$

$$\int x^2 \sqrt[3]{2-4x^3}\, dx = -\frac{1}{16}\,(2-4x^3)^{4/3} + C$$

8. $\int \frac{(x+1)}{(x^2+2x+5)^2}\, dx$

$u = x^2+2x+5$, $du = (2x+2)\,dx$

$= 2(x+1)\,dx$

$\frac{du}{2} = (x+1)\,dx$

$\therefore$

$$\int \frac{(x+1)\,dx}{(x^2+2x+5)^2} = \int \frac{du}{2u^2} = \frac{1}{2}\int \frac{du}{u^2} = \frac{1}{2}\,\frac{-1}{u}$$

$$= \frac{-1}{2}\,\frac{1}{x^2+2x+5}$$

$\therefore$

$$\int \frac{x+1}{(x^2+2x+5)^2}\, dx = -\frac{1}{2(x^2+2x+5)} + C$$

9. $\int \sin \frac{t}{3}\, dt$

$u = \frac{t}{3}$, $du = \frac{1}{3} dt$

$3du = dt$

$\therefore$

$\int \sin \frac{t}{3}\, dt = \int \sin u \; 3du = 3\int \sin u\, du =$

$= -3\cos u = -3\cos \frac{t}{3}$

$\therefore$

$\| \int \sin \frac{t}{3}\, dt = -3\cos \frac{t}{3} + C \|$

10. $\int \frac{\cos\sqrt{x}}{\sqrt{x}}\, dx$

$u = \sqrt{x}$, $du = \frac{1}{2\sqrt{x}}\, dx$

$2du = \frac{dx}{\sqrt{x}}$

$\therefore$

$\int \frac{\cos\sqrt{x}}{\sqrt{x}}\, dx = \int \cos u \; 2du = 2\int \cos u\, du$

$= 2\sin u = 2\sin\sqrt{x}$

$\therefore$

$\| \int \frac{\cos\sqrt{x}}{\sqrt{x}}\, dx = 2\sin\sqrt{x} + C \|$

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11. $\int \frac{x^3-1}{(x^4-4x)^{2/3}}\,dx$

$u = x^4-4x \quad , \quad du = (4x^3-4)\,dx$

$= 4(x^3-1)\,dx$

$\frac{du}{4} = (x^3-1)\,dx$

$\therefore$

$\int \frac{x^3-1}{(x^4-4x)^{2/3}}\,dx = \int \frac{1}{u^{2/3}}\,\frac{du}{4} = \frac{1}{4}\int u^{-2/3}\,du$

$= \frac{1}{4}\,\frac{u^{1/3}}{1/3} = \frac{3}{4}(x^4-4x)^{1/3}$

$\therefore$

$$\int \frac{x^3-1}{(x^4-4x)^{2/3}}\,dx = \frac{3}{4}(x^4-4x)^{1/3} + C$$

12. $\int \frac{1}{x^3}\left(1+\frac{1}{x^2}\right)^{5/3} dx$

$u = 1+\frac{1}{x^2} \quad , \quad du = \frac{-2}{x^3}\,dx$

$\frac{du}{-2} = \frac{1}{x^3}\,dx$

$\therefore$

$\int \frac{1}{x^3}\left(1+\frac{1}{x^2}\right)^{5/3} dx = \int u^{5/3}\,\frac{du}{-2} = -\frac{1}{2}\int u^{5/3}\,du$

$= -\frac{1}{2}\,\frac{u^{8/3}}{8/3} = \frac{-3}{16}\,u^{8/3}$

$\therefore$

$$\int \frac{1}{x^3}\left(1+\frac{1}{x^2}\right)^{5/3} dx = -\frac{3}{16}\left(1+\frac{1}{x^2}\right)^{8/3} + C$$

13. $\int (2-t^2)\sqrt[4]{6t-t^3}\, dt$

$u = 6t - t^3$, $du = (6-3t^2)\, dt$

$= 3(2-t^2)\, dt$

$\frac{du}{3} = (2-t^2)\, dt$

$\therefore$

$$\int (2-t^2)\sqrt[4]{6t-t^3}\, dt = \int \sqrt[4]{u}\, \frac{du}{3}$$

$$= \frac{1}{3}\int u^{1/4}\, du$$

$$= \frac{1}{3}\frac{u^{5/4}}{5/4} = \frac{4}{15} u^{5/4}$$

$\therefore$

$$\int (2-t^2)\sqrt[4]{6t-t^3}\, dt = \frac{4}{15}(6t-t^3)^{5/4} + C$$

14. $\int \frac{2-x^2}{(x^3-6x+1)^5}\, dx$

$u = x^3-6x+1$, $du = (3x^2-6)\, dx$

$= 3(x^2-2)\, dx$

$= -3(2-x^2)\, dx$

$\frac{du}{-3} = (2-x^2)\, dx$

$\therefore$

$$\int \frac{2-x^2}{(x^3-6x+1)^5}\, dx = \int \frac{1}{u^5}\frac{du}{-3} = -\frac{1}{3}\int \frac{du}{u^5}$$

$$= -\frac{1}{3}\frac{u^{-4}}{-4} = \frac{1}{12}\frac{1}{u^4}$$

$\therefore$

$$\boxed{\int \frac{2-x^2}{(x^3-6x+1)^5}\, dx = \frac{1}{12}\frac{1}{(x^3-6x+1)^4} + C}$$

15. $\int \left(1-\frac{1}{x^2}\right)\left(x+\frac{1}{x}\right)^{-3} dx$

$u = x+\frac{1}{x}$, $du = \left(1-\frac{1}{x^2}\right)dx$

$\therefore$

$$\int \left(1-\frac{1}{x^2}\right)\left(x+\frac{1}{x}\right)^{-3} dx = \int u^{-3}\,du = \frac{u^{-2}}{-2}$$

$$\therefore \qquad = -\frac{1}{2}\left(x+\frac{1}{x}\right)^{-2}$$

$$\int \left(1-\frac{1}{x^2}\right)\left(x+\frac{1}{x}\right)^{-3} dx = -\frac{1}{2}\left(x+\frac{1}{x}\right)^{-2} + C$$

16. $\int \frac{x}{\sqrt{x+3}}\,dx$

$u = x+3$, $\begin{cases} du = dx \\ x = u-3 \end{cases}$

$\therefore$

$$\int \frac{x\,dx}{\sqrt{x+3}} = \int \frac{u-3}{\sqrt{u}}\,du$$

$$= \int \left(\frac{u}{\sqrt{u}} - \frac{3}{\sqrt{u}}\right) du$$

$$= \int \left(u^{1/2} - 3u^{-1/2}\right) du$$

$$= \frac{u^{3/2}}{3/2} - 3\frac{u^{1/2}}{1/2} = \frac{2}{3}u^{3/2} - 6u^{1/2}$$

$\therefore$

$$\boxed{\int \frac{x\,dx}{\sqrt{x+3}} = \frac{2}{3}(x+3)^{3/2} - 6(x+3)^{1/2} + C}$$

17. $\int x\sqrt{x+2}\,dx$

$u = x+2$, $\begin{cases} du = dx \\ x = u-2 \end{cases}$

$$\int x\sqrt{x+2}\,dx = \int (u-2)\sqrt{u}\,du$$

$$= \int (u\sqrt{u} - 2\sqrt{u})\,du$$

$$= \int (u^{3/2} - 2u^{1/2})\,du$$

$$= \frac{u^{5/2}}{5/2} - 2\frac{u^{3/2}}{3/2}$$

$$= \frac{2}{5}u^{5/2} - \frac{4}{3}u^{3/2}$$

$\therefore$

$$\int x\sqrt{x+2}\,dx = \frac{2}{5}(x+2)^{5/2} - \frac{4}{3}(x+2)^{3/2} + C$$

18. $\int x^2\sqrt{x+4}\,dx$

$u = x+4$, $\begin{cases} du = dx \\ x = u-4 \end{cases}$

$$\int x^2\sqrt{x+4}\,dx = \int (u-4)^2\sqrt{u}\,du$$

$$= \int (u^2 - 8u + 16)\,u^{1/2}\,du$$

$$= \int \left(u^{5/2} - 8u^{3/2} + 16u^{1/2}\right)du$$

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$$= \frac{u^{7/2}}{7/2} - 8\frac{u^{5/2}}{5/2} + 16\frac{u^{3/2}}{3/2}$$

$$= \frac{2}{7}u^{7/2} - \frac{16}{5}u^{5/2} + \frac{32}{3}u^{3/2}$$

$$\therefore \boxed{\int x^2\sqrt{x+4}\,dx = \frac{2}{7}(x+4)^{7/2} - \frac{16}{5}(x+4)^{5/2} + \frac{32}{3}(x+4)^{3/2} + C}$$

19. $\displaystyle\int \frac{\sqrt{x}}{\sqrt{1+\sqrt{x}}}\,dx$

$u = 1+\sqrt{x}$, $\left\{ du = \frac{1}{2\sqrt{x}}\,dx \right.$

$2\sqrt{x}\,du = dx$

$2(u-1)\,du = dx$

$\left\{ \sqrt{x} = u-1 \right.$

$$\therefore \int \frac{\sqrt{x}}{\sqrt{1+\sqrt{x}}}\,dx = \int \frac{(u-1)\ 2(u-1)\,du}{\sqrt{u}}$$

$$= 2\int \frac{u^2-2u+1}{\sqrt{u}}\,du$$

$$= 2\int \left(u^{3/2} - 2u^{1/2} + u^{-1/2}\right)du$$

$$= 2\left(\frac{u^{5/2}}{5/2} - 2\frac{u^{3/2}}{3/2} + \frac{u^{1/2}}{1/2}\right)$$

$$= \frac{4}{5}u^{5/2} - \frac{8}{3}u^{3/2} + 4u^{1/2} \longrightarrow$$

$$\boxed{\int \frac{\sqrt{x}}{\sqrt{1+\sqrt{x}}}\,dx = \frac{4}{5}\left(1+\sqrt{x}\right)^{5/2} - \frac{8}{3}\left(1+\sqrt{x}\right)^{3/2} + 4\left(1+\sqrt{x}\right)^{1/2} + C}$$

20. $\displaystyle\int \frac{t^2}{\sqrt{t+2}}\,dt$

$$u = t+2 \quad , \quad \left\{\begin{array}{l} du = dt \\ t = u-2 \end{array}\right.$$

$\therefore$

$$\int \frac{t^2}{\sqrt{t+2}}\,dt = \int \frac{(u-2)^2}{\sqrt{u}}\,du$$

$$= \int \frac{u^2-4u+4}{\sqrt{u}}\,du$$

$$= \int \left(\frac{u^2}{\sqrt{u}} - 4\frac{u}{\sqrt{u}} + \frac{4}{\sqrt{u}}\right) du$$

$$= \int \left(u^{3/2} - 4u^{1/2} + 4u^{-1/2}\right) du$$

$$= \frac{u^{5/2}}{5/2} - 4\frac{u^{3/2}}{3/2} + 4\frac{u^{1/2}}{1/2}$$

$$= \frac{2}{5}u^{5/2} - \frac{8}{3}u^{3/2} + 8u^{1/2}$$

$\therefore$

$$\boxed{\int \frac{t^2}{\sqrt{t+2}}\,dt = \frac{2}{5}(t+2)^{5/2} - \frac{8}{3}(t+2)^{3/2} + 8(t+2)^{1/2} + C}$$

21. $\int \cos 7x \, dx$

$u = 7x \quad , \quad du = 7\,dx$

$\frac{du}{7} = dx$

$\therefore$

$$\int \cos 7x \, dx = \int \cos u \frac{du}{7} = \frac{1}{7}\int \cos u \, du$$

$$= \frac{1}{7} \sin u$$

$\therefore$

$$\boxed{\int \cos 7x \, dx = \frac{1}{7} \sin 7x + C}$$

22. $\int t^9 \sin(t^{10}) \, dt$

$u = t^{10} \quad , \quad du = 10\, t^9 \, dt$

$\frac{du}{10} = t^9 dt$

$\therefore$

$$\int \underline{t^9} \sin(t^{10}) \underline{dt} = \int \sin u \frac{du}{10} = \frac{1}{10} \int \sin u \, du$$

$$= \frac{1}{10}(-\cos u)$$

$$= \frac{-1}{10} \cos t^{10}$$

$\therefore$

$$\boxed{\int t^9 \sin t^{10} \, dt = \frac{-1}{10} \cos t^{10} + C}$$

23. $\int (\sin t)^6 \cos t \, dt$

$u = \sin t \quad , \quad du = \cos t \, dt$

$\therefore$

$$\int (\sin t)^6 \cos t \, dt = \int u^6 \, du = \frac{u^7}{7}$$

$\therefore$

$$\boxed{\int (\sin t)^6 \cos t \, dt = \frac{(\sin t)^7}{7} + C}$$

24. $\int \frac{\sin t}{(\cos t)^3} \, dt$

$u = \cos t \quad , \quad du = -\sin t \, dt$

$-du = \sin t \, dt$

$\therefore$

$$\int \frac{\sin t}{(\cos t)^3} \, dt = \int \frac{-du}{u^3} = -\int \frac{du}{u^3}$$

$$= -\frac{u^{-2}}{-2} = \frac{u^{-2}}{2}$$

$\therefore$

$$\boxed{\int \frac{\sin t}{(\cos t)^3} \, dt = \frac{1}{2(\cos t)^2} + C}$$

25. $\int \sqrt{\sin 2z}\ \cos 2z\ dz$

$u = \sin 2z \quad , \quad du = 2\cos 2z\ dz$

$\frac{du}{2} = \cos 2z\ dz$

$\therefore$

$$\int \sqrt{\sin 2z}\ \cos 2z\ dz = \int \sqrt{u}\ \frac{du}{2}$$

$$= \frac{1}{2}\int \sqrt{u}\ du = \frac{1}{2}\ \frac{u^{3/2}}{3/2}$$

$$= \frac{1}{3}u^{3/2}$$

$\therefore$

$$\boxed{\int \sqrt{\sin 2z}\ \cos 2z\ dz = \frac{1}{3}(\sin 2z)^{3/2} + C}$$

26. $\int \sin 3z \sqrt{1-\cos 3z}\ dz$

$u = 1-\cos 3z \quad , \quad du = 3\sin 3z\ dz$

$\frac{du}{3} = \sin 3z\ dz$

$\therefore$

$$\int \sin 3z \sqrt{1-\cos 3z}\ dz =$$

$$= \int \sqrt{u}\ \frac{du}{3} = \frac{1}{3}\int \sqrt{u}\ du = \frac{1}{3}\ \frac{u^{3/2}}{3/2}$$

$\therefore$

$$\boxed{\int \sin 3z \sqrt{1-\cos 3z}\ dz = \frac{2}{9}(1-\cos 3z)^{3/2} + C}$$

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27. $\int \frac{\sin z}{\cos^2 z}\, dz$

$u = \cos z \quad , \quad du = -\sin z\, dz$

$-du = \sin z\, dz$

$\therefore$

$$\int \frac{\sin z}{\cos^2 z}\, dz = \int \frac{-du}{u^2} = -\int \frac{du}{u^2} = -\frac{u^{-1}}{-1}$$

$$= \frac{1}{u}$$

$\therefore$

$$\boxed{\int \frac{\sin z}{\cos^2 z}\, dz = \frac{1}{\cos z} + C}$$

28. $\int \frac{1}{\sqrt{z}} \sec^2 \sqrt{z}\, dz$

$u = \sqrt{z} \quad , \quad du = \frac{1}{2\sqrt{z}}\, dz$

$2du = \frac{dz}{\sqrt{z}}$

$\therefore$

$$\int \frac{1}{\sqrt{z}} \sec^2 \sqrt{z}\, dz = \int \sec^2 u \; 2\, du$$

$$= 2\int \sec^2 u\, du$$

$$= 2\, tg\, u = 2\, tg \sqrt{z}$$

$\therefore$

$$\boxed{\int \frac{1}{\sqrt{z}} \sec^2 \sqrt{z}\, dz = 2\, tg \sqrt{z} + C}$$

29. $\int \frac{1}{z^2} \operatorname{cosec}^2 \frac{1}{z}\, dz$

$u = \frac{1}{z}$, $\begin{cases} du = -\frac{1}{z^2}\, dz \\ -du = \frac{1}{z^2}\, dz \end{cases}$

$\therefore$

$$\int \underbrace{\frac{1}{z^2}} \operatorname{cosec}^2 \frac{1}{z} \underbrace{dz} = \int -\operatorname{cosec}^2 u \, du$$

$$= -\int \operatorname{cosec}^2 u \, du$$

$$= \operatorname{cotg} u = \operatorname{cotg} \frac{1}{z}$$

$\therefore$

$$\boxed{\int \frac{1}{z^2} \operatorname{cosec}^2 \frac{1}{z}\, dz = \operatorname{cotg} \frac{1}{z} + c}$$

30. $\int \frac{1}{x-1}\, dx$

$u = x-1$, $du = dx$

$$\int \frac{1}{x-1}\, dx = \int \frac{1}{u}\, du = \ln u = \ln |x-1|$$

$\therefore$

$$\boxed{\int \frac{1}{x-1}\, dx = \ln |x-1| + c}$$

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31. $\int \frac{2}{1-4x}\,dx$

$u = 1-4x \quad , \quad du = -4\,dx$

$\frac{du}{-4} = dx$

$\therefore \int \frac{2}{1-4x}\,dx = \int \frac{2}{u}\,\frac{du}{-4} = -\frac{1}{2}\int \frac{du}{u}$

$= -\frac{1}{2}\ln|u|$

$= -\frac{1}{2}\ln|1-4x|$

$\therefore$

$$\boxed{\int \frac{2}{1-4x}\,dx = -\frac{1}{2}\ln|1-4x| + C}$$

32. $\int \frac{x}{x^2+4}\,dx$

$u = x^2+4 \quad , \quad du = 2x\,dx$

$\frac{du}{2} = x\,dx$

$\therefore$

$\int \frac{x}{x^2+4}\,dx = \int \frac{1}{u}\,\frac{du}{2} = \frac{1}{2}\int \frac{du}{u}$

$= \frac{1}{2}\ln|u|$

$= \frac{1}{2}\ln|x^2+4| + C$

$\therefore$

$$\boxed{\int \frac{x}{x^2+4}\,dx = \frac{1}{2}\ln|x^2+4| + C}$$

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33. $\int \frac{x^2}{1-x^3}\,dx$

$u = 1-x^3, \quad du = -3x^2\,dx$

$\frac{du}{-3} = x^2\,dx$

$\therefore$

$\int \frac{x^2}{1-x^3}\,dx = \int \frac{1}{u}\,\frac{du}{-3} = \frac{-1}{3}\int \frac{du}{u} = \frac{-1}{3}\ln|u|$

$= -\frac{1}{3}\ln|1-x^3|$

$\therefore$

$$\boxed{\int \frac{x^2}{1-x^3}\,dx = -\frac{1}{3}\ln|1-x^3| + C}$$

34. $\int \frac{\cos x}{1-\sin x}\,dx$

$u = 1-\sin x, \quad du = -\cos x\,dx$

$-du = \cos x\,dx$

$\therefore$

$\int \frac{\cos x}{1-\sin x}\,dx = \int \frac{-du}{u} = -\int \frac{du}{u} = -\ln|u|$

$= -\ln|1-\sin x|$

$\therefore$

$$\boxed{\int \frac{\cos x}{1-\sin x}\,dx = -\ln|1-\sin x| + C}$$

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35. $\int \frac{\sin x}{1-3\cos x}\,dx$

$u = 1-3\cos x$, $du = 3\sin x\,dx$

$\frac{du}{3} = \sin x\,dx$

$\therefore$

$$\int \frac{\sin x}{1-3\cos x}\,dx = \int \frac{1}{u}\frac{du}{3}$$

$$= \frac{1}{3}\int \frac{du}{u} = \frac{1}{3}\ln|u|$$

$$= \frac{1}{3}\ln|1-3\cos x|$$

$\therefore$

$$\boxed{\int \frac{\sin x}{1-3\cos x}\,dx = \frac{1}{3}\ln|1-3\cos x| + C}$$

36. $\int \frac{1}{x(1+\ln x)}\,dx$

$u = 1+\ln x$, $du = \frac{1}{x}\,dx$

$\therefore$

$$\int \frac{1}{x(1+\ln x)}\,dx = \int \frac{du}{u} = \ln|u|$$

$$= \ln|1+\ln x| + C$$

$\therefore$

$$\boxed{\int \frac{1}{x(1+\ln x)}\,dx = \ln|1+\ln x| + C}$$

37. $\displaystyle\int \frac{1}{\sqrt{x}\,(1+\sqrt{x})}\,dx$

$u = 1+\sqrt{x}$, $\begin{cases} du = \dfrac{1}{2\sqrt{x}}\,dx \\ 2\,du = \dfrac{dx}{\sqrt{x}} \end{cases}$

$$\int \frac{dx}{\sqrt{x}\,(1+\sqrt{x})} = \int \frac{2\,du}{u} = 2\int \frac{du}{u}$$

$$= 2\ln|u|$$

$$= 2\ln|1+\sqrt{x}|$$

$$\boxed{\int \frac{1}{\sqrt{x}\,(1+\sqrt{x})}\,dx = 2\ln(1+\sqrt{x}) + C}$$

38. $\displaystyle\int \frac{x+2}{x^2+4x-1}\,dx$

$u = x^2+4x-1$, $du = (2x+4)\,dx$

$\dfrac{du}{2} = (x+2)\,dx$

$$\int \frac{x+2}{x^2+4x-1}\,dx = \int \frac{1}{u}\,\frac{du}{2} = \frac{1}{2}\int \frac{du}{u}$$

$$= \frac{1}{2}\ln|u|$$

$$= \frac{1}{2}\ln|x^2+4x-1|$$

$$\boxed{\int \frac{x+2}{x^2+4x-1}\,dx = \frac{1}{2}\ln|x^2+4x-1| + C}$$

39. $\int \frac{\ln z}{z}\, dz$

$u = \ln z \quad , \quad du = \frac{1}{z}\, dz$

$\therefore$

$$\int \frac{\ln z}{z}\, dz = \int u\, du = \frac{u^2}{2} = \frac{(\ln z)^2}{2}$$

$\therefore$

$$\boxed{\int \frac{\ln z}{z}\, dz = \frac{(\ln z)^2}{2} + C}$$

40. $\int \frac{\ln(\ln t)}{t \ln t}\, dt$

$u = \ln(\ln t) \quad , \quad du = \frac{1}{\ln t}\frac{1}{t}\, dt$

$\therefore$

$$\int \frac{\ln(\ln t)}{t \ln t}\, dt = \int u\, du$$

$$= \frac{u^2}{2} = \frac{1}{2}(\ln(\ln t))^2$$

$\therefore$

$$\boxed{\int \frac{\ln(\ln t)}{t \ln t}\, dt = \frac{1}{2}(\ln(\ln t))^2 + C}$$

41. $\int \frac{1}{1+x^{1/3}} dx$

Seja $u = 1 + x^{1/3}$, $du = \frac{1}{3} x^{-2/3} dx$

$\Leftrightarrow$

$x^{1/3} = u - 1$

$3x^{2/3} du = dx$

$3\underbrace{(x^{1/3})}^2 du = dx$

$3(u-1)^2 du = dx$

$\therefore$

$$\int \frac{1}{1+x^{1/3}} dx = \int \frac{3(u-1)^2 du}{u}$$

$$= 3\int \frac{u^2 - 2u + 1}{u} du$$

$$= 3\int \left(u - 2 + \frac{1}{u}\right) du$$

$$= 3\frac{u^2}{2} - 6u + 3\ln|u|$$

$$= \frac{3}{2}(1+x^{1/3})^2 - 6(1+x^{1/3}) + 3\ln|1+x^{1/3}|$$

$\therefore$

$$\boxed{\int \frac{1}{1+x^{1/3}} dx = \frac{3}{2}(1+x^{1/3})^2 - 6(1+x^{1/3}) + 3\ln|1+x^{1/3}| + c}$$