

12.  $x, y \in (0, \frac{\pi}{2})$

$$\left. \begin{aligned} \sin x &= \frac{3}{5} & ; & \cos x = \sqrt{1 - \sin^2 x} = \frac{4}{5} \\ \cos y &= \frac{12}{13} & ; & \sin y = \sqrt{1 - \cos^2 y} = \frac{5}{13} \end{aligned} \right\}$$

a)  $\sin(x-y) =$

$$= \sin x \cos y - \underbrace{\sin y}_{\frac{5}{13}} \underbrace{\cos x}_{\frac{4}{5}}$$

$$= \frac{3}{5} \cdot \frac{12}{13} - \sqrt{1 - \cos^2 y} \sqrt{1 - \sin^2 x}$$

$$= \frac{36}{65} - \sqrt{1 - \frac{144}{169}} \sqrt{1 - \frac{9}{25}}$$

$$= \frac{36}{65} - \sqrt{\frac{25}{169}} \sqrt{\frac{16}{25}}$$

$$= \frac{36}{65} - \frac{5}{13} \cdot \frac{4}{5}$$

$$= \frac{36}{65} - \frac{20}{65} = \frac{16}{65}$$

b)  $\cos(x-y) = \cos x \cos y + \sin x \sin y$

$$= \sqrt{1 - \sin^2 x} \cdot \frac{12}{13} + \frac{3}{5} \sqrt{1 - \cos^2 y}$$

$$= \sqrt{1 - \frac{9}{25}} \cdot \frac{12}{13} + \frac{3}{5} \sqrt{1 - \frac{144}{169}}$$

$$= \frac{4}{5} \cdot \frac{12}{13} + \frac{3}{5} \cdot \frac{5}{13}$$

$$= \frac{48}{65} + \frac{15}{65}$$

$$= \frac{63}{65}$$

c)  $\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$

$$\tan x = \frac{\sin x}{\cos x} = \frac{3/5}{4/5} = \frac{3}{4}$$

$$\tan y = \frac{\sin y}{\cos y} = \frac{5/13}{12/13} = \frac{5}{12}$$

$$\tan(x+y) = \frac{\frac{3}{4} + \frac{5}{12}}{1 - \frac{3}{4} \cdot \frac{5}{12}}$$

$$= \frac{\frac{14}{12}}{1 - \frac{5}{16}} = \frac{\frac{14}{12}}{\frac{11}{16}}$$

$$= \frac{14}{12} \cdot \frac{16}{11}$$

$$= \frac{56}{33}$$

(5)