

29. $\cos \theta = -\frac{4}{5} ; \frac{\pi}{2} \leq \theta \leq \pi$

$$\sin \theta = \pm \sqrt{1 - \frac{16}{25}} = \pm \frac{3}{5}$$

$$\frac{\pi}{2} \leq \theta \leq \pi \Rightarrow \sin \theta > 0$$

Logo

$$\sin \theta = \frac{3}{5}$$

$$\begin{aligned} \sin 2\theta &= 2 \sin \theta \cos \theta \\ &= 2 \cdot \frac{3}{5} \left(-\frac{4}{5}\right) \\ &= -\frac{24}{25} \end{aligned}$$

$$\begin{aligned} \cos 2\theta &= \cos^2 \theta - \sin^2 \theta \\ &= \frac{16}{25} - \frac{9}{25} = \frac{7}{25} \end{aligned}$$

30. $\sin \theta = \frac{12}{13}, 0 \leq \theta \leq \frac{\pi}{2}$

$$\cos \theta = \pm \sqrt{1 - \frac{144}{169}} = \pm \sqrt{\frac{25}{169}} = \pm \frac{5}{13}$$

$$\cos \theta = \frac{5}{13} \text{ pois } 0 \leq \theta \leq \frac{\pi}{2}$$

Daí

$$\begin{aligned} \sin 2\theta &= 2 \sin \theta \cos \theta \\ &= 2 \cdot \frac{12}{13} \cdot \frac{5}{13} \\ &= \frac{120}{169} \end{aligned}$$

$$\begin{aligned} \cos 2\theta &= \cos^2 \theta - \sin^2 \theta \\ &= \frac{25}{169} - \frac{144}{169} = -\frac{119}{169} \end{aligned}$$

31. $\sin \theta = \frac{2}{3}, 0 \leq \theta \leq \frac{\pi}{2}$

$$\cos \theta = \sqrt{1 - \frac{4}{9}} = \frac{\sqrt{5}}{3}$$

$$\sin 4\theta = 2 \sin 2\theta \cos 2\theta$$

Logo

$$\begin{aligned} \sin 2\theta &= 2 \sin \theta \cos \theta \\ &= 2 \cdot \frac{2}{3} \cdot \frac{\sqrt{5}}{3} = \frac{4\sqrt{5}}{9} \end{aligned}$$

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta = \frac{5}{9} - \frac{4}{9} = \frac{1}{9}$$

Daí

$$\sin 4\theta = 2 \cdot \frac{4\sqrt{5}}{9} \cdot \frac{1}{9} = \frac{8\sqrt{5}}{81}$$