

36) e) Continuação

$$y = \cos^2 \frac{\pi}{16}$$

$$y^2 - y + \left(\frac{1}{8} - \frac{\sqrt{2}}{16}\right) = 0$$

$$y = \frac{1 \pm \sqrt{1 - 4\left(\frac{1}{8} - \frac{\sqrt{2}}{16}\right)}}{2}$$
$$= \frac{1 \pm \sqrt{1 - \frac{1}{2} + \frac{\sqrt{2}}{4}}}{2}$$

$$= \frac{1 \pm \sqrt{\frac{1}{2} + \frac{\sqrt{2}}{4}}}{2}$$

$$= \frac{1 \pm \frac{\sqrt{2 + \sqrt{2}}}{2}}{2}$$

$$y = \frac{2 \pm \sqrt{2 + \sqrt{2}}}{4}$$

$$\cos^2 \frac{\pi}{16} = \frac{2 \pm \sqrt{2 + \sqrt{2}}}{4}$$

$$\cos \frac{\pi}{16} = \sqrt{\frac{2 \pm \sqrt{2 + \sqrt{2}}}{4}}$$

$$\cos \frac{\pi}{16} = \frac{\sqrt{2 \pm \sqrt{2 + \sqrt{2}}}}{2}$$

Aqui temos que analisar

$$\cos \frac{\pi}{16} > \cos \frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

$\frac{\pi}{16} < 11.25^\circ$

$$\cos \frac{\pi}{16} > \cos \frac{\pi}{4}$$

logo

$$\cos \frac{\pi}{16} = \frac{\sqrt{2 + \sqrt{2 + \sqrt{2}}}}{2}$$

f) $\operatorname{tg} 33.75^\circ$

$$\operatorname{tg} 135^\circ = \operatorname{tg} 4 \times 33.75^\circ$$

Mas

$$\operatorname{tg} 4x = \operatorname{tg} 2 \cdot 2x$$

$$= \frac{2 \operatorname{tg} 2x}{1 - \operatorname{tg}^2 2x}$$

$$= \frac{2 \left(\frac{2 \operatorname{tg} x}{1 - \operatorname{tg}^2 x} \right)}{1 - \left(\frac{2 \operatorname{tg} x}{1 - \operatorname{tg}^2 x} \right)^2}$$

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